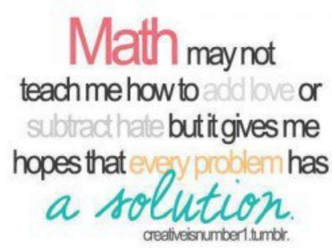
Section 6.2 Day 1 - Applications of Differential Equations 3/18/19

Homework:



- Page 430 #59 b, c (use *k* values from pt a only)
- Finish Section 6.2 Day 1 Worksheet
- 6.1-6.2 Quiz Thurs or Fri - 3/21 or 3/22

Objective: Solve application problems involving differential equations or exponential growth and decay models .

Do Now: *B*egin the *Do Now* at the top of the sheet and talk about the solution with your group!

Do Now

1. The rate of change of the amount of a substance W is directly proportional to the amount of the substance present at a given time t, meaning $\frac{dW}{dt} = kW$. Find the equation that models the amount of substance present at time t.

$W = Ce^{kt}$

2. Based on the equation found in #1, when would the model represent a growth function? When would it represent a decay function?

$k > 0$, $k < 0$

3. Find the particular solution of the equation in #1 if there are 250 gallons of the substance present when $t = 0$.

$W = 250e^{kt}$

The Law of Exponential Change

If y changes at a rate proportional to the amount present (that is, if $\frac{dy}{dt} = ky$), and if $y = y_0$ when $t = 0$, then

$$y = y_0 e^{kt}$$

The constant k is the **growth constant** if $k > 0$ or the **decay constant** if $k < 0$.

Class Work:

1. Suppose that the cholera bacteria in a colony grows unchecked according to the Law of Exponential Change. The colony starts with 1 bacterium and doubles in number every half hour. How many bacteria will the colony contain at the end of 24 hours?

$y = y_0 e^{kt}$

t = hours

$y = e^{kt}$

$(\frac{1}{2}, 2)$

$2 = e^{\frac{1}{2}k}$

$\ln 2 = \frac{1}{2}k$

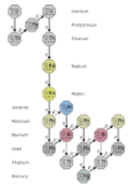
$k = 2\ln 2 = \ln 4$

$y = e^{t\ln 4} = e^{\ln 4^t}$

$\boxed{y = 4^t}$

$y(24) = 4^{24} = 2.8 \times 10^{14}$ bact.

Radon is a chemical element with symbol Rn and atomic number 86. It is a radioactive, colorless, odorless, tasteless noble gas, occurring naturally as a decay product of radium. Its most stable isotope, ^{222}Rn, has a half-life of 3.8 days.



2. The decay equation for radon-222 gas is known to be $y = y_0 e^{-0.18t}$, with t in days. About how long will it take the amount of radon in a sealed sample of air to decay to 90% of its original value?

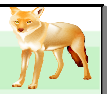
$$y = y_0 e^{-.18t}$$

$$y = .9 y_0 = y_0 e^{-.18t}$$

$$\ln(.9) = -.18t$$

$$t = \frac{\ln(.9)}{-.18}$$

$t = .585$ days



4. The rate of change of the number of coyotes $N(t)$ in a population is directly proportional to $650 - N(t)$, where t is the time in years. When $t = 0$, the population is 300, and when $t = 2$, the population has increased to 500. Find the population when $t = 3$.

$$\frac{dN}{dt} = k(650 - N)$$

$$\int \frac{dN}{650 - N} = \int k\,dt$$

$$-\ln|650 - N| = kt + C_1$$

$$\ln|650 - N| = -kt + C_2$$

$$|650 - N| = e^{-kt + C_2} = e^{-kt} e^{C_2}$$

$$650 - N = Ce^{-kt}, \quad C = \pm e^{C_2}$$

$$N = 650 - Ce^{-kt}$$

$$N(0) = 300$$

$$300 = 650 - Ce^0$$

$$-350 = -C$$

$$C = 350$$

$$N = 650 - 350e^{-kt}$$

$$N(2) = 500$$

$$500 = 650 - 350e^{-2k}$$

$$\frac{-150 = -350e^{-2k}}{-350}$$

$$\frac{3}{7} = e^{-2k}$$

$$\ln\left(\frac{3}{7}\right) = -2k$$

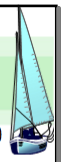
$$\frac{1}{2}\ln\left(\frac{3}{7}\right) = -k$$

$$N = 650 - 350e^{t\ln\sqrt{\frac{3}{7}}}$$

$$N = 650 - 350\left(\sqrt{\frac{3}{7}}\right)^t$$

$N(3) = 551.802$ coyotes

≈ 551 or 552 coyotes



5. Ignoring resistance, a sailboat starting from rest (meaning its initial velocity is zero) accelerates, $\frac{dV}{dt}$, at a rate proportional to the difference between the velocities of the wind (W) and the boat (V). The wind is blowing at 20 knots, and after 1 minute the boat is moving at 5 knots.

   a. Write the velocity v as a function of time t.

   b. Write the distance traveled by the boat as a function of time.

Knots $= \frac{nm}{hr}$

$$\frac{dv}{dt} = k(W - V)$$

$$\int \frac{dv}{W-V} = \int k\,dt$$

$$-\ln|W-V| = kt + C_1$$

$$\ln|W-V| = -kt - C_1$$

$$|W-V| = e^{-kt-C_1}$$

$$W - V = Ce^{-kt}, \quad C = \pm e^{-C_1}$$

$$V = W - Ce^{-kt}$$

$$V = 20 - Ce^{-kt}$$

$V(0) = 0$

$$0 = 20 - Ce^{0}$$

$$C = 20$$

$$V = 20 - 20e^{-kt}$$

$$V\left(\tfrac{1}{60}\right) = 5 = 20 - 20e^{-k/60}$$

$$-15 = -20e^{-k/60}$$

$$\frac{3}{4} = e^{-k/60}$$

$$\ln\left(\frac{3}{4}\right) = \frac{-k}{60} \Rightarrow -k = 60\ln\left(\frac{3}{4}\right)$$