Do Now:

1. Solve the differential equation $y' + 2y = 0$. Find the particular solution that goes through $\left(-1, e^2\right)$.

$\frac{dy}{dx} = -2y$

$\int \frac{1}{y}\,dy = \int -2\,dx$

$\ln|y| = -2x + C$

$|y| = e^{-2x+C}$

$y = \pm e^{-2x} e^{C}$

$y = Ce^{-2x}$

$e^{2} = Ce^{2}$

$1 = C$

$\boxed{y = e^{-2x}}$

2. Solve the differential equation $\frac{dy}{dx} = \cos(x)$. Explain the graphical relationship between $\frac{dy}{dx}$ and y.

$\int dy = \int \cos x\,dx$

$y = \sin x + C$

$\frac{dy}{dx}$ represents the slope of the tangent line of y at each point (x, y).

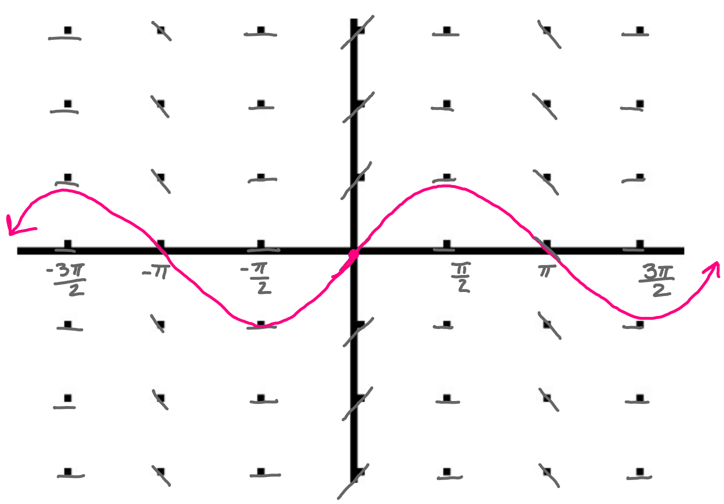
3. Find, if possible, the general solution to the differential equation $\frac{dy}{dx} = x + y$. If not possible, explain why.

Not possible to separate the variables → general solution can not be found.

Class Work:

A slope field, *or direction field,* is a way for us to "see" the solutions to a given differential equation. It is made up of tiny tangent lines that represent the slope of the *solution* to the differential equation at a given point.

1. Sketch a slope field for the following differential equation: $\frac{dy}{dx} = \cos(x)$. Label the axes. Draw the particular solution for $y(0) = 0$. Then, solve the differential with the given initial condition. Is there an observable pattern in the slope field? Why or why not?



