7.1 day 1 - Area of a Region Between Two Curves 5/9/19

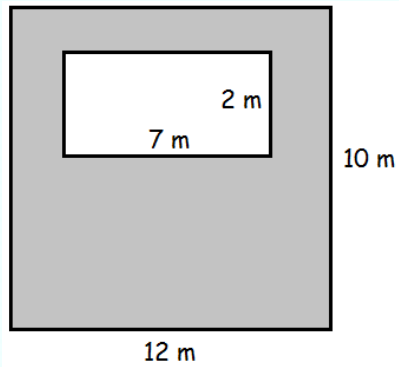
Homework: 7.1 A

Quiz 7.1 Wednesday 5/15

Objective: Find the area of a region between two curves using integration.

Do Now: Explain with a sentence what the following means. $\int_0^3 x^2 dx$

1 more Do Now: Find the area of the shaded region.



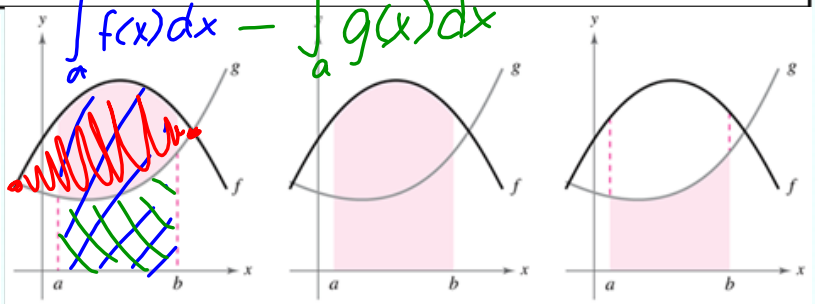
The **area between two curves** $f(x)$ and $g(x)$, where $f(x) > g(x)$, that is bounded by the vertical lines $x = a$ and $x = b$ is equal to $\int_a^b f(x) - g(x)\,dx$.

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

1.) Determine which is the upper ($f(x)$) curve and which is the lower ($g(x)$) curve.

2.) Find *a* and *b* (limits of integration). Sometimes they are given, and sometimes you need to find where the curves intersect.

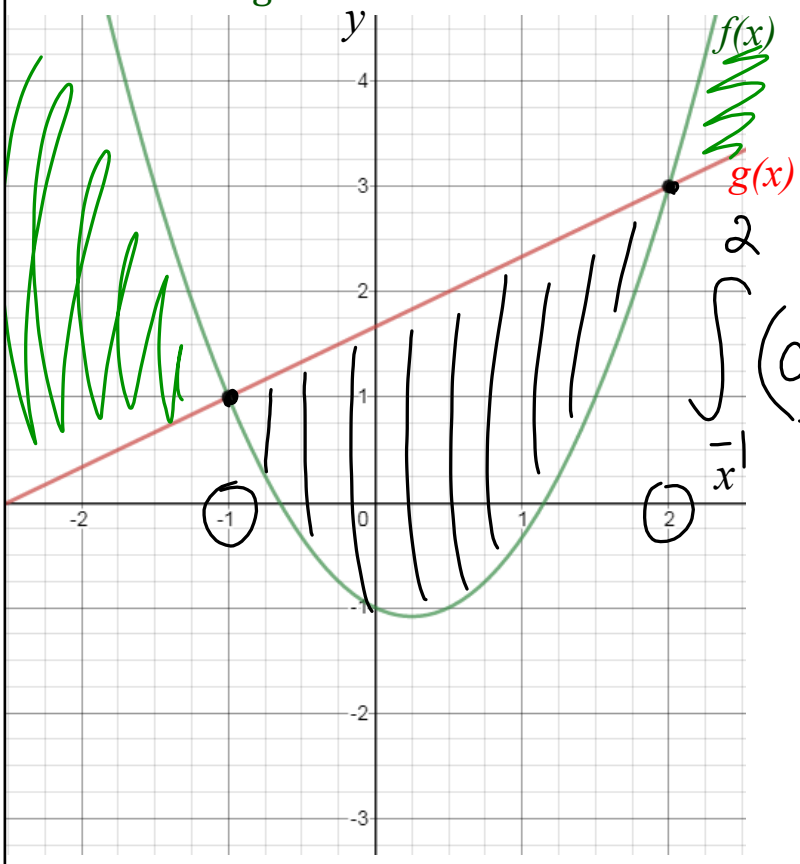
3.) Integrate $f(x) - g(x)$ from *a* to *b*. This will be the area of the region between the curves!



