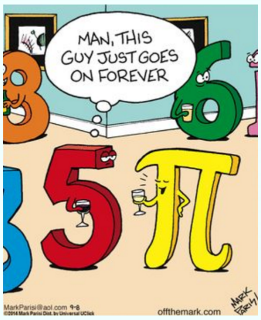
6.1 Day 2 - Slope Fields

3/14/19

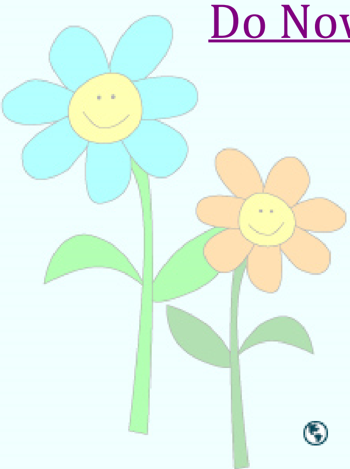


Homework:

- Text p. 410 #37-41 odd, 53-57, 59
- Complete 6.1 Day 2 & 3 Worksheet: Due Monday

Objective:

Use Slope Fields to approximate solutions of differential equations.

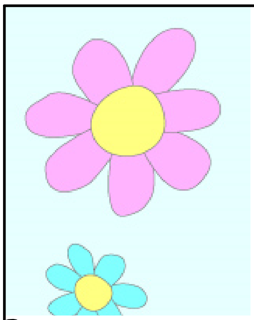


Do Now:

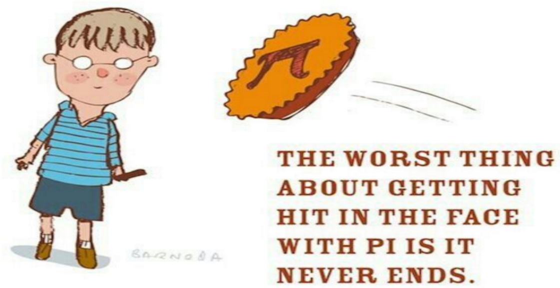
Complete #~~1~~2-~~2~~3 on today's worksheet with your group...

BUT... before we begin... for your entertainment... I bring you...

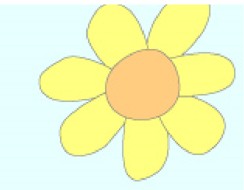
a musical π day song!



Do Now



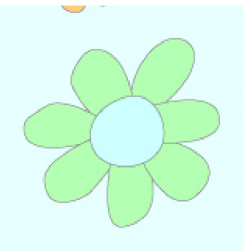
2. Solve the differential equation $\frac{dy}{dx} = \cos(x)$. Explain the graphical relationship between $\frac{dy}{dx}$ and y.



$y = \sin x + C$

3. Find, if possible, the general solution to the differential equation $\frac{dy}{dx} = x + y$. If not possible, explain why.

$\frac{dy}{dx} - y = x$ $\qquad \frac{dy}{dx} = xy$



Homework Questions

No rational person would do this.

Class Work:

Solving a differential equation analytically can be difficult or even impossible. However, there is a graphical approach you can use to learn a lot about the solution of a differential equation. A *slope field* is a way for us to "see" the solutions to a given differential equation. It is made up of tiny tangent lines that represent the slope of the *solution* to the differential equation at a given point.

Slope Fields

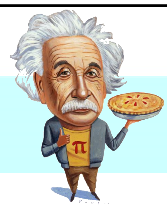
Consider a differential equation of the form: $y' = F(x, y)$.

At each point (x, y) in the xy-plane where F is defined, the differential equation determines the slope, $y' = F(x, y)$, of the solution at that point.

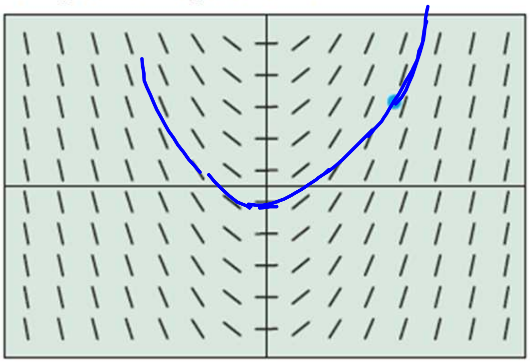
If you draw a short line segment with slope $F(x, y)$ at selected points (x, y) in the domain of F, then these line segments form a **slope field**, or a *direction field* for the differential equation.

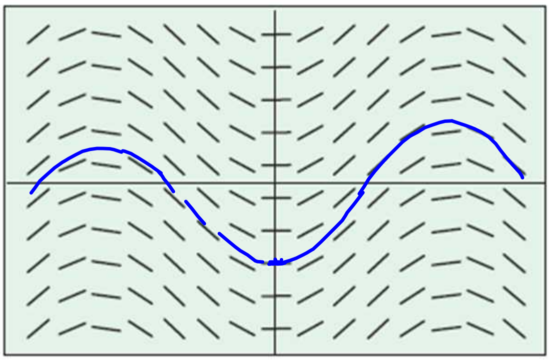
Each line segment has the same slope as the solution curve through that point.

