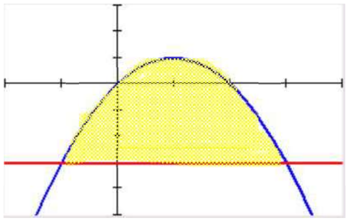
Name Answer Key Date

Calc I H – 7.1 Review Period

I. Find the area of the regions enclosed by the following lines and curves:

1. The curve $y = 2x - x^2$ and the line $y = -3$.



$2x - x^2 = -3$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3, -1$

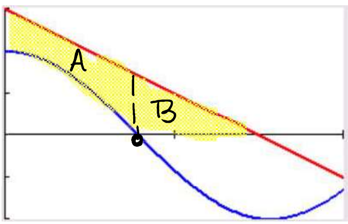
$A = \int_{-1}^{3} 2x - x^2 - (-3)\, dx = \int_{-1}^{3} (2x - x^2 + 3)\, dx$

$= \left(x^2 - \frac{x^3}{3} + 3x\right)\Big|_{-1}^{3} = \cancel{9-9} + 9 - \left(1 + \frac{1}{3} - 3\right)$

$= 9 - 1 - \frac{1}{3} + 3 = 11 - \frac{1}{3}$

$= \boxed{10\frac{2}{3} u^2}$

2. Below the line $y = 3 - 2x$ and above the curve $y = 2\cos(2x)$ in the first quadrant.



$2\cos(2x) = 0$

$\cos(2x) = 0$

$2x = \pi/2$

$x = \pi/4$

$3 - 2x = 0$

$x = 3/2$

$A = \underbrace{\int_{0}^{\pi/4} (3 - 2x - 2\cos(2x))\, dx}_{A} + \underbrace{\int_{\pi/4}^{3/2} (3 - 2x)\, dx}_{B}$

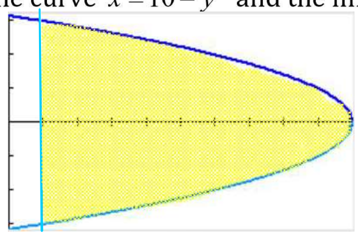
$= \left(3x - x^2 - \sin(2x)\right)\Big|_{0}^{\pi/4} + \left(3x - x^2\right)\Big|_{\pi/4}^{3/2}$

$= \cancel{\frac{3\pi}{4}} - \cancel{\frac{\pi^2}{16}} - \sin\frac{\pi}{2} + \frac{9}{2} - \frac{9}{4} - \cancel{\frac{3\pi}{4}} + \cancel{\frac{\pi^2}{16}} = \boxed{1\frac{1}{4} u^2}$

3. The curve $x = 10 - y^2$ and the line $x = 1$.

$y^2 = 10 - x$

$y = \pm\sqrt{10 - x}$



$10 - y^2 = 1$

$9 = y^2$

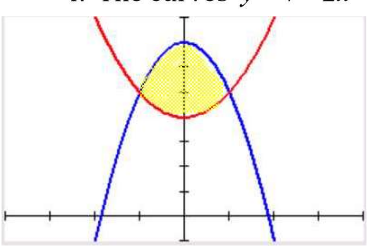
$y = \pm 3$

$A = \int_{-3}^{3} (10 - y^2 - 1)\, dy = \int_{-3}^{3} (9 - y^2)\, dy$

$= 2\int_{0}^{3} (9 - y^2)\, dy = 2\left(9y - \frac{y^3}{3}\right)\Big|_{0}^{3} = 2(27 - 9)$

$= \boxed{36 u^2}$

4. The curves $y = 7 - 2x^2$ and $y = x^2 + 4$.



$7 - 2x^2 = x^2 + 4$

$3 = 3x^2$

$x^2 = 1$

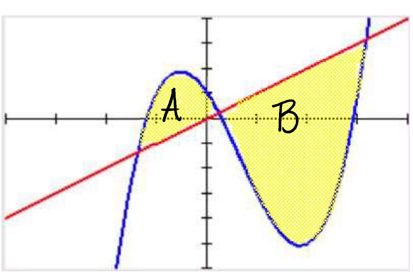
$x = \pm 1$

$A = 2\int_{0}^{1} (7 - 2x^2) - (x^2 + 4)\, dx$

$= 2\int_{0}^{1} (-3x^2 + 3)\, dx = 2\left(-x^3 + 3x\right)\Big|_{0}^{1}$

$= 2(-1 + 3) = \boxed{4 u^2}$

5. The line $y = x$ and the curve $y = x^3 - 2x^2 - 3x + 1$. (Hint: There are two regions.)



$x^3 - 2x^2 - 3x + 1 = x$

Point of Int on GC:

$x = -1.391, 0.227, 3.164$

$A = \underbrace{\int_{-1.391}^{0.227} (x^3 - 2x^2 - 3x + 1 - x)\, dx}_{A} + \underbrace{\int_{0.227}^{3.164} x - (x^3 - 2x^2 - 3x + 1)\, dx}_{B}$

$= \int_{-1.391}^{0.227} (x^3 - 2x^2 - 4x + 1)\, dx + \int_{0.227}^{3.164} (-x^3 + 2x^2 + 4x - 1)\, dx = \boxed{15.684 u^2}$

