Name___________________________________ **Date**_______________

Calc I H – 4.5 day 4 - U-Substitution with Trig Practice **Period** ____________

Evaluate each integral:

1) $\int_0^{\frac{\pi}{2}} \cos(3x)dx$

$u = 3x$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$x = 0, u = 0$

$x = \pi/2, u = \frac{3\pi}{2}$

$$\int_0^{3\pi/2} \cos u \cdot \frac{du}{3}$$

$$= \frac{1}{3} \sin u \Big|_0^{3\pi/2}$$

$$= \frac{1}{3}\left(\sin\frac{3\pi}{2} - \sin 0\right) = \frac{1}{3}(-1-0) = \boxed{-\frac{1}{3}}$$

2) $\int 3x\sin(3x^2+1)dx$

$u = 3x^2+1$

$\frac{du}{dx} = 6x$

$dx = \frac{du}{6x}$

$$\int \cancel{3x}^{\,1} \sin u \cdot \frac{du}{\cancel{6x}_{\,2}}$$

$$= -\frac{1}{2}\cos u + C$$

$$= \boxed{-\frac{1}{2}\cos(3x^2+1) + C}$$

3) $\int \sin^{10}(x)\cos(x)dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$$\int u^{10}\cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}}$$

$$= \frac{u^{11}}{11} + C$$

$$= \boxed{\frac{\sin^{11}(x)}{11} + C}$$

4) $\int 5\csc^2(x)\sqrt{\cot(x)}dx$

$u = \cot x$

$\frac{du}{dx} = -\csc^2 x$

$dx = \frac{du}{-\csc^2 x}$

$$5\int \cancel{\csc^2 x}\, u^{1/2} \cdot \frac{du}{-\cancel{\csc^2 x}}$$

$$= -5 \cdot \frac{2}{3} u^{3/2} + C$$

$$= -\frac{10}{3}\sqrt{\cot^3 x} +$$

5) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin(2x)\cos^3(2x)dx$

$u = \cos(2x)$

$\frac{du}{dx} = -2\sin(2x)$

$dx = \frac{du}{-2\sin(2x)}$

$x = \pi/4, u = 0$

$x = \pi/2, u = -1$

$$= \int_0^{-1} \cancel{\sin(2x)}\, u^3 \cdot \frac{du}{-2\cancel{\sin(2x)}}$$

$$= -\frac{1}{2}\int_0^{-1} u^3 du = \frac{1}{2}\int_{-1}^{0} u^3 du$$

$$= \frac{1}{2}\left[\frac{u^4}{4}\right]_{-1}^{0}$$

$$= \frac{1}{8}\left[0 - (-1)^4\right] = \boxed{-\frac{1}{8}}$$

6) $\int_0^{\frac{\pi}{12}} \sec(4x)\tan(4x)dx$

$u = 4x$

$\frac{du}{dx} = 4$

$dx = \frac{du}{4}$

$x = 0, u = 0$

$x = \pi/12, u = \pi/3$

$$\int_0^{\pi/3} \sec u \tan u \cdot \frac{du}{4}$$

$$= \frac{1}{4}\sec u \Big|_0^{\pi/3}$$

$$= \frac{1}{4}\left(\sec \pi/3 - \sec 0\right)$$

$$= \frac{1}{4}(2-1) = \boxed{\frac{1}{4}}$$

Solve the differential equation:

7) $\frac{dy}{dx}=\sin 4x\sqrt{\cos 4x+1}$

$\int dy=\int \sin 4x\,(\cos 4x+1)^{1/2}\,dx$

$u=\cos 4x+1$

$\frac{du}{dx}=-4\sin 4x$

$dx=\frac{du}{-4\sin 4x}$

$y=\int \cancel{\sin 4x}\cdot u^{1/2}\cdot\frac{du}{-4\cancel{\sin 4x}}$

$y=-\frac{1}{4}\cdot\frac{2}{3}u^{3/2}+C = \boxed{-\frac{1}{6}\sqrt{(\cos 4x+1)^3}+C}$

8) Find the particular solution that passes through the point $\left(\frac{\pi}{2},-1\right)$: $\frac{dy}{dx}=\sin\left(\frac{\pi}{2}-3x\right)$

$\int dy=\int \sin\left(\frac{\pi}{2}-3x\right)dx$

$u=\frac{\pi}{2}-3x$

$\frac{du}{dx}=-3 \Rightarrow dx=\frac{du}{-3}$

$y=\int \sin u\cdot\frac{du}{-3}$

$y=+\frac{1}{3}\cos u+C=\frac{1}{3}\cos\left(\frac{\pi}{2}-3x\right)+C$

$-1=\frac{1}{3}\cos\left(\frac{\pi}{2}-\frac{3\pi}{2}\right)+C$

$-1=\frac{1}{3}\cos(-\pi)+C$

$-1=-\frac{1}{3}+C$

$C=-2/3$

$\boxed{y=\frac{1}{3}\cos\left(\pi/2-3x\right)-2/3}$

9) $\int f'(\theta)\,d\theta=\sec\theta\tan\theta(\sec\theta-1)\,d\theta$

$u=\sec\theta-1$

$\frac{du}{d\theta}=\sec\theta\tan\theta \Rightarrow d\theta=\frac{du}{\sec\theta\tan\theta}$

$f(\theta)=\int \cancel{\sec\theta\tan\theta}\,(u)\cdot\frac{du}{\cancel{\sec\theta\tan\theta}}$

$f(\theta)=\frac{u^2}{2}+C$

$\boxed{f(\theta)=\frac{(\sec\theta-1)^2}{2}+C}$

10) Find the particular solution ~~that passes through the point $(0,\pi)$~~ : $\int f'(\theta)\,d\theta=\theta\sin\left(\theta^2\right)d\theta$

given $f(\pi)=0$

$u=\theta^2$

$\frac{du}{d\theta}=2\theta \Rightarrow d\theta=\frac{du}{2\theta}$

$f(\theta)=\int \cancel{\theta}\sin u\cdot\frac{du}{2\cancel{\theta}}$

$f(\theta)=-\frac{1}{2}\cos u+C$

$f(\theta)=-\frac{1}{2}\cos(\theta^2)+C$

$\pi=-\frac{1}{2}\cos(0)+C$

$C=\pi+\frac{1}{2}$

$\boxed{f(\theta)=-\frac{1}{2}\cos(\theta^2)+\pi+\frac{1}{2}}$