Evaluate the following integrals.

1. $\displaystyle\int_{e^2}^{e^8} \frac{dx}{x\ln(x)}$

$u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$, $dx = x\,du$

$x = e^2$, $u = 2$

$x = e^8$, $u = 8$

$\displaystyle\int_2^8 \frac{du}{u} = \left(\ln|u|\right)_2^8$

$= \ln 8 - \ln 2 = \boxed{\ln 4}$

2. $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-1}}$

$u = e^x$, $a = 1$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$\displaystyle\int \frac{du}{u\sqrt{u^2-1}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

$= \boxed{\operatorname{arcsec}(e^x) + C}$

3. $\displaystyle\int_{\sqrt{3}}^{2} \frac{dx}{\sqrt{4-x^2}}$

$a = 2$, $du = dx$

$u = x$

$\displaystyle\int_{\sqrt{3}}^{2} \frac{du}{\sqrt{a^2-u^2}} = \left(\arcsin\frac{u}{a}\right)_{\sqrt{3}}^{2}$

$= \left(\arcsin\frac{x}{2}\right)_{\sqrt{3}}^{2}$

$= \arcsin 1 - \arcsin\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3\cdot\pi}{3\cdot 2} - \frac{2\cdot\pi}{2\cdot 3} = \boxed{\frac{\pi}{6}}$

4. $\displaystyle\int \frac{(x^2+1)dx}{\sqrt{x-1}}$

$u = x-1$

$du = dx$

$x = u+1$

$x^2 = u^2+2u+1$

$\displaystyle\int \frac{u^2+2u+1+1}{\sqrt{u}}\,du$

$= \displaystyle\int \left(u^{3/2} + 2u^{1/2} + 2u^{-1/2}\right)du$

$= \frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2} + 4u^{1/2} + C$

$= \boxed{\frac{2}{5}(x-1)^{5/2} + \frac{4}{3}(x-1)^{3/2} + 4(x-1)^{1/2} + C}$

5. $\displaystyle\int \left(x\sqrt{2x^2+1}\right)dx$

$u = 2x^2+1$

$\frac{du}{dx} = 4x$

$dx = \frac{du}{4x}$

$= \frac{1}{4}\displaystyle\int \sqrt{u}\,du$

$= \frac{1}{4}\left(\frac{2}{3}u^{3/2}\right) + C$

$= \boxed{\frac{1}{6}(2x^2+1)^{3/2} + C}$

6. $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{\sin(x)}{1+\cos^2(x)}dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$x = \frac{\pi}{2}$, $u = 0$

$x = \pi$, $u = -1$

$a = 1$

$-\displaystyle\int_0^{-1} \frac{1}{1+u^2}\,du = \int_{-1}^{0} \frac{1}{1+u^2}\,du$

$= \left(\arctan u\right)_{-1}^{0}$

$= \arctan 0 - \arctan(-1)$

$= 0 - (-\pi/4) = \boxed{\frac{\pi}{4}}$

7. $\displaystyle\int_{\sqrt{3}}^{3} \frac{dx}{9+x^2}$

$u = x$, $du = dx$

$a = 3$

$\displaystyle\int_{\sqrt{3}}^{3} \frac{du}{a^2+u^2} = \left(\frac{1}{3}\arctan\frac{u}{3}\right)_{\sqrt{3}}^{3}$

$= \frac{1}{3}\left[\arctan 1 - \arctan\frac{\sqrt{3}}{3}\right]$

$= \frac{1}{3}\left[\frac{3\pi}{3\cdot 4} - \frac{2\pi}{2\cdot 6}\right] = \frac{1}{3}\left(\frac{\pi}{12}\right)$

$= \boxed{\frac{\pi}{36}}$

8. $\displaystyle\int \sqrt{\tan x}\sec^2 x\,dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\displaystyle\int \sqrt{u}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{3}\tan^{3/2}x + C}$

