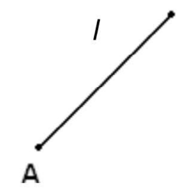
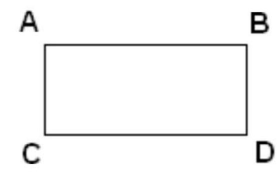
Do Now: Use the shapes to the right to answer questions #1 and 2.

1) What shape is formed when line l is rotated around point A? If line l has a length of 5 inches, what is the area of the shape formed when it makes one complete rotation around point A?



2) If rectangle ABCD is rotated 360° around $\overline{CD}$, what shape is formed? What volume does this shape have if $\overline{CD}$ is 8 meters long and $\overline{AC}$ is 3 meters long? What volume does this shape have if rectangle ABCD is rotated around $\overline{AC}$?



Notes:

$$\textbf{Volume of solid} = \pi\int_a^b [R(x)]^2\,dx.$$

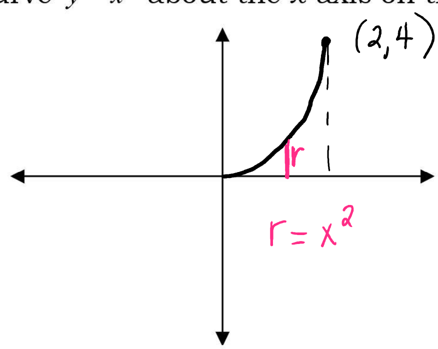
Examples

1) Find the volume of the solid formed by revolving the curve $y = x^2$ about the x-axis on the interval [0, 2].

$$V = \pi\int_0^2 (x^2)^2\,dx$$

$$V = \pi\int_0^2 x^4\,dx = \pi\frac{x^5}{5}\Big|_0^2$$

$$= \pi\left(\frac{32}{5} - 0\right) = \boxed{\frac{32\pi}{5}}$$

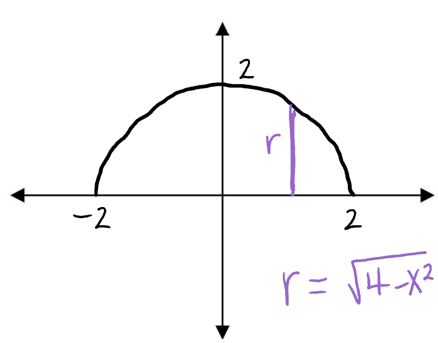


2) Find the volume of the solid formed by revolving the region between the curve $y = \sqrt{4 - x^2}$ and the x-axis about the x-axis.

$$V = \pi\int_{-2}^{2}\left(\sqrt{4-x^2}\right)^2 dx = 2\pi\int_0^2 (4 - x^2)\,dx$$

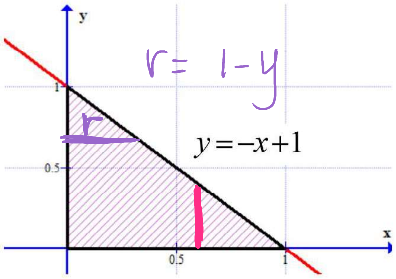
$$V = 2\pi\left(4x - \frac{x^3}{3}\right)\Big|_0^2 = 2\pi\left(8 - \frac{8}{3} - 0\right)$$

$$= 2\pi\left(\frac{16}{3}\right) = \boxed{\frac{32\pi}{3}}$$



3) Set-up and evaluate the integral that gives the volume of the solid formed by revolving the region about the given axes.

a)



$r = 1-y$

$r=-x+1$

About the y-axis:

$r = 1-y$

$V=\pi\int_0^1 (1-y)^2\,dy = \pi\int_0^1 (1-2y+y^2)\,dy$

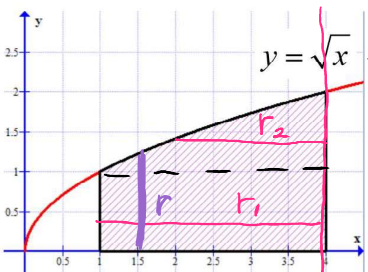
$= \pi\left(y-y^2+\frac{y^3}{3}\right)\Big|_0^1 = \pi\left(1-1+\frac{1}{3}\right) = \boxed{\frac{\pi}{3}}$

About the x-axis:

$V=\pi\int_0^1 (-x+1)^2\,dx = \pi\int_0^1 (x^2-2x+1)\,dx$

$= \pi\left(\frac{x^3}{3}-x^2+x\right)\Big|_0^1 = \pi\left(\frac{1}{3}-1+1\right) = \boxed{\frac{\pi}{3}}$

b)



$\Rightarrow x=y^2$

$r=\sqrt{x}$

$r_1 = 4-1=3$

$r_2 = 4-x$

$= 4-y^2$

About the x-axis:

$V=\pi\int_1^4 (\sqrt{x})^2\,dx = \pi\int x\,dx = \pi\left(\frac{x^2}{2}\right)\Big|_1^4$

$= \pi\left(\frac{16}{2}-\frac{1}{2}\right) = \boxed{\frac{15\pi}{2}}$

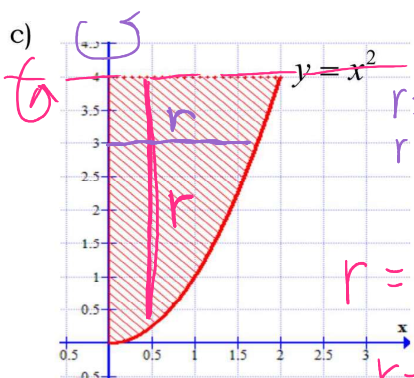
About the line $x=4$:

$V=\pi\int_0^1 3^2\,dy + \pi\int_1^2 (4-y^2)^2\,dy$

$V=\pi\int_0^1 9\,dy + \pi\int_1^2 (16-8y^2+y^4)\,dy$

$V=\pi(9y)\Big|_0^1 + \pi\left(16y-\frac{8}{3}y^3+\frac{y^5}{5}\right)\Big|_1^2 = \boxed{\frac{188\pi}{15}}$

c)



$r=x$

$r=\sqrt{y}$

$r=4-y$

$r=4-x^2$

About the y-axis:

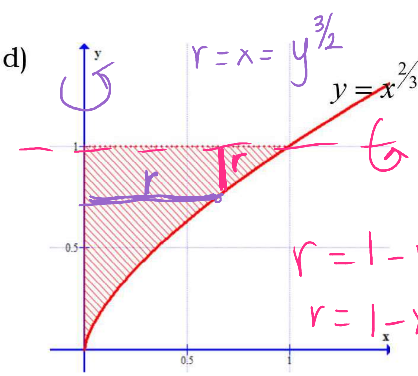
$V=\pi\int_0^4 (\sqrt{y})^2\,dy = \pi\int_0^4 y\,dy = \pi\left(\frac{y^2}{2}\right)\Big|_0^4 = \boxed{8\pi}$

About the line $y=4$:

$V=\pi\int_0^2 (4-x^2)^2\,dx = \pi\int_0^2 (16-8x^2+x^4)\,dx$

$= \pi\left(16x-\frac{8}{3}x^3+\frac{x^5}{5}\right)\Big|_0^2 = \boxed{\frac{256\pi}{15}}$

d)



$r=x=y^{3/2}$

$r=1-y$

$r=1-x^{2/3}$

About the y-axis:

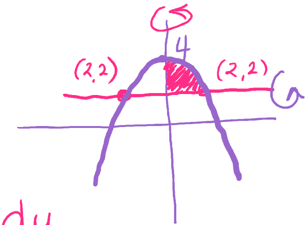
$V=\pi\int_0^1 (y^{3/2})^2\,dy = \pi\int_0^1 y^3\,dy = \pi\frac{y^4}{4}\Big|_0^1 = \boxed{\frac{\pi}{4}}$

About the line $y=1$:

$V=\pi\int_0^1 (1-x^{2/3})^2\,dx = \pi\int_0^1 (1-2x^{2/3}+x^{4/3})\,dx$

$= \pi\left(x-\frac{6}{5}x^{5/3}+\frac{3}{7}x^{7/3}\right)\Big|_0^1 = \boxed{\frac{8\pi}{35}}$

e) The region bounded by $y = 2$, $y = 4 - \frac{x^2}{2}$ & the y-axis revolved about:



i) the y-axis

$2 = 4 - \frac{x^2}{2}$

$-2 = -\frac{x^2}{2}$

$-4 = -x^2$

$x = \pm 2$

$\frac{x^2}{2} = 4 - y$

$x^2 = 8 - 2y$

$x = \pm\sqrt{8-2y}$

$x = \sqrt{8-2y}$

$r = x$

$r = \sqrt{8-2y}$

$V = \pi \int_2^4 (\sqrt{8-2y})^2 \, dy$

$V = \pi \int_2^4 (8-2y)\,dy = \pi\left(8y - y^2\right)\Big|_2^4$

$= \pi(16 - 12) = \boxed{4\pi}$

ii) $y = 2$

$r = 4 - \frac{x^2}{2} - 2$

$r = 2 - \frac{x^2}{2}$

$V = \pi \int_2^4 \left(2 - \frac{x^2}{2}\right)^2 dx$

$V = \pi \int_2^4 \left(4 - 2x^2 + \frac{x^4}{4}\right) dx$

$V = \pi \left(4x - \frac{2x^3}{3} + \frac{x^5}{20}\right)\Big|_2^4 = \boxed{\frac{304\pi}{15}}$