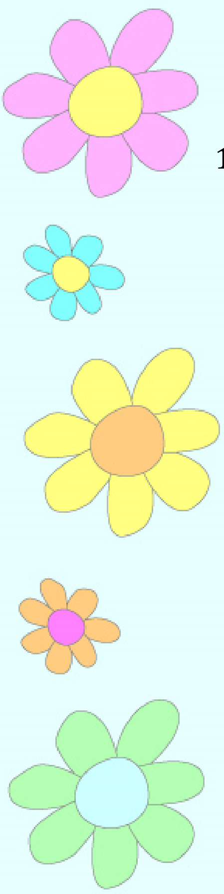
A slope field shows the general shape of all the solutions.



Examples of Slope Fields:





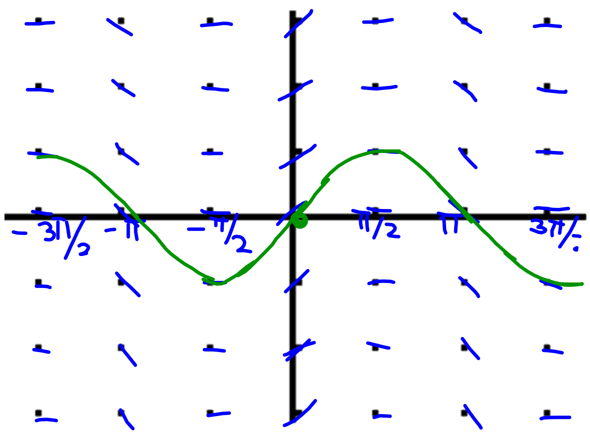


Examples

1. Sketch a slope field for the following differential equation: $\dfrac{dy}{dx} = \cos x$

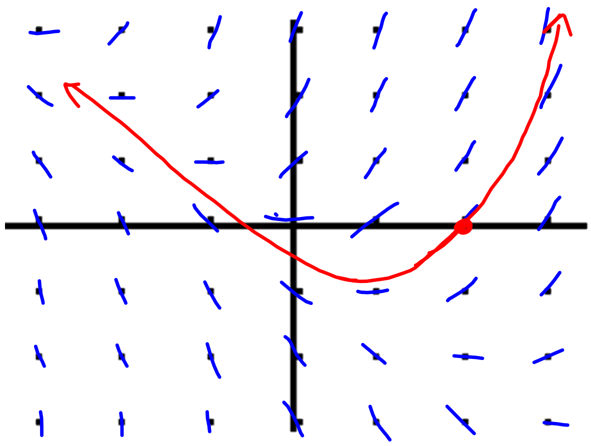
Label the axes. Draw the particular solution for $y(0) = 0$.

Is there an observable pattern in the slope field? Why or why not?

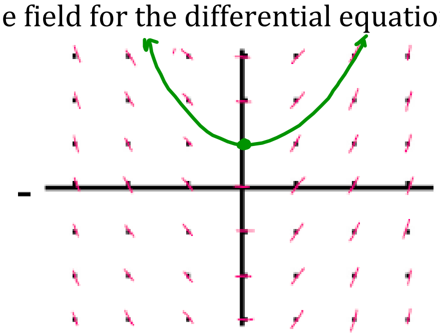


Examples

2. Sketch a slope field for the differential equation $\frac{dy}{dx} = x + y$ and sketch the graph of the particular solution that passes through the point (2, 0).



3. Sketch a slope field for the differential equation $y' = x$.



a) On what intervals does the solution appear to be increasing? Decreasing?

$(0, \infty)$ $(-\infty, 0)$

b) What pattern is observable in the slope field? What can this be attributed to?

Vertical $\frac{dy}{dx}$ dep. on x only.

c) Where do the tangent lines appear to have a slope of zero? What does this tell you about the solution?

$x=0$ min at $x=0$

d) What "family of functions" does this slope field appear to represent? Solve the differential equation to confirm your answer.

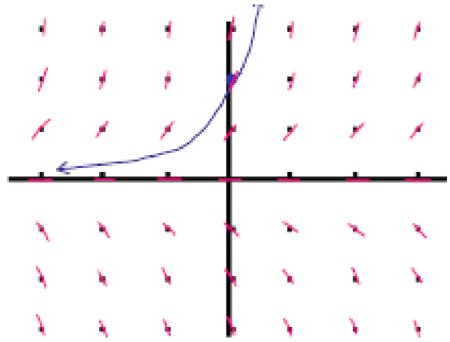
$y = \frac{x^2}{2} + C$

e) Sketch in the particular solution that satisfies the initial condition (0, 1).

4. Consider the differential equation $\frac{dy}{dx} = y$.

a. Explain what this differential equation means analytically in terms of the slope of the curve at any given point.

b. Sketch a slope field for the given differential equation in the space below.



c. What pattern is observable in the slope field? What can this be attributed to?

horiz $\frac{dy}{dx}$ dep on y only.

d. Use the slope field to sketch the specific solution that satisfies the initial condition $y(0) = 2$.

e. Solve the differential equation analytically and compare the result to the solution that you sketched above. Remember that we are using the initial condition $y(0) = 2$.