Name Answer Key Date________________

Calc I H – 4.5 day 5 - U-Substitution Focused Practice Period ____________

U-Substitution with Indefinite Integrals – Evaluate each:

1. $\int \frac{4x^2+5}{(4x^3+15x)^3}dx$

$u = 4x^3+15x$

$\frac{du}{dx} = 12x^2+15$

$dx = \frac{du}{3(4x^2+5)}$

$\int \frac{\cancel{4x^2+5}}{u^3}\cdot\frac{du}{3(\cancel{4x^2+5})} = \frac{1}{3}\int u^{-3}du$

$= \frac{1}{3}\cdot\frac{u^{-2}}{-2}+C = -\frac{1}{6u^2}+C$

$= \boxed{-\frac{1}{6(4x^3+15x)^2}+C}$

2. $\int \frac{9}{2}x^2\sqrt{3x^3+4}\,dx$

$u = 3x^3+4$

$\frac{du}{dx} = 9x^2$

$dx = \frac{du}{9x^2}$

$\int \frac{\cancel{9}}{2}\cancel{x^2}\,u^{1/2}\cdot\frac{du}{\cancel{9x^2}}$

$= \frac{1}{2}\int u^{1/2}du$

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$

$= \boxed{\frac{1}{3}(3x^3+4)^{3/2}+C}$

3. $\int\left(1+\frac{1}{x}\right)^3\frac{1}{x^2}dx$

$u = 1+\frac{1}{x}$

$\frac{du}{dx} = -\frac{1}{x^2}$

$dx = -x^2du$

$\int u^3\cdot\frac{1}{\cancel{x^2}}\cdot -\cancel{x^2}du$

$= -\int u^3du = \frac{-u^4}{4}+C$

$= \boxed{-\frac{1}{4}\left(1+\frac{1}{x}\right)^4+C}$

U-Substitution with Definite Integrals – Evaluate each:

4. $\int_{-1}^{3}(5x+4)^5dx$

$u = 5x+4$

$\frac{du}{dx} = 5 \rightarrow dx = \frac{du}{5}$

$x=-1,\ u=-1$

$x=3,\ u=19$

$\frac{1}{5}\int_{-1}^{19}u^5du = \frac{1}{5}\cdot\frac{1}{6}u^6\Big|_{-1}^{19}$

$= \frac{1}{30}\left(19^6-(-1)^6\right)$

$= \boxed{1,568,196}$

5. $\int_0^1\frac{x}{\sqrt{x^2+1}}dx$

$u = x^2+1$

$\frac{du}{dx} = 2x \rightarrow dx = \frac{du}{2x}$

$x=0,\ u=1$

$x=1,\ u=2$

$\int_1^2\frac{\cancel{x}}{u^{1/2}}\cdot\frac{du}{2\cancel{x}} = \frac{1}{2}\int_1^2u^{-1/2}du$

$= \frac{1}{2}\cdot 2u^{1/2}\Big|_1^2$

$= \boxed{\sqrt{2}-1 \approx .414}$

6. $\int_1^3(3x^2+1)\sqrt{x^3+x}\,dx$

$u = x^3+x$

$\frac{du}{dx} = 3x^2+1 \rightarrow dx = \frac{du}{3x^2+1}$

$x=1,\ u=2$

$x=3,\ u=30$

$\int_2^{30}(\cancel{3x^2+1})\sqrt{u}\cdot\frac{du}{\cancel{3x^2+1}}$

$\int_2^{30}u^{1/2}du = \frac{2}{3}u^{3/2}\Big|_2^{30}$

$= \frac{2}{3}\left(30^{3/2}-2^{3/2}\right)$

$= \boxed{107.659}$

U-Substitution with Trig – Evaluate each:

7. $\int \cos(3x+4)dx$

$u = 3x+4$

$\frac{du}{dx} = 3 \rightarrow dx = \frac{du}{3}$

$\int \cos(u) \cdot \frac{du}{3}$

$= \frac{1}{3}\sin u + C$

$= \boxed{\frac{1}{3}\sin(3x+4) + C}$

8. $\int 10x\sin(5x^2)\cos(5x^2)dx$

$u = \cos(5x^2)$ OR $u = \sin(5x^2)$

$\frac{du}{dx} = -10x\sin(5x^2)$ $\quad$ $\frac{du}{dx} = 10x\cos(5x^2)$

$dx = \frac{du}{-10x\sin(5x^2)}$ $\quad$ $dx = \frac{du}{10x\cos(5x^2)}$

$\int \cancel{10x\sin(5x^2)} \cdot u \cdot \frac{du}{-\cancel{10x\sin(5x^2)}}$

$= \int -u\,du = \frac{-u^2}{2} + C$

$= \boxed{-\frac{\cos^2(5x^2)}{2} + C = \frac{\sin^2(5x^2)}{2} + C}$

9. $\int 3x^3\sec(x^4)\tan(x^4)dx$

$u = x^4$

$\frac{du}{dx} = 4x^3 \rightarrow dx = \frac{du}{4x^3}$

$\int 3\cancel{x^3}\sec(u)\tan(u) \cdot \frac{du}{4\cancel{x^3}}$

$= \frac{3}{4}\int \sec u \tan u\, du$

$= \frac{3}{4}\sec u + C$

$= \boxed{\frac{3}{4}\sec(x^4) + C}$

10. $\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{\sec^2(2x)}{\tan^3(2x)}dx$

$u = \tan(2x)$

$\frac{du}{dx} = 2\sec^2(2x) \rightarrow dx = \frac{du}{2\sec^2(2x)}$

$x = \pi/8$, $u = \tan(\pi/4) = 1$

$x = \pi/12$, $u = \tan(\pi/6) = \frac{1}{\sqrt{3}}$

$\int_{1/\sqrt{3}}^{1} \frac{\cancel{\sec^2(2x)}}{u^3} \cdot \frac{du}{2\cancel{\sec^2(2x)}} = \frac{1}{2}\int_{1/\sqrt{3}}^{1} u^{-3}du$

$= \frac{1}{2} \cdot \frac{u^{-2}}{-2}\Big|_{1/\sqrt{3}}^{1}$

$= -\frac{1}{4}\left(1^2 - (\sqrt{3})^2\right)$

$= -\frac{1}{4}(-2) = \boxed{\frac{1}{2}}$

11. $\int_{\frac{2\pi}{15}}^{\frac{\pi}{20}} \csc^2(5x)dx$

$u = 5x$

$\frac{du}{dx} = 5 \rightarrow dx = \frac{du}{5}$

$x = \frac{2\pi}{15}$, $u = \frac{2\pi}{3}$

$x = \frac{\pi}{20}$, $u = \frac{\pi}{4}$

$\int_{2\pi/3}^{\pi/4} \csc^2 u \cdot \frac{du}{5}$

$= \frac{1}{5}\left(-\cot u\right)_{2\pi/3}^{\pi/4}$

$= -\frac{1}{5}\left(\cot(\pi/4) - \cot(2\pi/3)\right)$

$= -\frac{1}{5}\left(1 - \left(\frac{1}{-\sqrt{3}}\right)\right) = -\frac{1 + \frac{1}{\sqrt{3}}}{5}$

$\boxed{\approx -.316}$