

4.5 day 3 - Trig Integration using u-substitution

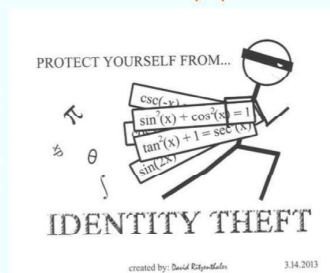
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Homework:

- Assignment # 108
- Quiz 4.5 Tuesday 5/8

Objective:

Integrate trig functions using u-substitution

Do Now: Evaluate

$$\int 42x(x^2 + 4)^{20} dx$$

$$u = x^2 + 4$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$\int 4\cancel{2x}^{21} u^{20} \cdot \frac{du}{\cancel{2x}} = u^{21} + C$$

$$= (x^2 + 4)^{21} + C$$

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HW Questions?

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Integrate Trig functions using u-substitution:

$$1) \int 5 \cos(5x) dx = \sin u + C$$

$$u = 5x$$

$$\frac{du}{dx} = 5$$

$$dx = \frac{du}{5}$$

$$\int \cancel{5} \cos u \cdot \frac{du}{\cancel{5}} = \boxed{\sin(5x) + C}$$

$$2) \int \sin^2(3x) \cos(3x) dx$$

$$u = \sin(3x)$$

$$\frac{du}{dx} = 3 \cos(3x)$$

$$dx = \frac{du}{3 \cos(3x)}$$

$$= \int u^2 \cancel{\cos(3x)} \cdot \frac{du}{\cancel{3 \cos(3x)}} = \frac{u^3}{3} + C = \boxed{\frac{\sin^3(3x)}{3} + C}$$

$$\frac{d}{dx} \left(\frac{\sin^3(3x)}{3} + C \right) = \frac{3 \sin^2(3x) \cos(3x) \cdot 3}{9}$$

$$= \sin^2(3x) \cos(3x) \checkmark$$

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Integrate Trig functions using u-substitution:

$$3) \int \cos^2(x) \sin(x) dx$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{du}{-\sin x}$$

$$= \int u^2 \cancel{\sin x} \cdot \frac{du}{-\cancel{\sin x}} = - \int u^2 du = -\frac{u^3}{3} + C$$

$$= \boxed{-\frac{\cos^3 x}{3} + C}$$

$$\frac{d}{dx} \left(-\frac{\cos^3 x}{3} + C \right)$$

$$= \frac{-3 \cos^2 x (-\sin x)}{3}$$

$$= \cos^2 x \sin x$$

$$4) \int x \cos(x^2 + 1) dx$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$\int \cancel{x} \cos(u) \frac{du}{\cancel{2x}} = \frac{1}{2} \sin u + C$$

$$= \boxed{\frac{1}{2} \sin(x^2 + 1) + C}$$

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This is an interesting problem:

What should we choose as our u value?

$$\begin{aligned}
 & 5) \int \sin(2x) \cos(2x) dx \\
 & u = \sin(2x) \\
 & \frac{du}{dx} = 2 \cos(2x) \\
 & dx = \frac{du}{2 \cos(2x)} \\
 & = \int u \cos(2x) \cdot \frac{du}{2 \cos(2x)} \\
 & = \frac{u^2}{4} du \\
 & = \frac{\sin^2(2x)}{4} + C
 \end{aligned}$$

$$\begin{aligned}
 & u = \cos(2x) \\
 & \frac{du}{dx} = -2 \sin(2x) \\
 & dx = \frac{du}{-2 \sin(2x)}
 \end{aligned}$$

$$\begin{aligned}
 & \int \sin(2x) u \cdot \frac{du}{-2 \sin(2x)} \\
 & = -\frac{u^2}{4} + C \\
 & = -\frac{\cos^2(2x)}{4} + C
 \end{aligned}$$



Answers on next page

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This is an interesting problem:

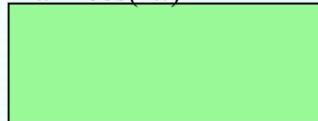
What should we choose as our u value?

$$5) \int \sin(2x) \cos(2x) dx$$

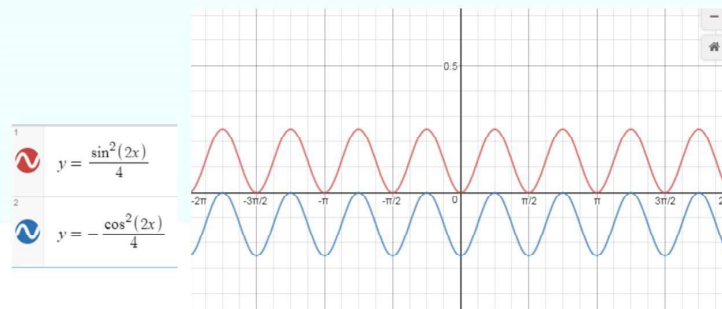
$$u = \sin(2x)$$



$$u = \cos(2x)$$



Why did we get different answers? Is this a problem? Can we even trust math anymore?



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Integrate Trig functions using u-substitution:

$$6) \int \frac{\sec^2(4x)}{\tan^3(4x)} dx = \int \frac{\cancel{\sec^2(4x)}}{u^3} \cdot \frac{du}{4\cancel{\sec^2(4x)}}$$

$$u = \tan(4x)$$

$$\frac{du}{dx} = 4 \sec^2(4x)$$

$$dx = \frac{du}{4 \sec^2(4x)}$$

$$= \frac{1}{4} \int u^{-3} du$$

$$= \frac{1}{4} \cdot \frac{u^{-2}}{-2} + C = -\frac{1}{8u^2} + C$$

$$= \frac{-1}{8 \tan^2(4x)} + C$$



$$8 \tan^2(4x) + C = -1$$

$$= \frac{\sec^2(4x)}{\tan^3(4x)} \cdot 4$$

$$= \frac{\sec^2(4x)}{\tan^3(4x)} \checkmark$$

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