Name_____ **Date**_____

Calc I H – 5.1-5.4 Derivative Day 2 and 3 Practice **Period**_____

Find the derivative of each:

1) $y = \ln(5x^3 + 2x^2 - 1)$

$y' = \frac{1}{5x^3+2x^2-1} \cdot 15x^2 + 4x = \boxed{\frac{15x^2+4x}{5x^3+2x^2-1}}$

2) $y = e^{3x^2-5}$

$y' = e^{3x^2-5} \cdot 6x = \boxed{6xe^{3x^2-5}}$

3) $y = \ln\sqrt{x} = \ln x^{\frac{1}{2}} = \frac{1}{2}\ln x$

$\boxed{y' = \frac{1}{2x}}$

4) $y = 6x^2e^{2x}$

$y' = 12xe^{2x} + 6x^2e^{2x} \cdot 2 = 12xe^{2x} + 12x^2e^{2x}$

$\boxed{y' = 12xe^{2x}(1+x)}$

5) $f(x) = -5x^2e^{-3x^2}$

$f'(x) = -10xe^{-3x^2} + (-5x^2)(e^{-3x^2} \cdot -6x)$

$f'(x) = -10xe^{-3x^2} + 30x^3e^{-3x^2}$

$\boxed{f'(x) = -10xe^{-3x^2}(1-3x^2)}$

6) $g(x) = (3\sin x)(e^{2x})$

$g'(x) = 3\cos x e^{2x} + 3\sin x e^{2x}(2)$

$g'(x) = 3\cos x e^{2x} + 6\sin x e^{2x}$

$\boxed{g'(x) = 3e^{2x}(\cos x + 2\sin x)}$

7) $y = \ln(x^4 + 3x^2 + 1)$

$y' = \frac{1}{x^4+3x^2+1} \cdot 4x^3 + 6x$

$\boxed{y' = \frac{4x^3+6x}{x^4+3x^2+1} \text{ OR } \frac{2x(2x^2+3)}{x^4+3x^2+1}}$

8) $f(x) = \ln\frac{x(x^3+2)^2}{\sqrt{x+1}} = \ln x + 2\ln(x^3+2) - \frac{1}{2}\ln(x+1)$

$f'(x) = \frac{1}{x} + \frac{2(3x^2)}{x^3+2} - \frac{1}{2(x+1)}$

$\boxed{f'(x) = \frac{1}{x} + \frac{6x^2}{x^3+2} - \frac{1}{2(x+1)}}$

9) $y = \ln e^{x^4}$

$y = x^4$

$\boxed{y' = 4x^3}$

10) $f(x) = \ln\frac{2xe^{3x}}{\sqrt{3x-2}}$ Note: $\ln e^{3x} = 3x$

$f(x) = \ln 2 + \ln x + 3x - \frac{1}{2}\ln(3x-2)$

$f'(x) = \frac{1}{x} + 3 - \frac{1(3)}{2(3x-2)}$

$\boxed{f'(x) = \frac{1}{x} + 3 - \frac{3}{2(3x-2)}}$

11) $f(x) = \frac{\ln x^2}{x^2} = \frac{2\ln x}{x^2}$ Quotient Rule

$f'(x) = \frac{\frac{2}{x}(x^2) - 2x(2\ln x)}{(x^2)^2}$

$f'(x) = \frac{2x - 4x\ln x}{x^4} = \frac{2x(1-2\ln x)}{x^4}$

$\boxed{f'(x) = \frac{2(1-2\ln x)}{x^3} = \frac{2(1-\ln x^2)}{x^3}}$

12) $y = \ln(x^3(2x+1)^4)$

$y = 3\ln x + 4\ln(2x+1)$

$y' = \frac{3}{x} + \frac{4(2)}{2x+1}$

$\boxed{y' = \frac{3}{x} + \frac{8}{2x+1}}$

13) $y = 7xe^{-4x}$

$y' = 7e^{-4x} + 7x e^{-4x}(-4)$

$y' = 7e^{-4x} - 28x e^{-4x}$

$\boxed{y' = 7e^{-4x}(1-4x)}$

14) $f(x) = \ln\frac{\sqrt{x}}{5-x}$ $= \frac{1}{2}\ln x - \ln(5-x)$

$f'(x) = \frac{1}{2x} - \frac{1}{5-x}(-1)$

$\boxed{f'(x) = \frac{1}{2x} + \frac{1}{5-x}}$

Find an equation of the tangent line to the given curve at the given point:

15) $y = xe^x - e^x$, $(1,0)$

$y' = e^x + xe^x - e^x$

$y' = xe^x \Rightarrow m = y'(1) = 1e^1 = e$

$\boxed{y = e(x-1)}$

16) $y = \ln(4x+5)$, $(-1,0)$

$y' = \frac{1}{4x+5}(4) = \frac{4}{4x+5}$

$m = y'(-1) = \frac{4}{4(-1)+5} = 4$

$\boxed{y = 4(x+1)}$

17) $y = \ln(x^{12})$, $(1,0)$

$y = 12\ln x$

$y' = \frac{12}{x}$ $\quad m = y'(1) = 12$

$\boxed{y = 12(x-1)}$

18) $y = e^{\sin(x)}$, $x = 0$

$y' = e^{\sin x}(\cos x) = \cos x\, e^{\sin x}$

$y'(0) = \cos(0)\, e^{\sin(0)} = 1e^0 = 1$

$y(0) = e^{\sin(0)} = e^0 = 1$

$m = 1$, $(0,1)$

$\boxed{y - 1 = x}$

19) $y = 2xe^{3x^2}$, $(1, 2e^3)$

$y' = 2e^{3x^2} + 2x e^{3x^2}(6x)$

$= 2e^{3x^2} + 12x^2 e^{3x^2} = 2e^{3x^2}(1+6x^2)$

$y'(1) = 2e^3(1+6) = 14e^3$

$\boxed{y - 2e^3 = 14e^3(x-1)}$

20) $y = \ln(5x^2)$, $x = 4$

$y = \ln 5 + 2\ln x$

$y' = \frac{2}{x}$ $\quad y'(4) = \frac{2}{4} = \frac{1}{2}$

$y(4) = \ln(5 \cdot 4^2) = \ln(80)$

$(4, \ln(80))$, $m = \frac{1}{2}$

$\boxed{y - \ln(80) = \frac{1}{2}(x-4)}$

Find the second derivative of each:

21) $y = xe^x$

$y' = e^x + xe^x = e^x(1+x)$

$y'' = e^x(1+x) + e^x$

$\boxed{y'' = e^x(2+x)}$

22) $f(x) = \ln(x^2) = 2\ln x$

$f'(x) = \frac{2}{x} = 2x^{-1}$

$\boxed{f'(x) = \frac{-2}{x^2}}$

23) $g(x) = e^{(3x^2+1)}$

$g'(x) = e^{(3x^2+1)}(6x) = 6xe^{3x^2+1}$

$g''(x) = 6e^{3x^2+1} + 6xe^{3x^2+1}(6x)$

$g''(x) = 6e^{3x^2+1} + 36x^2e^{3x^2+1}$

$\boxed{g''(x) = 6e^{3x^2+1}(1+6x^2)}$

24) $f(x) = \ln\left(\frac{x+1}{2x}\right) = \ln(x+1) - \ln 2 - \ln x$

$f'(x) = \frac{1}{x+1} - \frac{1}{x} = (x+1)^{-1} + x^{-1}$

$\boxed{f''(x) = \frac{-1}{(x+1)^2} + \frac{1}{x^2}}$

Implicit Differentiation. Find the derivative of each:

25) $e^y\cos(x) = x+1$

$\frac{d}{dx}$

$e^y\cos x\frac{dy}{dx} + e^y(-\sin x) = 1$

$e^y\cos x\frac{dy}{dx} = 1 + e^y\sin x$

$\boxed{\frac{dy}{dx} = \frac{1+e^y\sin x}{e^y\cos x}}$

26) $e^{x+y} = x^2$

$\frac{d}{dx}$

$e^{x+y}\left(1+\frac{dy}{dx}\right) = 2x$

$1 + \frac{dy}{dx} = \frac{2x}{e^{x+y}}$

$\boxed{\frac{dy}{dx} = \frac{2x}{e^{x+y}} - 1 \text{ OR } \frac{2x-e^{x+y}}{e^{x+y}}}$

27) $\ln(4x+3y)=6y$

$\frac{d}{dx}\left(\ln(4x+3y)=6y\right)$

$\frac{1}{4x+3y}\left(4+3\frac{dy}{dx}\right)=6\frac{dy}{dx}$

$4+3\frac{dy}{dx}=6(4x+3y)\frac{dy}{dx}$

$4=6(4x+3y)\frac{dy}{dx}-3\frac{dy}{dx}$

$4=\frac{dy}{dx}(24x+18y-3)$

$\boxed{\frac{dy}{dx}=\frac{4}{24x+18y-3}}$

28) $\ln\left(4x^2y\right)=5$

$\frac{d}{dx}(\ln 4+2\ln x+\ln y=5)$

$\frac{2}{x}+\frac{1}{y}\frac{dy}{dx}=0$

$\frac{1}{y}\frac{dy}{dx}=-\frac{2}{x}$

$\boxed{\frac{dy}{dx}=\frac{-2y}{x}}$