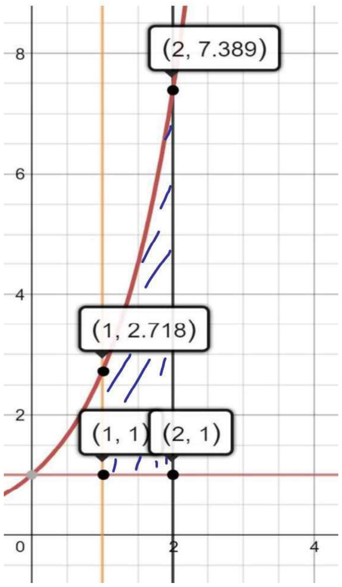
Do Now:

Set-up, <u>but do not solve</u>, the integral that will determine the volume of the solid obtained by rotating the region bounded by $y = e^x$, $x = 1$, $x = 2$, $y = 1$ around. . .

a) the x-axis **b)** the y-axis **c)** $x = 2$ **d)** $y = 1$ **e)** $x = 4$ **f)** $y = -2$



a) $\pi\int_1^2 (e^x)^2 - 1^2\,dx = \pi\int_1^2 e^{2x} - 1\,dx$

b) $\pi\int_1^e 2^2 - 1^2\,dy + \pi\int_e^{e^2} 2^2 - (\ln y)^2\,dy$

c) $\pi\int_1^e 1^2\,dy + \pi\int_e^{e^2} (2 - \ln y)^2\,dy$

d) $\pi\int_1^2 (e^x - 1)^2\,dx$

e) $\pi\int_1^e 3^2 - 2^2\,dy + \pi\int_e^{e^2} (4 - \ln y)^2 - 2^2\,dy$

f) $\pi\int_1^2 (e^x + 2)^2 - 3^2\,dx$

Directions: Show all work! *<u>A calculator is only allowed for #4a, 5 & 7.</u>*

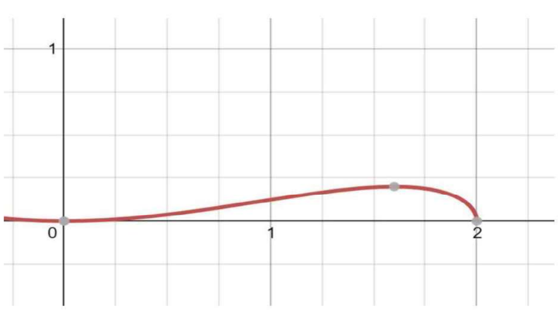
1) A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{8}x^2\sqrt{2-x}$ and the x-axis about the x-axis, where x and y are measured in meters. Find the tank's volume.

$x \le 2$ $\quad$ $x \ge 0$

$$\pi\int_0^2 \left(\frac{1}{8}x^2\sqrt{2-x}\right)^2 dx = \frac{\pi}{64}\int_0^2 x^4(2-x)\,dx = \frac{\pi}{64}\int_0^2 2x^4 - x^5\,dx = \frac{\pi}{64}\left[\frac{2}{5}x^5 - \frac{1}{6}x^6\right]_0^2$$

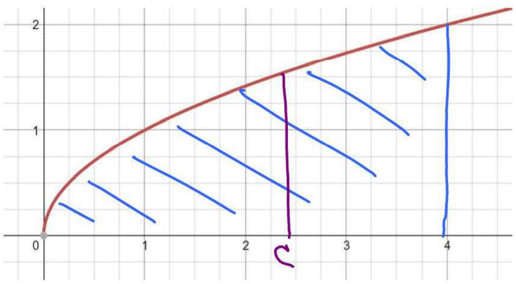
$$= \frac{\pi}{64}\left(\frac{2}{5}\cdot 32 - \frac{1}{6}\cdot 64\right) = \frac{\pi}{64}\left(\frac{64}{5} - \frac{32}{3}\right) = \frac{\pi}{64}\left(\frac{192-160}{15}\right)$$

$$= \frac{\pi}{\cancel{64}_{\,2}}\left(\frac{\cancel{32}^{\,1}}{15}\right) = \frac{\pi}{30}$$



2) The region bounded by $y=\sqrt{x}$, $y=0$, $x=0$, $x=4$ is revolved about the x-axis. Find the value of x in the interval [0,4] that divides the solid into two parts of equal volume.