$\int dy = \int \cos x\,dx$

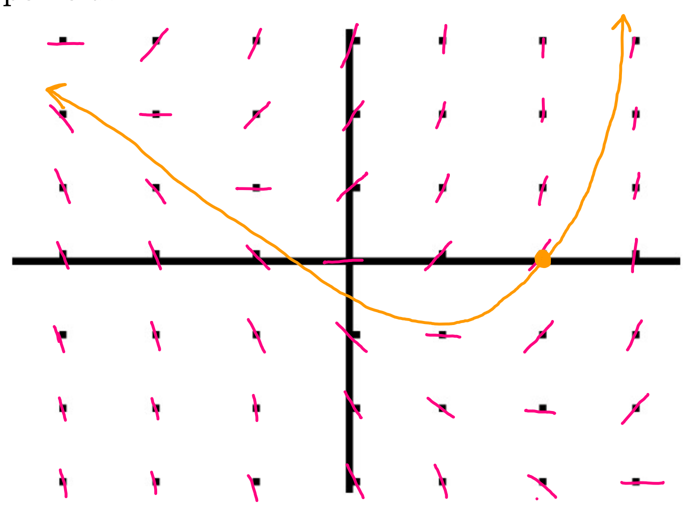
$y = \sin x + C$ → $y = \sin x$

$0 = \sin 0 + C$

$0 = C$

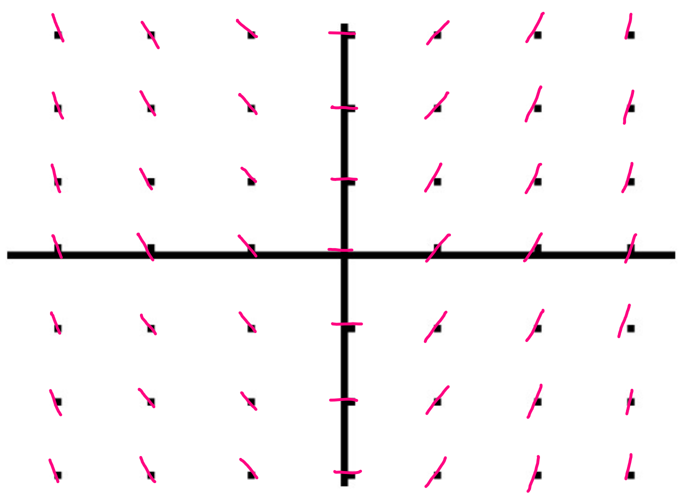
vertical slope pattern → $\frac{dy}{dx}$ dependent on only x!

2. Sketch a slope field for the differential equation $\frac{dy}{dx} = x + y$ and sketch the graph of the particular solution that passes through the point $(2, 0)$. Is there an observable pattern in the slope field?



diagonal slope pattern → dependent on x & y

3. Sketch a slope field for the differential equation $y' = x$.



a. On what intervals does the solution appear to be increasing? Decreasing?

$(0, \infty)$ $(-\infty, 0)$

b. What pattern is observable in the slope field? What can this be attributed to? vertical slope pattern b/c $\frac{dy}{dx}$ is in terms of x.

c. Where do the tangent lines appear to have a slope of zero? What does this tell you about the solution?

$x=0$ → possible extrema when $x=0$

d. What "family of functions" does this slope field appear to represent? Solve the differential equation to confirm your answer.

$y = ax^2 + c$

$\int dy = \int x\,dx$

$y = \frac{1}{2}x^2 + C$

e. Sketch in the particular solution that satisfies the initial condition (0, 1).

$y = \frac{1}{2}x^2 + C$

$1 = \frac{1}{2}(0)^2 + C$

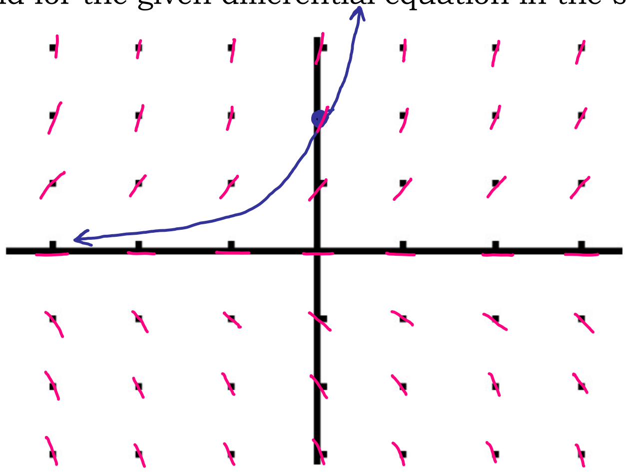
$1 = C$

$y = \frac{1}{2}x^2 + 1$

4. Consider the differential equation $\frac{dy}{dx} = y$.

   a. Explain what this differential equation means analytically in terms of the slope of the curve at any given point. Slope of the curve is equivalent to the y-coordinate at that point

   b. Sketch a slope field for the given differential equation in the space below.



   c. What pattern is observable in the slope field? What can this be attributed to?

   horizontal slope pattern → $\frac{dy}{dx}$ is dependent on only y

   d. Use the slope field to sketch the specific solution that satisfies the initial condition $y(0) = 2$.

   e. Solve the differential equation $\frac{dy}{dx} = y$ analytically and compare the result to the solution that you sketched above. Remember that we are using the initial condition $y(0) = 2$.

