Do Now:

Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 1$ with <u>semicircle</u> cross sections taken perpendicular to the x-axis.

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

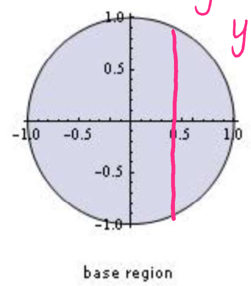
d

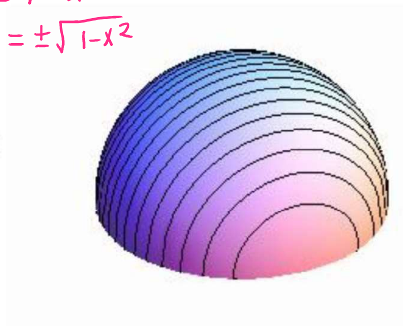
$d = 2y$

$d = 2\sqrt{1-x^2}$

$A = \frac{\pi}{2} r^2 = \frac{\pi}{2}(\sqrt{1-x^2})$

$A = \frac{\pi}{2}(1-x^2)$





$V = \int_{-1}^{1} \frac{\pi}{2}(1-x^2)\,dx$

OR

$V = 2 \cdot \frac{\pi}{2} \int_0^1 (1-x^2)\,dx = \pi\left(x - \frac{x^3}{3}\right)\Big|_0^1 = \pi\left(1 - \frac{1}{3}\right)$

$V = \boxed{\frac{2\pi}{3}}$

Class Work:

For cross sections of area $A(x)$ taken perpendicular to the x-axis: $Volume = \int_a^b A(x)dx$.

For cross sections of area $A(y)$ taken perpendicular to the y-axis: $Volume = \int_c^d A(y)dy$

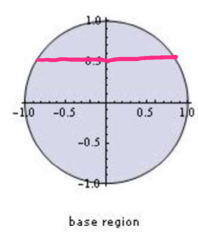
1) Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 1$ with <u>square</u> cross sections taken perpendicular to the y-axis.

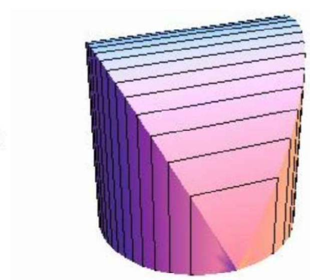
s

$s = 2x$

$V = s^2 = 4x^2$

$V = 4(1-y^2)$





$V = \int_{-1}^{1} 4(1-y^2)\,dy$

OR

$V = 2 \cdot 4 \int_0^1 (1-y^2)\,dy = 8\left(y - \frac{y^3}{3}\right)\Big|_0^1 = 8\left(1 - \frac{1}{3}\right)$

$V = \boxed{\frac{16}{3}}$

2) Find the volume of the solid whose base is the region bounded by $f(x)=2\left(x-\frac{1}{2}\right)^2-\frac{1}{2}$ and $y=x$ on the interval $[0,1]$ with <u>semicircle</u> cross sections taken perpendicular to the x-axis.

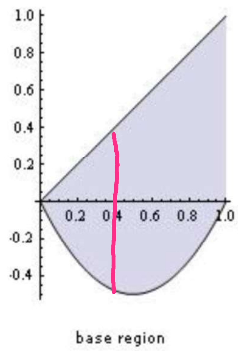
$d = x - \left[2\left(x-\frac{1}{2}\right)^2-\frac{1}{2}\right]$

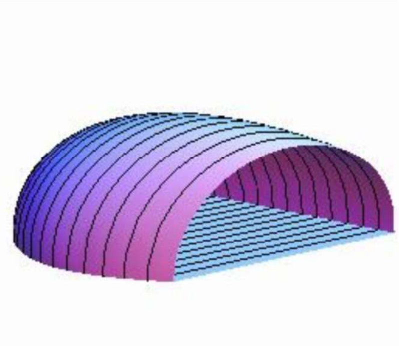
$d = x - 2\left(x-\frac{1}{2}\right)^2+\frac{1}{2}$