$\pi\int_0^4 (\sqrt{x})^2\,dx = \pi\int_0^4 x\,dx = \pi\left[\frac{1}{2}x^2\right]_0^4 = \frac{\pi}{2}\cdot 16 = 8\pi$



$\pi\int_0^c (\sqrt{x})^2\,dx = 4\pi$

$\pi\int_0^c x\,dx = 4\pi$

$\pi\left[\frac{1}{2}x^2\right]_0^c = 4\pi$

$\frac{1}{2}c^2 = 4$

$c^2 = 8$

$c = \pm 2\sqrt{2}$

$\boxed{2\sqrt{2}}$

3) Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

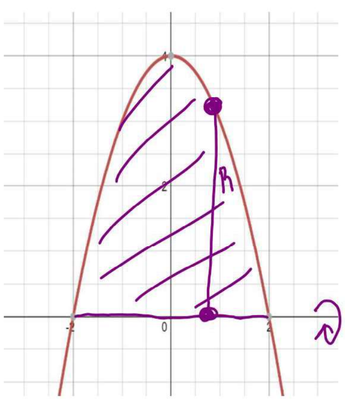
a) $y=4-x^2$, $y=0$

$\pi\int_{-2}^{2}(4-x^2)^2\,dx = 2\pi\int_0^2 16-8x^2+x^4\,dx$

$= 2\pi\left[16x-\frac{8}{3}x^3+\frac{x^5}{5}\right]_0^2$

$= 2\pi\left[32-\frac{8\cdot 8}{3}+\frac{32}{5}\right]$

$= \frac{512\pi}{15}$



b) $y=2$, $4-\frac{x^2}{4}$

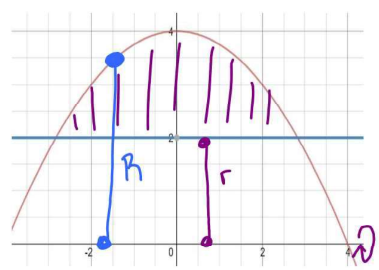
$4-\frac{x^2}{4}=2$

$16-x^2=8$

$x^2=8$

$x=\pm 2\sqrt{2}$

$2\pi\int_0^{2\sqrt{2}}\left(4-\frac{x^2}{4}\right)^2-2^2\,dx = 132.694$



4) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y=4$.

a) $y=\sec x$, $y=0$, $0\le x\le \frac{\pi}{3}$

$\pi\int_0^{\frac{\pi}{3}} 4^2-(4-\sec x)^2\,dx = 27.657$

b) $y=\frac{1}{2}x^3$, $y=4$, $x=0$

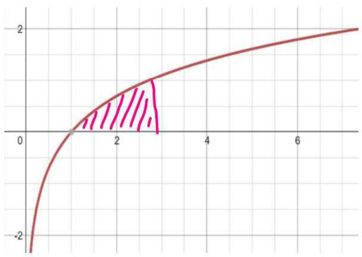
$\pi\int_0^2 \left(4-\frac{1}{2}x^3\right)^2\,dx = 14.362$

5) Region G is bounded by the curve $y = \ln x$, $x = e$, and the x-axis. Order from smallest to largest the volumes determined when G is rotated about the axes:

I. $y = 0$ II. $y = 1$ III. $y = e$

a) III < II < I b) III < I < II c) II < I < III d) I < II < III e) I < III < II

$y=0$ $\pi\int_1^e (\ln x)^2 dx = 2.257$



$y=1$ $\pi\int_1^e 1^2 - (1-\ln x)^2 dx = 4.027$

$y=e$ $\pi\int_1^e e^2 - (e-\ln x)^2 dx = 14.823$

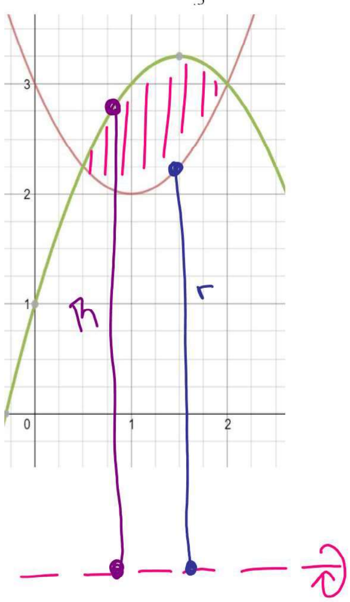
6) Which of the following expressions represents the volume of the solid generated when the region bounded by $y = x^2 - 2x + 3$ and $y = 1 + 3x - x^2$ is rotated about the line $y = -2$?

