Do Now:
Evaluate the following integrals.

1. $\int x^{1/2}\left(x^{3/2}+2\right)^9 dx = \int x^{1/2} u^9 \frac{2du}{3x^{1/2}}$

$u = x^{3/2}+2$

$\frac{du}{dx} = \frac{3}{2}x^{1/2}$

$du = \frac{3}{2}x^{1/2}dx$

$\frac{2du}{3x^{1/2}} = dx$

$= \frac{2}{3}\int u^9 du$

$= \frac{2}{3}\cdot\frac{1}{10}u^{10} + C$

$\boxed{= \frac{1}{15}\left(x^{3/2}+2\right)^{10} + C}$

2. $\int_0^{\sqrt{\pi/2}} t\sin\left(\pi - t^2\right) dt = \int_{\pi}^{\frac{\pi}{2}} t\sin(u)\frac{du}{-2t}$

$u = \pi - t^2$

$\frac{du}{dt} = -2t$

$du = -2t\,dt$

$\frac{du}{-2t} = dt$

$= -\frac{1}{2}\int_{\pi}^{\frac{\pi}{2}}\sin(u)du$

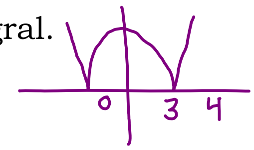
$= -\frac{1}{2}\left[-\cos(u)\right]_{\pi}^{\frac{\pi}{2}}$

$= \frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right) - \cos(\pi)\right]$

$= \frac{1}{2}\left(0-(-1)\right) = \boxed{\frac{1}{2}}$

Practice Problems:
Evaluate the indefinite integral.

1. $\int_0^4 \left|9-x^2\right| dx =$



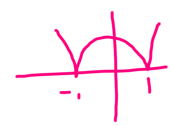
$\int_0^3 9-x^2 dx + \int_3^4 -9+x^2 dx$

$\left[9x - \frac{1}{3}x^3\right]_0^3 + \left[\frac{1}{3}x^3 - 9x\right]_3^4$

$9(3) - \frac{1}{3}(3)^3 - 0 + \frac{1}{3}(4)^3 - 9(4) - \frac{1}{3}(3)^3 + 9(3)$

$27 - 9 + \frac{64}{3} - 36 - 9 + 27 = 54 - 54 + \frac{64}{3} = \boxed{\frac{64}{3}}$

2. $\int_{-2}^{2} \left|1-x^2\right| dx =$



$2\int_0^1 1-x^2 dx + 2\int_1^2 x^2-1\,dx =$

$2\left[x - \frac{1}{3}x^3\right]_0^1 + 2\left[\frac{1}{3}x^3 - x\right]_1^2 =$

$2\left(1-\frac{1}{3}\right) + 2\left(\frac{8}{3} - 2 - \frac{1}{3} + 1\right) =$

$2\left(\frac{2}{3}\right) + 2\left(\frac{7}{3} - 1\right) = \frac{4}{3} + \frac{14}{3} - \frac{6}{3} = \frac{12}{3} \boxed{= 4}$

3. $\int \frac{x-5}{\sqrt{x-6}} dx = \int \frac{u+6-5}{u^{1/2}} du$

$u = x-6 \rightarrow u+6 = x$

$\frac{du}{dx} = 1$

$du = dx$

$= \int u^{1/2} + u^{-1/2} du$

$= \frac{2}{3}u^{3/2} + 2u^{1/2} + C$

$\boxed{= \frac{2}{3}(x-6)^{3/2} + 2(x-6)^{1/2} + C}$

4. $\int_0^{\pi/6} (1+\sin x\cos x)\, dx = \int_0^{\frac{\pi}{6}} 1\,dx + \int_0^{\frac{\pi}{6}} \sin x\cos x\, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\, dx$

$\frac{du}{\cos x} = dx$

$= x\Big]_0^{\frac{\pi}{6}} + \int_0^{\frac{1}{2}} u\cos(x)\frac{du}{\cos x}$

$= \frac{\pi}{6} + \int_0^{1/2} u\, du$

$= \frac{\pi}{6} + \left[\frac{1}{2}u^2\right]_0^{1/2} = \frac{\pi}{6} + \frac{1}{2}\left(\frac{1}{2}\right)^2$

$\boxed{= \frac{\pi}{6} + \frac{1}{8}}$

5. $\int_0^{\pi/3} \cos x\sqrt{1-\cos^2 x}\; dx = \int_0^{\frac{\pi}{3}} \cos x\sqrt{\sin^2 x}\; dx$

$= \int_0^{\frac{\pi}{3}} \cos x\sin x\, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\frac{du}{\cos x} = dx$

$= \int_0^{\frac{\sqrt{3}}{2}} \cancel{\cos x}\, u \frac{du}{\cancel{\cos x}}$

$= \frac{1}{2}u^2\Big]_0^{\sqrt{3}/2} = \frac{1}{2}\left(\frac{3}{4}\right) \boxed{= \frac{3}{8}}$