9. $\displaystyle\int 4x^3\cos(x^4)\,dx$

$u = x^4$

$\frac{du}{dx} = 4x^3$

$dx = \frac{du}{4x^3}$

$\displaystyle\int \cos u\,du$

$= \sin u + C$

$= \boxed{\sin(x^4) + C}$

10. $\displaystyle\int \frac{e^{2x}}{4+e^{4x}}\,dx$

$a=2$

$u=e^{2x}$

$\frac{du}{dx}=2e^{2x}$

$dx=\frac{du}{2e^{2x}}$

$$=\frac{1}{2}\int\frac{du}{a^2+u^2}$$

$$=\frac{1}{2}\left[\frac{1}{a}\arctan\frac{u}{a}\right]+C$$

$$=\frac{1}{2}\left[\frac{1}{2}\arctan\left(\frac{e^{2x}}{2}\right)\right]+C$$

$$=\boxed{\frac{1}{4}\arctan\left(\frac{e^{2x}}{2}\right)+C}$$

11. $\displaystyle\int_{\frac{\pi}{15}}^{\frac{\pi}{5}}\left(\cos^2 5x\sin 5x\right)dx$

$u=\cos 5x$

$\frac{du}{dx}=-5\sin 5x$

$dx=\frac{du}{-5\sin 5x}$

$x=\frac{\pi}{15},\ u=\frac{1}{2}$

$x=\frac{\pi}{5},\ u=-1$

$$-\frac{1}{5}\int_{1/2}^{-1}u^2\,du=\frac{1}{5}\int_{-1}^{1/2}u^2\,du$$

$$=\frac{1}{5}\cdot\frac{u^3}{3}\Big|_{-1}^{1/2}$$

$$=\frac{1}{15}\left(\frac{1}{8}-(-1)\right)=\frac{1}{15}\left(\frac{9}{8}\right)$$

$$=\boxed{\frac{3}{40}}$$

12. $\displaystyle\int\frac{\sin e^{-x}}{e^{x}}\,dx$

$u=e^{-x}$

$\frac{du}{dx}=-e^{-x}$

$dx=\frac{du}{-e^{-x}}=-e^{x}du$

$$=-\int\sin u\,du$$

$$=\cos u+C$$

$$=\boxed{\cos\left(\tfrac{1}{e^{x}}\right)+C}$$

13. $\displaystyle\int\frac{1}{x^2-2x+5}\,dx$

$$=\int\frac{1}{(x^2-2x+1)+5-1}\,dx$$

$$=\int\frac{1}{(x-1)^2+4}\,dx$$

$a=2$

$u=x-1$

$du=dx$

$$=\int\frac{1}{u^2+a^2}\,du$$

$$=\frac{1}{a}\arctan\frac{u}{a}+C$$

$$=\boxed{\frac{1}{2}\arctan\left(\frac{x-1}{2}\right)+C}$$

14. $\displaystyle\int\frac{x}{(\ln 3)(x^2+4)}\,dx$

$u=x^2+4$

$\frac{du}{dx}=2x$

$dx=\frac{du}{2x}$

$$\frac{1}{2\ln(3)}\int\frac{1}{u}\,du$$

$$=\frac{1}{\ln 9}\cdot\ln|u|+C$$

$$=\boxed{\frac{\ln(x^2+4)}{\ln 9}+C}$$

15. $\displaystyle\int_{9}^{16}\frac{dx}{\left(2+\sqrt{x}\right)^3}$

$u=2+\sqrt{x}$

$\frac{du}{dx}=\frac{1}{2\sqrt{x}}$

$dx=2\sqrt{x}\,du$

$x=9,\ u=5$

$x=16,\ u=6$

$$=\int_5^6\frac{2\sqrt{x}}{u^3}\,du=2\int_5^6\frac{u-2}{u^3}\,du$$

$$=2\int_5^6\left(u^{-2}-2u^{-3}\right)du$$

$$=2\left(-\frac{1}{u}+\frac{1}{u^2}\right)\Big|_5^6$$

$$=2\left(-\frac{1}{6}+\frac{1}{36}+\frac{1}{5}-\frac{1}{25}\right)$$

$$=2\left(\frac{19}{900}\right)=\boxed{\frac{19}{450}}$$