$A = \frac{\pi}{2}r^2$

$A = \frac{\pi}{2}\left(\frac{x-2\left(x-\frac{1}{2}\right)^2+\frac{1}{2}}{2}\right)^2$

$A = \frac{\pi}{8}\left(x-2\left(x-\frac{1}{2}\right)^2+\frac{1}{2}\right)^2$





$V = \frac{\pi}{8}\int_0^1 \left(x-2\left(x-\frac{1}{2}\right)^2+\frac{1}{2}\right)^2 dx$

$x^2 - x + \frac{1}{4}$

$V = \frac{\pi}{8}\int_0^1 \left(x - 2x^2 + 2x - \frac{1}{2} + \frac{1}{2}\right)^2 = \frac{\pi}{8}\int_0^1 (3x-2x^2)^2 dx$

$V = \frac{\pi}{8}\int_0^1 (9x^2 - 12x^3 + 4x^4)\,dx = \frac{\pi}{8}\left(3x^3 - 3x^4 + \frac{4}{5}x^5\right)\Big|_0^1 = \boxed{\frac{\pi}{10}}$

3) The base of a solid is the region between the curve $y=2\sqrt{\sin x}$ and the interval $[0,\pi]$ on the x-axis. The cross sections parallel to the y-axis are equilateral triangle with bases running from the x-axis to the curve as shown below. Find the volume of the region.

$A = \frac{\sqrt{3}}{4}s^2$

$S = 2\sqrt{\sin x}$

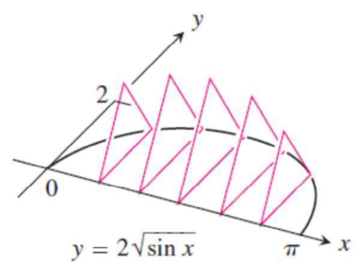
$A = \frac{\sqrt{3}}{4}\left(2\sqrt{\sin x}\right)^2$

$A = \sqrt{3}\sin x$

$V = \int_0^{\pi} \sqrt{3}\sin x\, dx$

$V = \sqrt{3}\left(-\cos x\right)\Big|_0^{\pi}$

$V = \sqrt{3}\left(-\cos\pi + \cos 0\right) = \boxed{2\sqrt{3}}$



4) A solid has its base given by the region bounded by the lines $x+2y=6$, $x=0$ and $y=0$; and the cross sections taken perpendicular to the:

$2y = 6 - x$

$y = 3 - \frac{x}{2}$

a) x-axis are semi-circles. Find the volume of the solid.

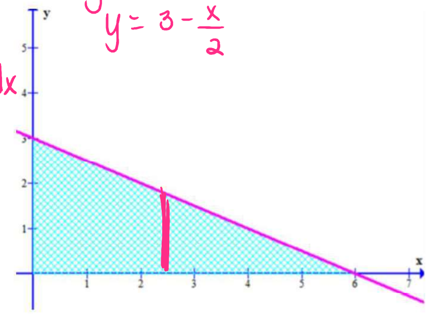
$d = 3 - \frac{x}{2}$

$A = \frac{\pi r^2}{2} = \frac{\pi}{2}\left(\frac{3-\frac{x}{2}}{2}\right)^2$

$A = \frac{\pi}{8}\left(3 - \frac{x}{2}\right)^2$

$V = \int_0^6 \frac{\pi}{8}\left(3 - \frac{x}{2}\right)^2 dx = \frac{\pi}{8}\int_0^6 \left(9 - 3x + \frac{x^2}{4}\right)dx$

$= \frac{\pi}{8}\left(9x - \frac{3}{2}x^2 + \frac{x^3}{12}\right)\Big|_0^6 = \boxed{\frac{9\pi}{4}}$



b) y-axis are circles. Find the volume of the solid.