6. $\int_0^1 x\sqrt{ax^2+b}\; dx = \int_b^{a+b} x\sqrt{u}\frac{du}{2ax} = \frac{1}{2a}\int_b^{a+b} u^{1/2} du$

$u = ax^2+b$

$\frac{du}{dx} = 2ax$

$du = 2ax\,dx$

$dx = \frac{du}{2ax}$

$= \frac{1}{2a}\left[\frac{2}{3}u^{3/2}\right]_b^{a+b}$

$= \frac{1}{3a}\left[(a+b)^{3/2} - b^{3/2}\right]$

OR

$= \frac{1}{3a}\left[(a+b)\sqrt{a+b} - b\sqrt{b}\right]$

7. $\int_{-2}^{2} x^2(x^3-1)\,dx = \int_{-9}^{7} \cancel{x^2}\, u \,\frac{du}{3\cancel{x^2}}$

$u = x^3-1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

$= \frac{1}{3}\int_{-9}^{7} u\,du$

$= \frac{1}{3}\left[\frac{1}{2}u^2\right]_{-9}^{7}$

$= \frac{1}{6}\left[7^2 - (-9)^2\right]$

$= \frac{1}{6}\left[49-81\right]$

$= \frac{1}{6}\left[32\right] = \frac{-32}{6} = \boxed{\frac{-16}{3}}$

8. $\int \sin^3 x \cos x\,dx = \int u^3 \cancel{\cos x}\,\frac{du}{\cancel{\cos x}}$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$\frac{du}{\cos x} = dx$

$= \int u^3\,du$

$= \frac{1}{4}u^4 + C$

$\boxed{= \frac{1}{4}\sin^4 x + C}$

9. $\int \frac{2e^x + 24x^5}{\left(e^x + 2x^6\right)^5}dx = \int \frac{2\cancel{(e^x+12x^5)}}{u^5}\,\frac{du}{\cancel{e^x+12x^5}}$

$u = e^x + 2x^6$

$\frac{du}{dx} = e^x + 12x^5$

$dx = \frac{du}{e^x+12x^5}$

$= 2\int u^{-5}\,du$

$= 2\left(-\frac{1}{4}\right)u^{-4} + C$

$= -\frac{1}{2u^4} + C$

$\boxed{= \frac{-1}{2(e^x+2x^6)^4} + C}$

10. $\int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2+4}}dx = \int_4^9 \frac{\cancel{x}}{u^{1/2}}\,\frac{du}{2\cancel{x}}$

$u = x^2+4$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \frac{1}{2}\int_4^9 u^{-1/2}\,du$

$= \frac{1}{2}\left[2u^{1/2}\right]_4^9$

$= \left[u^{1/2}\right]_4^9$

$= \sqrt{9} - \sqrt{4} = 3-2 \boxed{= 1}$

11. $\int x^2\sqrt{1-4x^3}\,dx = \int \cancel{x^2}\, u^{1/2}\,\frac{du}{-12\cancel{x^2}}$

$u = 1-4x^3$

$\frac{du}{dx} = -12x^2$

$dx = \frac{du}{-12x^2}$

$= -\frac{1}{12}\int u^{1/2}\,du$

$= -\frac{1}{12}\left[\frac{2}{3}u^{3/2}\right] + C$

$\boxed{= -\frac{1}{18}\left(1-4x^3\right)^{3/2} + C}$

12. $\int_{\pi^2/4}^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int_{\pi/2}^{\pi} \frac{\sin(u)}{\sqrt{x}}\cdot 2\sqrt{x}\,du$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$= 2\int_{\pi/2}^{\pi} \sin(u)\,du$

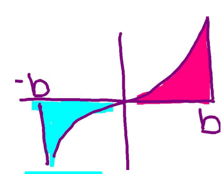
$= -2\cos(u)\Big]_{\pi/2}^{\pi}$

$= -2\cos(\pi) + 2\cos\left(\frac{\pi}{2}\right)$

$= -2(-1)$

$\boxed{= 2}$

13. If $\int_0^2 f(x)dx = \frac{11}{3}$, $\int_0^6 f(x)dx = 15$ and $f(x)$ is an odd function, find the following:



a. $\int_{-2}^{0} f(x)dx = \boxed{\frac{-11}{3}}$ b. $\int_{-2}^{2} f(x)dx \boxed{= 0}$ c. $\int_0^2 -f(x)dx = \boxed{\frac{-11}{3}}$ d. $\int_{-2}^{0} 3f(x)dx =$ e. $\int_0^2 f(3x)dx \boxed{= 5}$

$3\left(\frac{-11}{3}\right) = \boxed{-11}$

$u = 3x$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$\frac{1}{3}\int_0^6 f(u)\,du = \frac{15}{3} = 5$