$\int \frac{1}{y}\,dy = \int dx$

$\ln|y| = x + c$

$|y| = e^{x+c}$

$y = \pm e^x e^c$

$y = Ce^x$

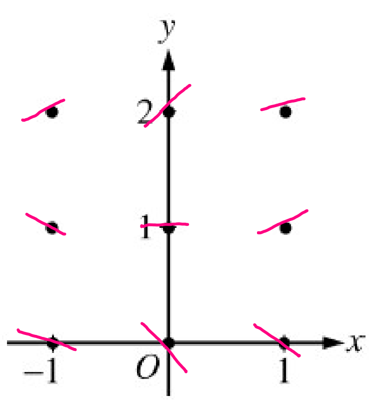
$2 = Ce^0$

$2 = C$

$y = 2e^x$

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

   a. On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



b. Find $\dfrac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

$$\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx}$$
$$= \frac{1}{2} + \frac{1}{2}x + y - 1$$
$$= -\frac{1}{2} + \frac{1}{2}x + y > 0$$
$$y > -\frac{1}{2}x + \frac{1}{2}$$

c. Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

$f'(x) = \frac{1}{2}x + y - 1$

$f'(0) = 0$

$f''(x) = -\frac{1}{2} + \frac{1}{2}x + y$

$f''(0) = -\frac{1}{2} + 0 + 1 > 0$

Since $f'(0) = 0$ and $f''(0) = \frac{1}{2} > 0$ there is a relative minimum at $x = 0$.

d. Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

$\frac{1}{2}x + y - 1 = m$

$y' = m$

$y = -\frac{1}{2}x + 1 + m = mx + b$

$-\frac{1}{2} = m$

$1 + m = b$

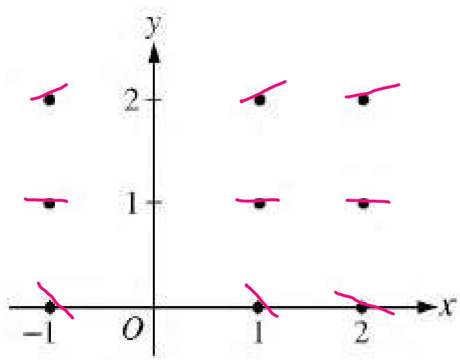
$1 - \frac{1}{2} = b$

$\frac{1}{2} = b$

$m = -\frac{1}{2}$ and $b = \frac{1}{2}$

6. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{y-1}{x^2}$, where $x \neq 0$.

a. On the axes provided, sketch a slope field for the given differential equation at the 9 points indicated.



b. Find the particular solution $y = f(x)$ to the differential equation with initial condition $f(2) = 0$.

$\int \frac{1}{y-1}\,dy = \int \frac{1}{x^2}\,dx$

$\ln|y-1| = -\frac{1}{x} + C$

$|y-1| = e^{-\frac{1}{x}+C}$

$y = \pm e^{-\frac{1}{x}} e^{C} + 1 = Ce^{-\frac{1}{x}} + 1$

$0 = Ce^{-\frac{1}{2}} + 1$

$-1 = Ce^{-1/2}$

$\frac{-1}{e^{-1/2}} = C$

$-e^{1/2} = C$

$C = -\sqrt{e}$

$y = -\sqrt{e}e^{-\frac{1}{x}} + 1$

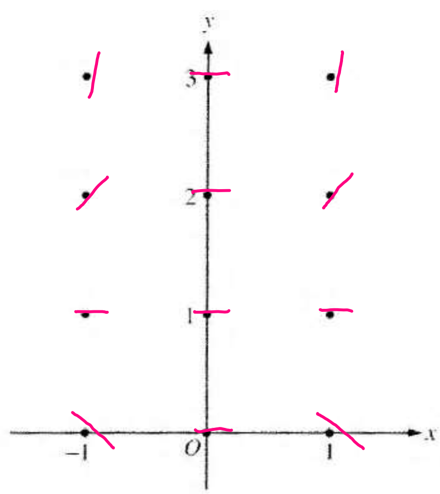
c. For the particular solution, $y = f(x)$ described in part (b), find the $\lim\limits_{x\to\infty} f(x)$.