Area of region between f and g	=	Area of region under f	−	Area of region under g
$\int_a^b [f(x) - g(x)]\,dx$	=	$\int_a^b f(x)\,dx$	−	$\int_a^b g(x)\,dx$

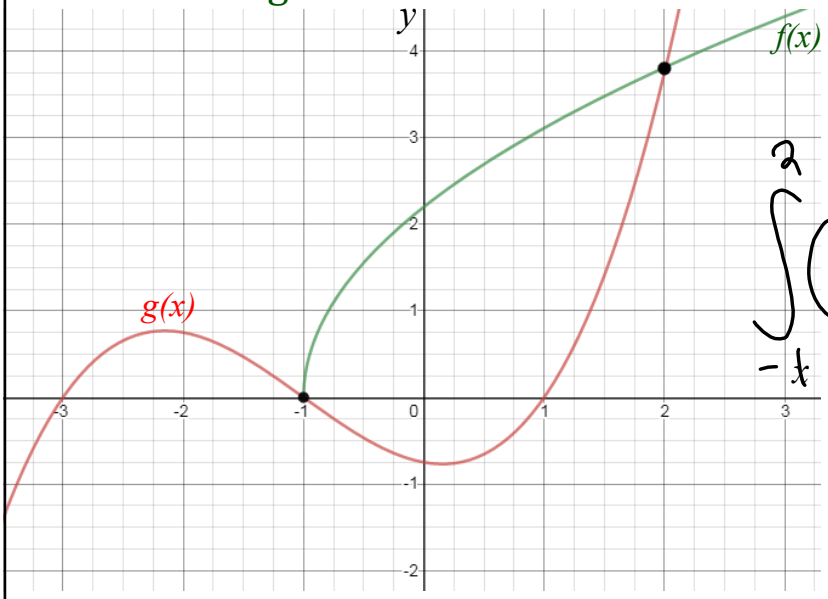
In a dx perspective, this means that the area is equal to the integral of the top curve minus the bottom curve!

Write the integral that will evaluate the area between the 2 curves.



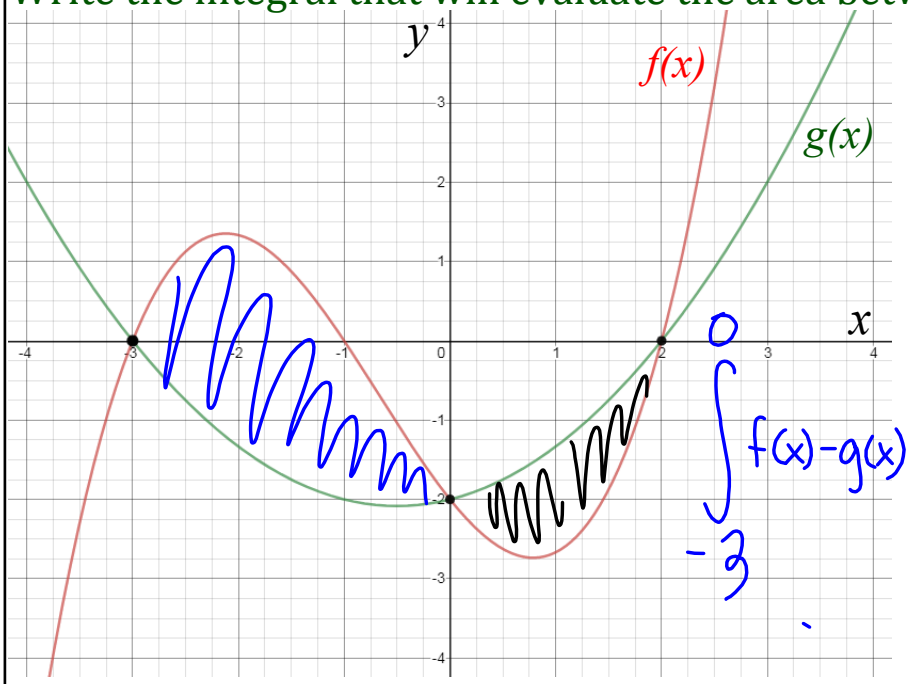
$$\int_{-1}^{2} (g(x) - f(x))\,dx$$

Write the integral that will evaluate the area between the 2 curves.



$$\int_{-1}^{2} (f(x) - g(x))\,dx$$

Write the integral that will evaluate the area between the 2 curves.



$$\int_{-3}^{0} f(x) - g(x)\,dx + \int_{0}^{2} g(x) - f(x)\,dx$$

Steps:

1) Find the points of intersection.

2) Draw the functions.

3) Set up a definite integral (Top function - Bottom function).

4) Simplify.

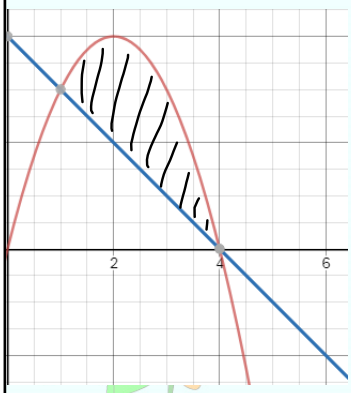
5) Integrate.

