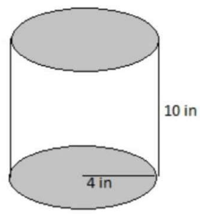
Name Answer Key　　　　　　　　　　　　　　　　　　　　　　Date________________

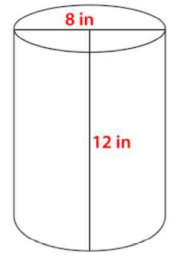
Calc I H – 7.3 Volume by Shell days 1-3　　　　　　　　　　　　　　**Period ____________**

DO NOW:

1. Find the surface area of the cylinders below if the top and bottom are open.

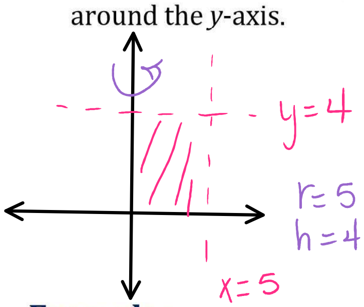


$SA = 2\pi r h$
$= 2\pi(4)(10)$
$= 80\pi\ u^2$



$SA = 2\pi r h$
$= 2\pi(4)(12)$
$= 96\pi\ u^2$

2. Find the volume of a cylinder created by rotating the region bounded by $x = 0$, $x = 5$, $y = 0$, and $y = 4$ around the y-axis.

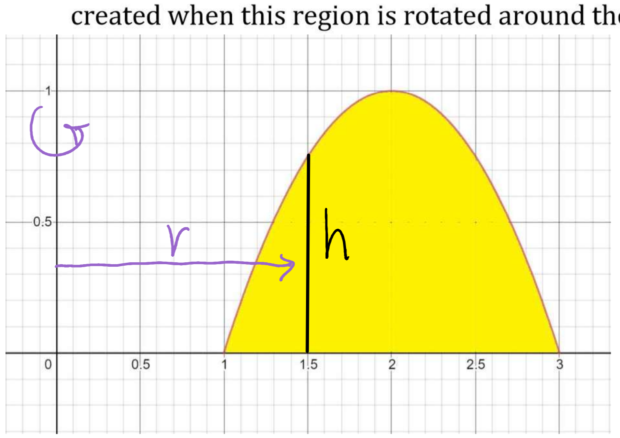


$r=5$
$h=4$

$V = \pi r^2 h$
$= \pi(5)^2(4)$
$= 100\pi\ u^3$

Examples:

1. Sketch the region bounded by $y = -x^2 + 4x - 3$ and the x-axis. What is the volume of the solid that is created when this region is rotated around the y-axis?



$r = x$

$h = -x^2+4x-3$

$V = 2\pi \int_1^3 x(-x^2+4x-3)\,dx$

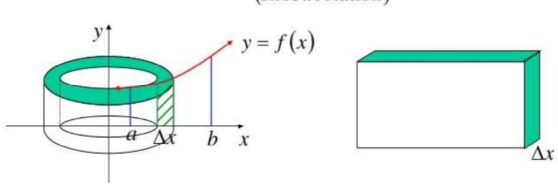
$= 2\pi \int_1^3 (-x^3+4x^2-3x)\,dx$

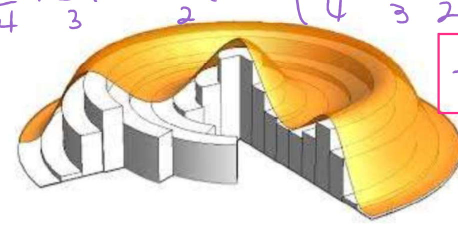
$= 2\pi\left(-\frac{x^4}{4}+\frac{4}{3}x^3-\frac{3}{2}x^2\right)\Big|_1^3$

$= 2\pi\left(\frac{-3^4}{4}+\frac{4}{3}(3)^3-\frac{3}{2}(3)^2-\left(-\frac{1}{4}+\frac{4}{3}-\frac{3}{2}\right)\right) =$ $\boxed{\frac{16\pi}{3}u^3}$