$\lim\limits_{x\to\infty} -\frac{1}{x} = 0$

$\lim\limits_{x\to\infty} -\sqrt{e}e^{-\frac{1}{x}} + 1 = -\sqrt{e} + 1$

7. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

a. On the axes provided, sketch a slope field for the differential equation at the 12 points indicated.



b. While the slope fields in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane. Describe all points in the *xy*-plane for which the slopes are positive.

$x^2(y-1) > 0$

always positive if $x \neq 0$

$\therefore x^2(y-1) > 0$ when $y-1>0$

$\therefore y > 1$ and $x \neq 0$

c. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

$\int \frac{1}{y-1}\,dy = \int x^2\,dx$

$\ln|y-1| = \frac{1}{3}x^3 + C$

$|y-1| = e^{\frac{1}{3}x^3 + C}$

$y = Ce^{\frac{1}{3}x^3} + 1$

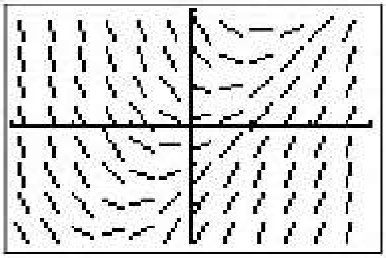
$3 = Ce^{0} + 1$

$3 = C + 1$

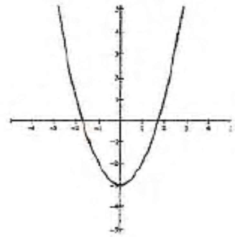
$2 = C$

$y = 2e^{\frac{1}{3}x^3} + 1$

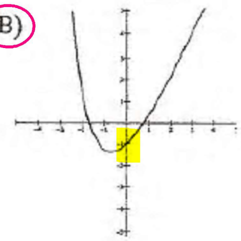
8. The slope field for $y' = 2x - y$ is given below. Which graph could be a solution to the differential equation? Justify your answer.



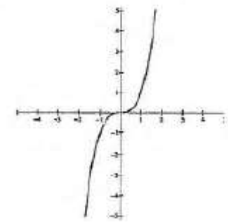
(A)



(B)



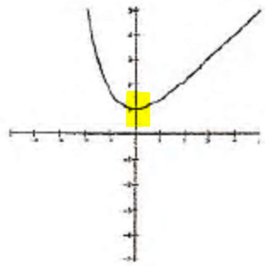
(C)



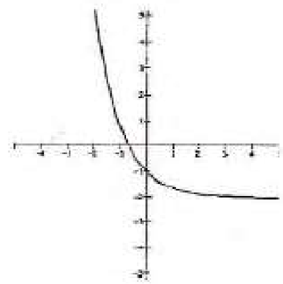
$y'\big|_{(0,-1)} = 2(0)-(-1)=1$

$y'\big|_{(0,1)} = 2(0)-1=-1$

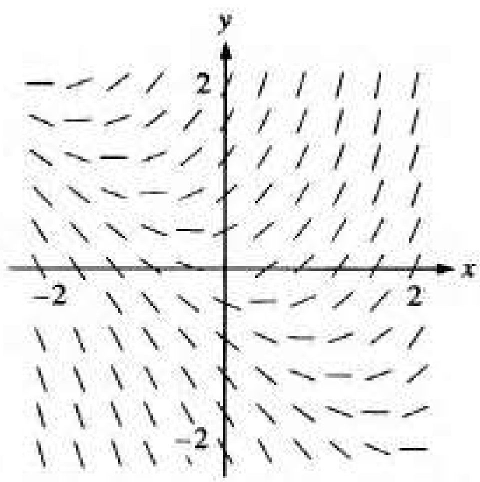
(D)



(E)



9.



diagonal pattern → $\frac{dy}{dx}$ in terms of x & y

$\frac{dy}{dx}=0$ along $y=-x$

. Shown above is a slope field for which of the following differential equations?

