Name Answer Key Date_______________

Calc I H – 5.2-5.4 day 4 – Integration of Exponential and Natural Log Practice **Period ____________**

Integrate the following functions:

1. $\int \tan 5\theta \, d\theta$ $= \int \frac{\sin(5\theta)}{\cos(5\theta)} d\theta$

$u = \cos(5\theta)$

$\frac{du}{d\theta} = -5\sin(5\theta)$

$d\theta = \frac{du}{-5\sin(5\theta)}$

$= \int \frac{\cancel{\sin(5\theta)}}{u} \cdot \frac{du}{-5\cancel{\sin(5\theta)}} = -\frac{1}{5}\int \frac{1}{u} du$

$= -\frac{1}{5}\ln|u| + C$

$= \boxed{-\frac{1}{5}\ln|\cos(5\theta)| + C}$

2. $\int \frac{e^{\frac{3}{x}}}{x^2} dx$

$u = \frac{3}{x}$

$\frac{du}{dx} = \frac{-3}{x^2} \rightarrow dx = \frac{x^2 du}{-3}$

$= \int \frac{e^u}{\cancel{x^2}} \cdot \frac{\cancel{x^2} du}{-3} = -\frac{1}{3}\int e^u du$

$= -\frac{1}{3}e^u + C = \boxed{-\frac{1}{3}e^{\frac{3}{x}} + C}$

3. $\int \frac{x-1}{x^2-2x} dx$

$u = x^2 - 2x$

$\frac{du}{dx} = 2x - 2 \rightarrow dx = \frac{du}{2x-2}$

$\int \frac{\cancel{x-1}}{u} \cdot \frac{du}{\cancel{2x-2}} = \frac{1}{2}\int \frac{1}{u} du$

$= \frac{1}{2}\ln|u| + C$

$= \boxed{\frac{1}{2}\ln|x^2-2x| + C}$

4. $\int \frac{\cos x}{1-\sin x} dx$

$u = 1 - \sin x$

$\frac{du}{dx} = -\cos x \rightarrow dx = \frac{du}{-\cos x}$

$\int \frac{\cancel{\cos x}}{u} \cdot \frac{du}{-\cancel{\cos x}} = -\int \frac{1}{u} du$

$= -\ln|u| + C$

$= \boxed{-\ln|1-\sin x| + C}$

Evaluate the following definite integrals:

5. $\int_{-1}^{1} \frac{1}{x+2} dx$

$u = x + 2$

$\frac{du}{dx} = 1 \rightarrow dx = du$

$x = -1, u = 1$

$x = 1, u = 3$

$\int_1^3 \frac{1}{u} du = \ln|u| \Big|_1^3$

$= \ln|3| - \ln|1| = \boxed{\ln 3}$

6. $\int_3^4 (2x-2)\left(e^{x^2-2x}\right) dx$

$u = x^2 - 2x$

$\frac{du}{dx} = 2x - 2 \rightarrow dx = \frac{du}{2x-2}$

$x = 3, u = 3$

$x = 4, u = 8$

$\int_3^8 \cancel{(2x-2)} e^u \cdot \frac{du}{\cancel{2x-2}} = \int_3^8 e^u du$

$= \boxed{e^8 - e^3 \approx 2960.872}$

7. Find the area of the region bounded by the curve $y = e^{-2x} + 2$, the x-axis, $x = 0$, and $x = 2$.

$$\int_0^2 (e^{-2x} + 2)\,dx \qquad u = -2x \qquad \begin{array}{l} x=0,\ u=0 \\ x=2,\ u=-4 \end{array}$$

$$\frac{du}{dx} = -2 \rightarrow dx = \frac{du}{-2}$$

$$\int_0^{-4} (e^u + 2)\frac{du}{-2} = -\frac{1}{2}\int_0^{-4} (e^u + 2)\,du = \frac{1}{2}\int_{-4}^{0} (e^u + 2)\,du$$

$$= \frac{1}{2}\left[e^u + 2u\right]_{-4}^{0} = \frac{1}{2}\left[e^0 + 2(0) - \left(e^{-4} + 2(-4)\right)\right]$$

$$= \frac{1}{2}\left(1 - e^{-4} + 8\right) = \boxed{\frac{1}{2}\left(9 - e^{-4}\right) \approx 4.491}$$