Volumes By Cylindrical Shells

(slice‖rotation)





Volumes of Solids using Shell Method

Rotation around a Vertical Line

$$V = 2\pi\int_c^d \text{radius} \bullet \text{height } dy$$

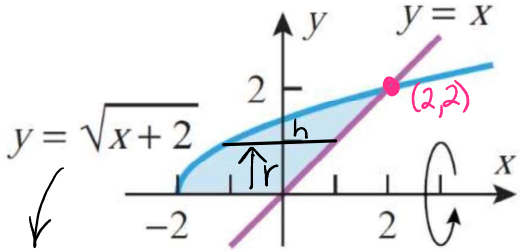
$$V = 2\pi\int_c^d r(y)h(y)\, dy$$

Rotation around a Horizontal Line

$$V = 2\pi\int_a^b \text{radius} \bullet \text{height } dx$$

$$V = 2\pi\int_a^b r(x)h(x)\, dx$$

2. Consider the region below rotated around the x -axis. What is the best method for finding the volume of the solid that is created? What is the volume of that solid?



$y^2 = x+2$

$x = y^2 - 2$

$y^2 - 2 = y$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0 \quad y = 2, -1$

$r = y, \quad h = y - (y^2 - 2) = y - y^2 + 2$

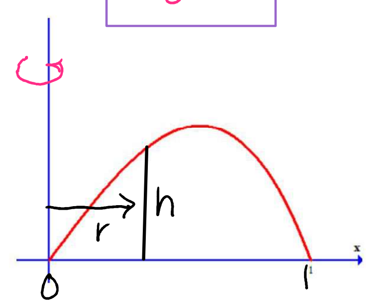
$V = 2\pi\int_0^2 y(y - y^2 + 2)\, dy$

$V = 2\pi\int_0^2 (y^2 - y^3 + 2y)\, dy$

$= 2\pi\left(\frac{y^3}{3} - \frac{y^4}{4} + y^2\right)\Big|_0^2 = 2\pi\left(\frac{2^3}{3} - \frac{2^4}{4} + 2^2 - 0\right)$

$= \boxed{\frac{16\pi}{3} u^3}$

3. Find the volume of the solid of revolution formed by revolving the region bounded by $y = x - x^3$ and the x-axis ($0 \le x \le 1$) about the y-axis.

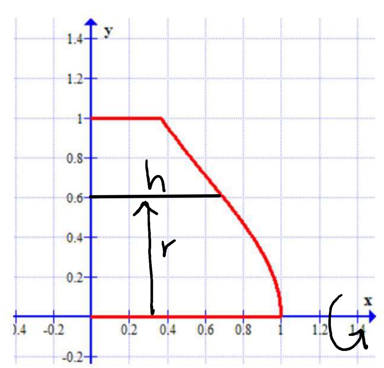


$r = x, \quad h = x - x^3$

$V = 2\pi\int_0^1 x(x - x^3)\, dx = 2\pi\int_0^1 (x^2 - x^4)\, dx$

$= 2\pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = 2\pi\left(\frac{1}{3} - \frac{1}{5} - 0\right) = \boxed{\frac{4\pi}{15} u^3}$

4. Find the volume of the solid of revolution formed by revolving the region bounded by the graph of $x = e^{-y^2}$ and the y-axis ($0 \le y \le 1$) about the x-axis.