6) Evaluate using the Fundamental Theorem of Calculus.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Example 1

Find the area of the region between the graphs of $f(x) = -x^2 + 4x$ and $g(x) = -x + 4$.



$$-x^2 + 4x = -x + 4$$
$$+x^2 \quad -4x \quad +x^2 - 4x$$
$$0 = x^2 - 5x + 4$$
$$0 = (x-4)(x-1)$$
$$x = 1, 4$$

$$\int_1^4 -x^2 + 4x - (-x+4)\,dx$$

$$\int_1^4 (-x^2 + 5x - 4)\,dx$$

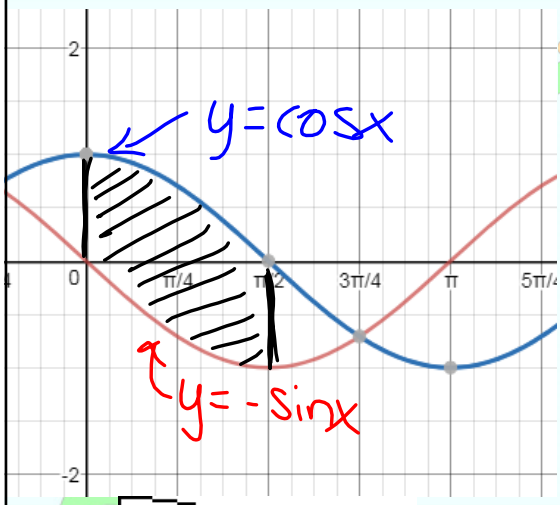
$$= \left[-\frac{x^3}{3} + \frac{5}{2}x^2 - 4x\right]_1^4$$

$$= \frac{-64}{3} + \frac{5}{2}(16) - 16 - \left(\frac{-1}{3} + \frac{5}{2} - 4\right)$$

$$= 4.5$$

Example 2

Find the area of the region between $y = \cos x$ and $y = -\sin x$ from $x = 0$ to $x = \frac{\pi}{2}$.



$$\int_0^{\pi/2} (\cos x + \sin x)\,dx$$

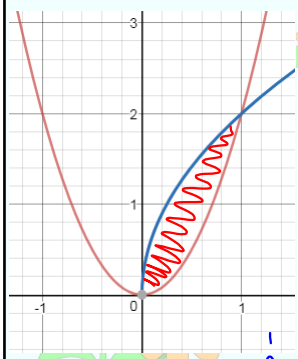
$$= \left[\sin x - \cos x\right]_0^{\pi/2}$$

$$= \sin \pi/2 - \cos \pi/2 - (\sin 0 - \cos 0)$$

$$= 1 + 1 = \boxed{2}$$

Example 3

Find the area of the region between $f(x) = 2x^2$ and $g(x) = 2\sqrt{x}$



$$2x^2 = 2\sqrt{x}$$

$$(x^2)^2 = (\sqrt{x})^2$$

$$x^4 = x$$

$$x^4 - x = 0$$

$$x(x^3 - 1) = 0 \qquad x = 0, 1$$

$$\int_0^1 2\sqrt{x} - 2x^2\,dx$$

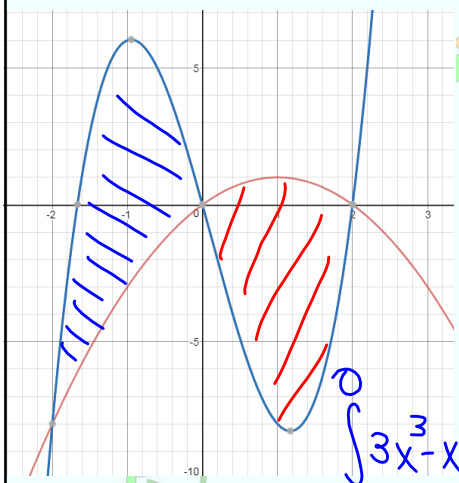
$$2\int_0^1 (x^{1/2} - x^2)\,dx$$

$$2\left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1$$

$$2\left[\frac{2}{3} - \frac{1}{3} - 0\right] = \boxed{\frac{2}{3}}$$

Example 4

Find the area of the region between the graphs of $f(x)=3x^3-x^2-10x$ and $g(x)=-x^2+2x$.



$3x^3-x^2-10x=-x^2+2x$

$3x^3-12x=0$

$3x(x^2-4)=0$

$x=0,\pm 2$

$\int_{-2}^{0} 3x^3-x^2-10x-(-x^2+2x)\,dx$

$+\int_{0}^{2} -x^2+2x-(3x^3-x^2-10x)\,dx$

$\int_{-2}^{0}(3x^3-12x)\,dx+\int_{0}^{2}(-3x^3+12x)\,dx$

Practice

Find the area of the region bounded by the graphs of $f(x)=2-x^2$ and $g(x)=x$.