

4.5 day 2 - Integration using u-substitution

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Homework:

- Assignment # 107
- Quiz 4.5 Tuesday 5/8

Objective:

Find particular solutions and evaluate definite integrals using u-substitution

Do Now: Find the general solution:

$$\int \frac{-15x^2 - 2}{(5x^3 + 2x)^4} dx$$

$$u = 5x^3 + 2x$$

$$\frac{du}{dx} = 15x^2 + 2$$

$$dx = \frac{du}{15x^2 + 2}$$

$$\int \frac{-\cancel{(15x^2 + 2)}}{u^4} \cdot \frac{du}{\cancel{15x^2 + 2}}$$

$$= - \int u^{-4} du = \frac{5}{u^5} + C$$

$$= \frac{5}{(5x^3 + 2x)^5} + C$$

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HW Questions?

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Particular Solutions requiring U-substitution:

1) $\frac{dy}{dx} = x\sqrt{4-x^2}$ (2,2) is a point on the original function

$$\int dy = \int x\sqrt{u} \cdot \frac{du}{-2x}$$

$$u = 4 - x^2$$

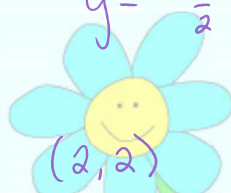
$$\frac{du}{dx} = -2x$$

$$dx = \frac{du}{-2x}$$

$$y = -\frac{1}{2} \int u^{1/2} du$$

$$y = -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C$$

$$y = -\frac{1}{3} (4-x^2) + 2$$



$$y = -\frac{1}{3} (4-x^2) + C$$

$$y = -\frac{1}{3} (4-2)^2 + C = 2$$

$$C = 2$$

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Particular Solutions requiring U-substitution:

You try these

2) $\frac{dy}{dx} = x^2(x^3-1)^2$ (1,4) is a point on the original function

$$4 = \frac{(1-1)^2}{9} + C \Rightarrow C = 4$$

$$\int dy = \int x^2 u^2 \cdot \frac{du}{3x^2}$$

$$y = \frac{u^3}{9} + C = \frac{(x^3-1)^2}{9} + C$$

$$y = \frac{(x^3-1)^2}{9} + 4$$

$$u = x^3 - 1$$

$$\frac{du}{dx} = 3x^2$$

$$dx = \frac{du}{3x^2}$$

3) $\frac{dy}{dx} = \frac{-48}{(3x+5)^3}$

(-1,3) is a point on the original function

$$u = 3x+5$$

$$\frac{du}{dx} = 3$$

$$dx = \frac{du}{3}$$

$$\int dy = \int \frac{-48 u^{-3} du}{3}$$

$$y = \frac{8}{u^2} + C = \frac{8}{(3x+5)^2} + C$$

$$y = \frac{8}{(3x+5)^2} + 1$$

$$3 = \frac{8}{(-3+5)^2} + C$$

$$3 = 2 + C \quad C = 1$$

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Let's try u -substitution with definite integrals:

$$4) \int_0^2 (12x^2 - 4x)(2x^3 - x^2 + 3)^2 dx$$

$$u = 2x^3 - x^2 + 3$$

$$\frac{du}{dx} = 6x^2 - 2x$$

$$dx = \frac{du}{6x^2 - 2x}$$

$$x=0, u=3$$

$$x=2, u=15$$

$$\int_3^{15} 2(6x^2 - 2x) u^2 \cdot \frac{du}{6x^2 - 2x}$$

$$= \frac{2}{3} u^3 \Big|_3^{15} = \frac{2}{3} (15^3 - 3^3) = \boxed{2232}$$

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Let's do that again:

$$5) \int_0^1 x \sqrt{2x^2 + 4} dx$$

$$u = 2x^2 + 4$$

$$\frac{du}{dx} = 4x$$

$$dx = \frac{du}{4x}$$

$$x=0, u=4$$

$$x=1, u=6$$

$$= \int_4^6 x u^{1/2} \cdot \frac{du}{4x} = \frac{1}{4} \cdot \frac{2}{3} u^{3/2} \Big|_4^6$$

$$\frac{1}{6} (\sqrt{6^3} - \sqrt{4^3}) = \frac{1}{6} (6\sqrt{6} - 8)$$

$$= \boxed{\sqrt{6} - \frac{4}{3}}$$



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Try this one:

$$6) \int_0^2 3x^2(x^3 - 2)^3 dx$$

$$u = x^3 - 2$$
$$\frac{du}{dx} = 3x^2$$

$$dx = \frac{du}{3x^2}$$

$$x=0, u=-2$$

$$x=2, u=6$$

$$\int_{-2}^6 3x^2 u^3 \cdot \frac{du}{3x^2}$$

$$= \left. \frac{u^4}{4} \right|_{-2}^6 = \frac{1}{4} (6^4 - (-2)^4)$$

$$= \boxed{320}$$



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