$r = y, \quad h = e^{-y^2}$

$V = 2\pi\int_0^1 y e^{-y^2}\, dy$

$u = -y^2$

$\frac{du}{dy} = -2y \Rightarrow dy = \frac{du}{-2y}$

$y = 0, u = 0$

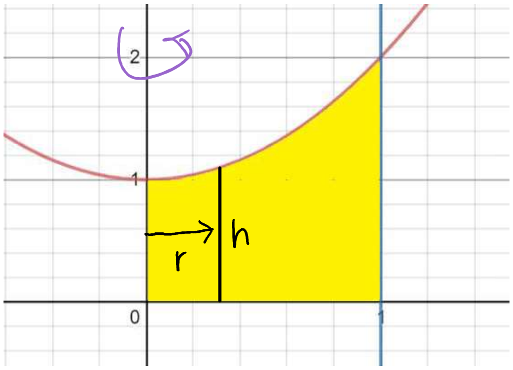
$y = 1, u = -1$

$V = \cancel{2}\pi\int_0^{-1} \cancel{y} e^u \cdot \frac{du}{-\cancel{2}\cancel{y}}$

$V = -\pi\int_0^{-1} e^u\, du = \pi\int_{-1}^0 e^u\, du$

$= \pi e^u\Big|_{-1}^0 = \pi(e^0 - e^{-1}) = \boxed{\pi\left(1 - \frac{1}{e}\right) u^3}$

5. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.



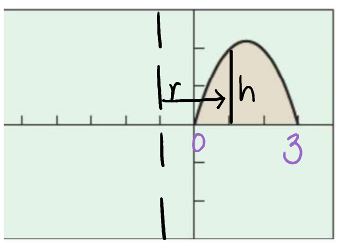
$r = x, \quad h = x^2 + 1$

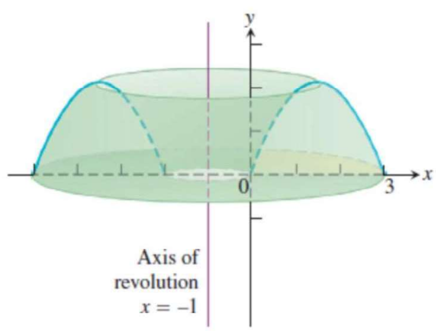
$V = 2\pi \int_0^1 x(x^2+1)\,dx = 2\pi \int_0^1 (x^3 + x)\,dx$

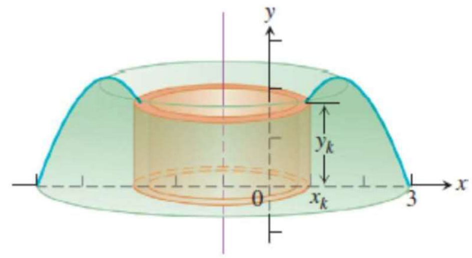
$= 2\pi \left(\frac{x^4}{4} + \frac{x^2}{2} \right)\Big|_0^1 = 2\pi\left(\frac{1}{4} + \frac{1}{2} - 0\right) = 2\pi\left(\frac{3}{4}\right)$

$= \boxed{\frac{3\pi}{2} u^3}$

6. The region enclosed by the x-axis and $f(x) = 3x - x^2$ is revolved about the line $x = -1$ to generate the shape of a bundt cake. Find the volume of the cake.







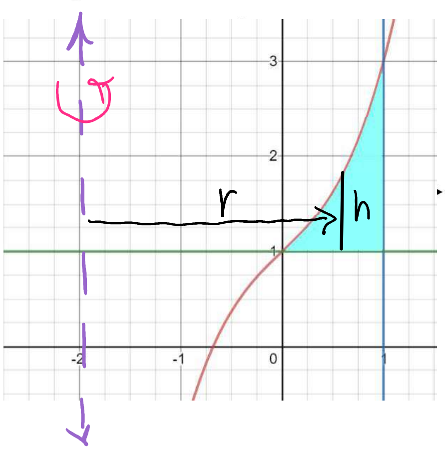
$r = x - (-1) = x + 1$

$h = 3x - x^2$

$V = 2\pi \int_0^3 (x+1)(3x - x^2)\,dx = 2\pi \int_0^3 (2x^2 - x^3 + 3x)\,dx = 2\pi\left(\frac{2}{3}x^3 - \frac{x^4}{4} + \frac{3}{2}x^2\right)\Big|_0^3$

$= 2\pi\left(\frac{2}{3}(3)^3 - \frac{3^4}{4} + \frac{3}{2}(3)^2 - 0\right) = \boxed{22.5\pi\ u^3}$

7. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = -2$.