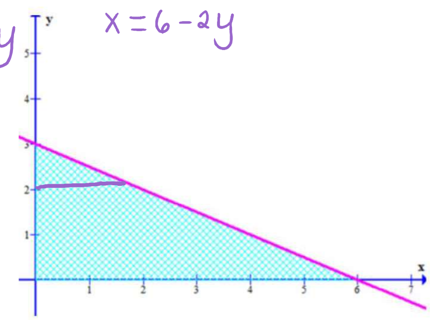
$d = 6 - 2y$

$x = 6 - 2y$

$A = \pi r^2 = \pi(3-y)^2$

$V = \int_0^3 \pi(3-y)^2 dy = \pi\int_0^3 \left(9 - 6y + y^2\right)dy$

$V = \pi\left(9y - 3y^2 + \frac{y^3}{3}\right)\Big|_0^3 = \boxed{9\pi}$



5) A solid has its base given by the region bounded by the lines $x+y=4$, $x=0$ and $y=0$; and the cross section is parallel to the:

a) y-axis are equilateral triangles. Find the volume of the solid.

$s = 4 - x$

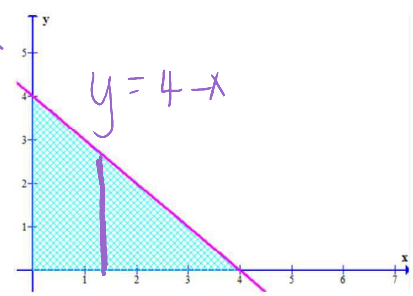
$y = 4 - x$

$A = \frac{\sqrt{3}}{4}s^2$

$A = \frac{\sqrt{3}}{4}(4-x)^2$

$V = \int_0^4 \frac{\sqrt{3}}{4}(4-x)^2 d = \frac{\sqrt{3}}{4}\int_0^4 \left(16 - 8x + x^2\right)dx$

$V = \frac{\sqrt{3}}{4}\left(16x - 4x^2 + \frac{x^3}{3}\right)\Big|_0^4 = \boxed{\frac{16\sqrt{3}}{3}}$



b) x-axis are isosceles right triangles with *each strip* being the hypotenuse of the triangle. Find the volume of the solid.

$\frac{s}{\sqrt{2}}$, h, $45°$, s

$\cancel{\tan 45°} = \frac{h}{s/2}$, 1

$\frac{s}{2} = h$

$x = 4 - y$

$A = \frac{1}{2}s \cdot \frac{s}{2}$

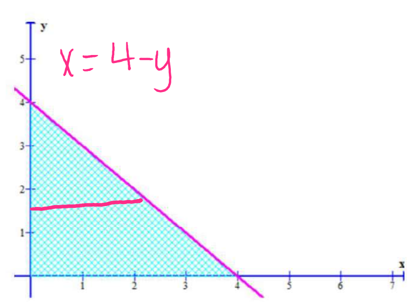
$A = \frac{s^2}{4} = \frac{(4-y)^2}{4}$

$V = \int_0^4 \frac{1}{4}(4-y)^2 dy$

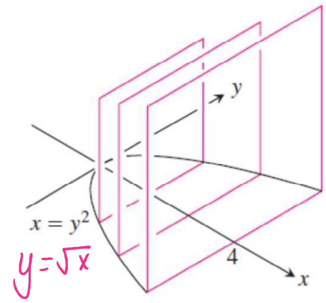
$V = \frac{1}{4}\int_0^4 \left(16 - 8y + y^2\right)dy$

$V = \frac{1}{4}\left(16y - 4y^2 + \frac{y^3}{3}\right)\Big|_0^4$

$V = \frac{1}{4} \cdot \frac{64}{3} = \boxed{\frac{16}{3}}$



6) The solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections perpendicular to the x-axis between these planes run from $y = \sqrt{x}$ and $y = -\sqrt{x}$. Find the volume of this solid if the cross sections are squares with bases in the xy-plane.



s

$S = \sqrt{x} - (-\sqrt{x})$

$S = 2\sqrt{x}$

$A = S^2 = (2\sqrt{x})^2 = 4X$

$V = \int_0^4 (4x)\,dx$

$V = (2x^2)\Big|_0^4$

$V = \boxed{32}$