a) $V = \pi\int_{.5}^{2}\left[\left(1+3x-x^2\right)-\left(x^2-2x+3\right)\right]dx$

b) $V = \pi\int_{.5}^{2}\left[\left(-1+3x-x^2\right)^2-\left(x^2-2x+1\right)^2\right]dx$

c) $V = \pi\int_{.5}^{2}\left[\left(3+3x-x^2\right)^2-\left(x^2-2x+5\right)^2\right]dx$

d) $V = \pi\int_{.5}^{2}\left[\left(3+3x-x^2\right)-\left(x^2-2x+5\right)\right]dx$



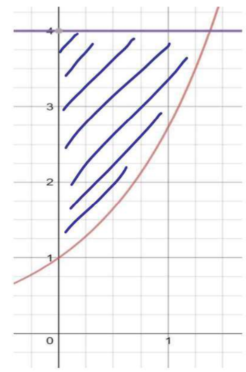
e) $V = \pi\int_{.5}^{2}\left[\left(1+3x-x^2\right)-\left(x^2-2x+3\right)\right]^2 dx$

$x^2-2x+3 = 1+3x-x^2$
$2x^2-5x+2=0$
$(2x-1)(x-2)=0$
$x=\frac{1}{2}$ $x=2$

$\pi\int_{1/2}^{2} (1+3x-x^2-(-2))^2 - (x^2-2x+3-(-2))^2 dx$

$\pi\int_{1/2}^{2} (3+3x-x^2)^2 - (x^2-2x+5)^2 dx$

7) Let R be the region bounded by $f(x) = e^x + 1$, the y-axis and $y = 4$.



$e^x + 1 = 4$

$e^x = 3$

$x = \ln 3$

a) Find the area of the region R.

$$\int_0^{\ln 3} 4 - e^x - 1\,dx = \int_0^{\ln 3} 3 - e^x\,dx$$

$$= 3x - e^x \Big]_0^{\ln 3}$$

$$= 3\ln 3 - e^{\ln 3} - 0 + e^0 = \ln 27 - 3 + 1$$

$$= \ln 27 - 2$$

b) A vertical line $x = h$, where $h > 0$ is chosen so that the area of the region bounded by $f(x)$, the y-axis, $y = 4$, and $x = h$ is half the area of region R. What is the value of h?

$$\int_0^h 4 - (e^x + 1)\,dx = \frac{\ln 27 - 2}{2}$$

$$\int_0^h 3 - e^x\,dx = \frac{\ln 27 - 2}{2}$$

$$3x - e^x \Big]_0^h = \frac{\ln 27 - 2}{2}$$

$$3h - e^h + e^0 = \frac{\ln 27 - 2}{2}$$

$$3h - e^h + 1 = \frac{\ln 27 - 2}{2}$$

$h = .3608,\ 1.6911$

$$\boxed{h = .3608}$$

c) Find the volume of the solid formed when region R is rotated about the line $y = 4$..

$$\pi \int_0^{\ln 3} (4 - e^x - 1)^2\,dx = \pi \int_0^{\ln 3} (3 - e^x)^2\,dx = 5.930$$

d) A horizontal line $y = k$, where $k > 4$, is chosen so that the volume of the solid formed when region R is rotated about $y = k$ is twice the volume of the solid in part *c*. Set up, but **do not evaluate**, an integral expression that represents the volume of the solid.

$$\pi \int_0^{\ln 3} (k - e^x - 1)^2 - (k - 4)^2\,dx = 2\pi \int_0^{\ln 3} (3 - e^x)^2\,dx$$