$r = x - (-2) = x + 2$

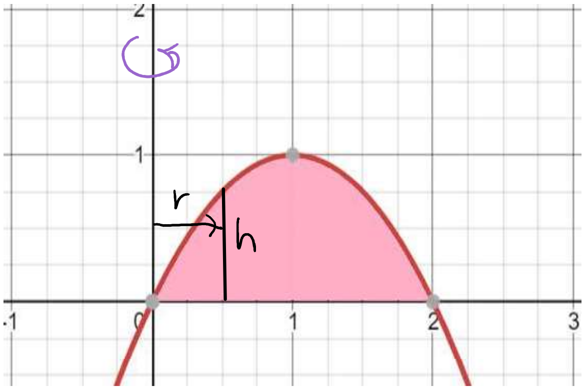
$h = x^3 + x + 1 - 1 = x^3 + x$

$V = 2\pi \int_0^1 (x+2)(x^3 + x)\,dx = 2\pi \int_0^1 (x^4 + 2x^3 + x^2 + 2x)\,dx$

$= 2\pi\left(\frac{x^5}{5} + \frac{x^4}{2} + \frac{x^3}{3} + x^2\right)\Big|_0^1$

$= 2\pi\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{3} + 1 - 0\right) = \boxed{\frac{61\pi}{15} u^3}$

8. Find the volume of the solid obtained by rotating the region bounded by $y = 2x - x^2$ and $y = 0$ about the y-axis.



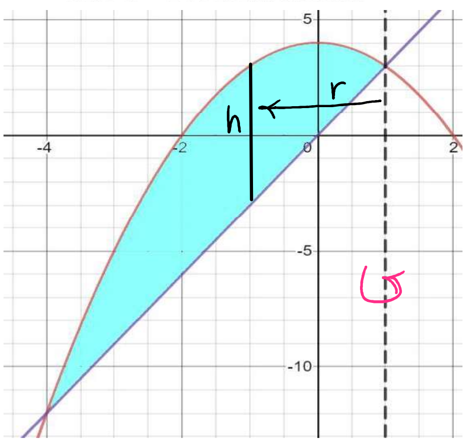
$r = x, \quad h = 2x - x^2$

$V = 2\pi \int_0^2 x(2x - x^2)\,dx = 2\pi \int_0^2 (2x^2 - x^3)\,dx$

$= 2\pi \left(\frac{2}{3}x^3 - \frac{x^4}{4}\right)\Big|_0^2 = 2\pi\left(\frac{2}{3}(2)^3 - \frac{2^4}{4} - 0\right)$

$= \boxed{\frac{8\pi}{3}u^3}$

9. Find the volume of the region bounded by the curves $y = 4 - x^2$, $y = 3x$ when it is revolved about the line $x = 1$ to form a solid.



$r = 1 - x, \quad h = 4 - x^2 - 3x$

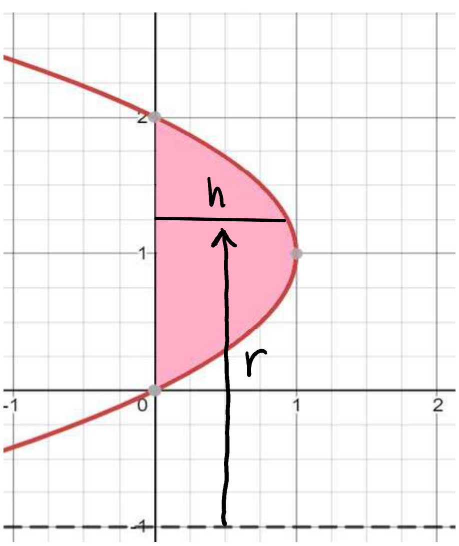
$V = 2\pi \int_{-4}^1 (1 - x)(4 - x^2 - 3x)\,dx = 2\pi \int_{-4}^1 (x^3 + 2x^2 - 7x + 4)\,dx$