~~(A)~~ $\frac{dy}{dx} = 1 + x$ ~~(B)~~ $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ ~~(E)~~ $\frac{dy}{dx} = \ln y$

10. The slope field for the differential equation $\frac{dy}{dx} = \frac{x^2y + y^2x}{3x + y}$ will have horizontal segments when

a) $x = 0$ or $y = 0$, only b) $y = -x$, only c) $y = -3x$, only

d) $y = 5$, only e) $x = 0$, $y = -x$ or $y = 0$

$x^2y + y^2x = 0$

$xy(x+y) = 0$

$x=0 \quad y=0 \quad -x=y$

11. Match the slope fields with their differential equations:

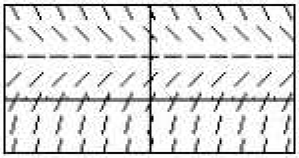
i. $\frac{dy}{dx} = \sin x$ C vertical pattern

ii. $\frac{dy}{dx} = x - y$ D diagonal pattern $y = x \rightarrow \frac{dy}{dx} = 0$

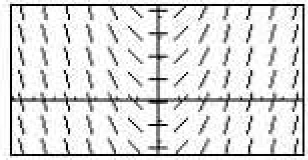
iii. $\frac{dy}{dx} = 2 - y$ A horizontal pattern

iv. $\frac{dy}{dx} = x$ B vertical pattern

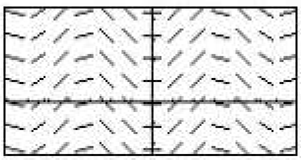
(A)



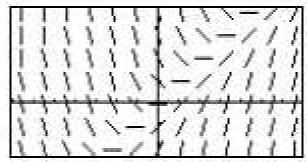
(B)



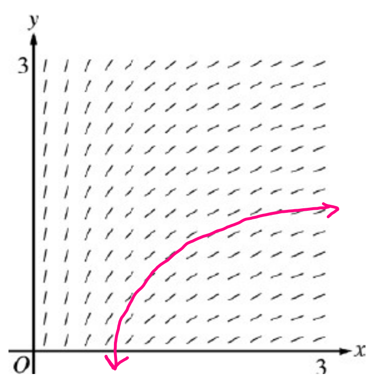
(C)



(D)



From the May 2008 *AP Calculus Course Description:*

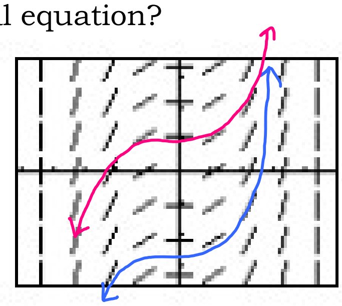


12. The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

a) $y = x^2$ b) $y = e^x$ c) $y = e^{-x}$ d) $y = \cos x$ e) $y = \ln x$

13. The slope field for the differential equation $\frac{dy}{dx} = f(x)$ is shown for $-4 < x < 4$ and $-4 < y < 4$.

Which statement is true for all possible solutions of the differential equation?
I. For $x < 0$ all solutions are decreasing. False
II. All solutions level off near the y-axis. True
III. For $x > 0$ all solutions are increasing. True



a) I only b) II only c) III only d) II & III only e) I, II, and III

14. Match the slope fields with their differential equations:

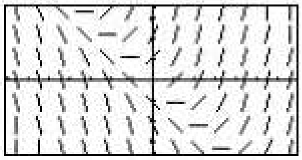
i. $\frac{dy}{dx} = \frac{1}{2}x - 1$ Vertical pattern B

ii. $\frac{dy}{dx} = \frac{1}{2}y$ Horizontal Pattern C

iii. $\frac{dy}{dx} = -\frac{x}{y}$ $\frac{dy}{dx} = 0$ when $x = 0$ D

iv. $\frac{dy}{dx} = x + y$ $\frac{dy}{dx} = 0$ when $y = -x$ A

(A)



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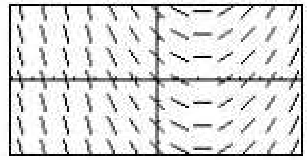

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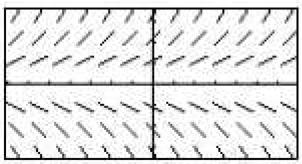

(D)