$x = y(3-y)$

6. The curve $x = 3y - y^2$ and the line $x + y = 3$.

$\Rightarrow x = 3 - y$

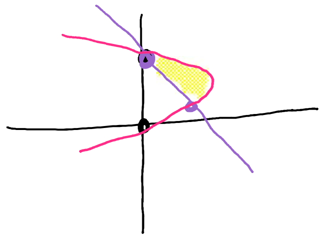
$y = -x + 3$

$3y - y^2 = 3 - y$

$0 = y^2 - 4y + 3$

$0 = (y-3)(y-1)$

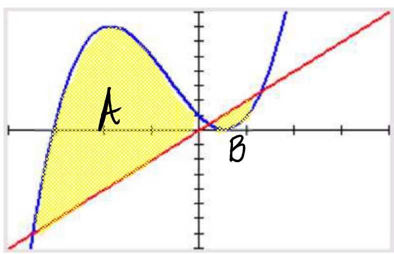
$y = 3, 1$



$$A = \int_1^3 3y - y^2 - (3-y)\,dy = \int_1^3 (4y - y^2 - 3)\,dy$$

$$= \left(2y^2 - \frac{y^3}{3} - 3y\right)\Big|_1^3 = \cancel{2(9) - 9 - 9} - \left(2 - \frac{1}{3} - 3\right) = -2 + \frac{1}{3} + 3 = \boxed{\frac{4}{3}u^2}$$

7. The line $y = 2x$ and the curve $y = x^3 + 2x^2 - 3x + 1$. (Hint: There are two regions.)



Point of Int (on GC):

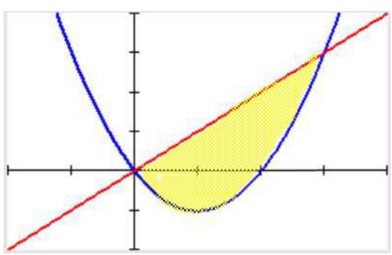
$x = -3.507$, 0.222, 1.285

$$A = \underbrace{\int_{-3.507}^{0.222} (x^3 + 2x^2 - 3x + 1 - 2x)\,dx}_{A} + \underbrace{\int_{0.222}^{1.285} (2x - (x^3 + 2x^2 - 3x + 1))\,dx}_{B}$$

$$A = \int_{-3.507}^{0.222} (x^3 + 2x^2 - 5x + 1)\,dx + \int_{0.222}^{1.285} (-x^3 - 2x^2 + 5x - 1)\,dx$$

$$= \boxed{26.153\,u^2}$$

8. The curve $y = x^2 - 2x$ and the line $y = x$.



$x^2 - 2x = x$

$x^2 - 3x = 0$

$x(x-3) = 0$

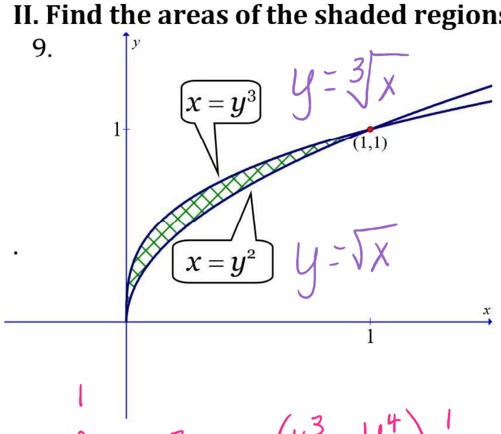
$x = 0, 3$

$$A = \int_0^3 x - (x^2 - 2x)\,dx = \int_0^3 (3x - x^2)\,dx$$

$$= \left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^3 = \frac{3}{2}(9) - 9 = \frac{27}{2} - \frac{18}{2} = \boxed{\frac{9}{2}u^2}$$

II. Find the areas of the shaded regions:

9.



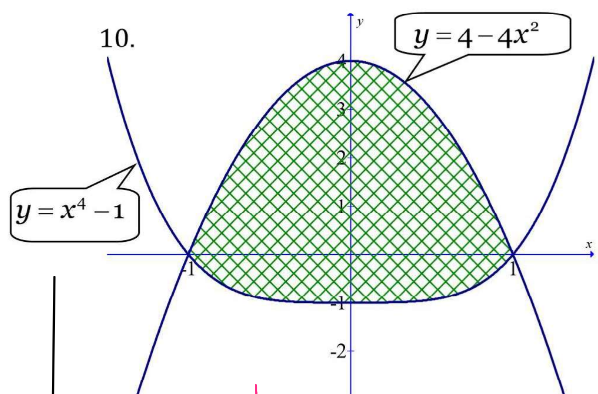
$$A = \int_0^1 y^2 - y^3\,dy = \left(\frac{y^3}{3} - \frac{y^4}{4}\right)\Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}u^2}$$

OR $$A = \int_0^1 (x^{1/3} - x^{1/2})\,dx = \left(\frac{3}{4}x^{4/3} - \frac{2}{3}x^{3/2}\right)\Big|_0^1$$

$$= \frac{3}{4} - \frac{2}{3} = \boxed{\frac{1}{12}u^2}$$

10.



$$A = 2\int_0^1 (4 - 4x^2) - (x^4 - 1)\,dx$$

$$= 2\int_0^1 (5 - 4x^2 - x^4)\,dx = 2\left(5x - \frac{4}{3}x^3 - \frac{x^5}{5}\right)\Big|_0^1$$

$$= 2\left(5 - \frac{4}{3} - \frac{1}{5}\right) = \boxed{\frac{104}{15}u^2}$$