$= 2\pi\left(\frac{x^4}{4} + \frac{2}{3}x^3 - \frac{7}{2}x^2 + 4x\right)\Big|_{-4}^1$

$= 2\pi\left(\frac{1}{4} + \frac{2}{3} - \frac{7}{2} + 4 - \left(64 - \frac{128}{3} - 56 - 16\right)\right)$

$= \boxed{104.167\pi\, u^3} = \frac{625}{6}\pi\, u^3$

10. Sketch the region bounded by $x = 2y - y^2$ and the y-axis. What is the volume of the solid that is created when this region is rotated around $y = -1$?



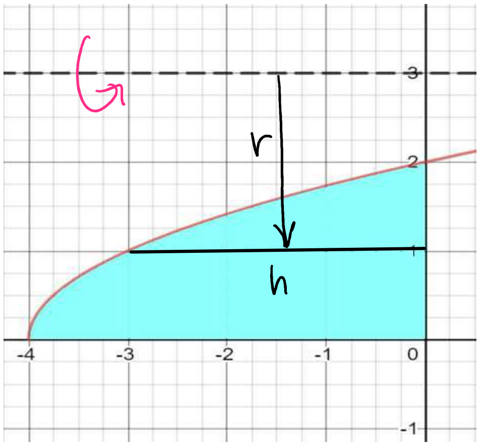
$r = y - (-1) = y + 1, \quad h = 2y - y^2$

$V = 2\pi \int_0^2 (y+1)(2y - y^2)\,dy = 2\pi \int_0^2 (-y^3 + y^2 + 2y)\,dy$

$= 2\pi\left(-\frac{y^4}{4} + \frac{y^3}{3} + y^2\right)\Big|_0^2$

$= 2\pi\left(-\frac{16}{4} + \frac{8}{3} + 4 - 0\right) = \boxed{\frac{16\pi}{3}u^3}$

11. Sketch the region bounded by $y=\sqrt{x+4}$, the y-axis, and the x-axis. What is the volume of the solid that is created when this region is rotated around $y=3$?



$y^2 = x+4$

$x = y^2-4$

$r = 3-y$, $h = 0-(y^2-4)$

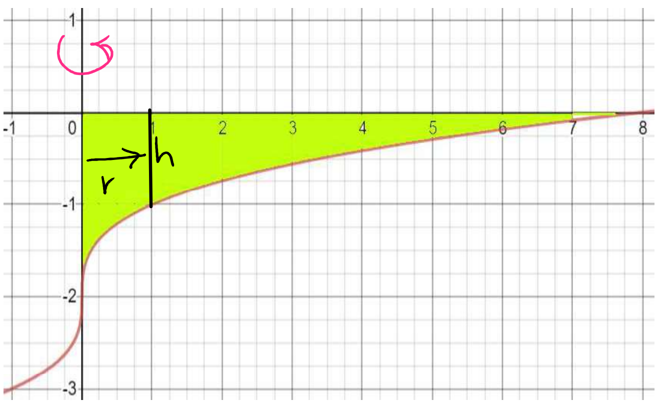
$= -y^2+4$

$V = 2\pi\int_0^2 (3-y)(-y^2+4)\,dy = 2\pi\int_0^2 (y^3-3y^2-4y+12)\,dy$

$= 2\pi\left(\frac{y^4}{4} - y^3 - 2y^2 + 12y\right)\Big|_0^2 = 2\pi\left(\frac{16}{4} - 8 - 8 + 24 - 0\right)$

$= \boxed{24\pi\ u^3}$

12. Sketch the region bounded by $y=x^{1/3}-2$, $y=0$, $x=0$ and $x=8$. What is the volume of the solid that is created when this region is rotated about the y-axis?



$r = x$, $h = 0-(x^{1/3}-2)$

$= -x^{1/3}+2$

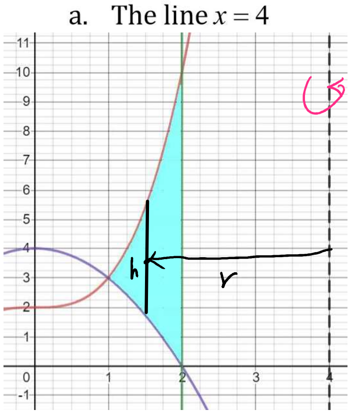
$V = 2\pi\int_0^8 x(-x^{1/3}+2)\,dx = 2\pi\int_0^8 (-x^{4/3}+2x)\,dx$

$= 2\pi\left(-\frac{3}{7}x^{7/3}+x^2\right)\Big|_0^8 = 2\pi\left(-\frac{3}{7}(8)^{7/3}+64-0\right)$

$= \boxed{18.286\pi\ u^3}$

13. Use the best method to find the volume found when the region bounded by $y=x^3+2$, $y=4-x^2$ and $x=2$ is rotated about:

a. The line $x=4$



Shell (dx)

$r = 4-x$

$h = x^3+2-(4-x^2)$

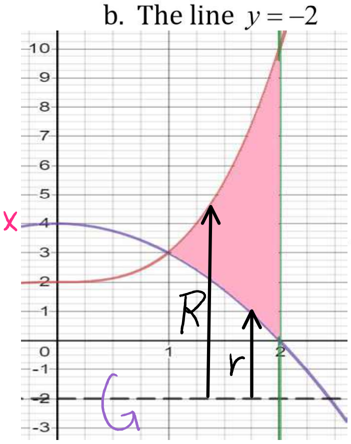
$= x^3+x^2-2$

$V = 2\pi\int_1^2 (4-x)(x^3+x^2-2)\,dx$

$= \boxed{18.767\pi\ u^3}$

$= \frac{563}{30}\pi\ u^3$

b. The line $y=-2$



Washer (dx)

$R = x^3+2-(-2)$

$= x^3+4$

$r = 4-x^2-(-2)$

$= 6-x^2$

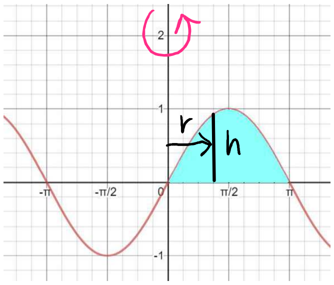
$V = \pi\int_1^2 (x^3+4)^2-(6-x^2)^2\,dx$

$= \boxed{49.943\pi\ u^3}$

Choose the best Method for finding the volume

14. Set-up, but do not solve, an integral to find the volume obtained by rotating the region bounded by $y = \sin x$ and $y = 0$ from $x = 0$ to $x = \pi$ around each line:

a. y-axis

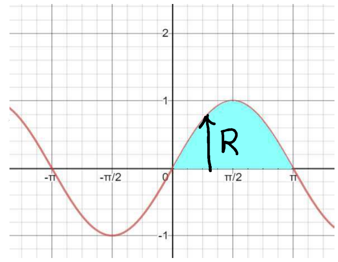


Shell (dx)

$r = x,\ h = \sin x$

$V = 2\pi \int_0^{\pi} x \sin x \, dx$

b. x-axis

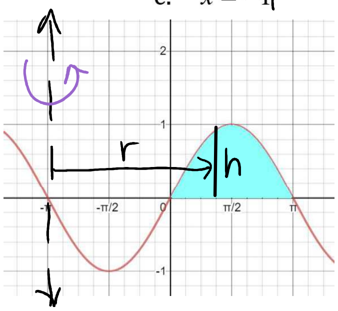


disk (dx)

$R = \sin x$

$V = \pi \int_0^{\pi} \sin^2 x \, dx$

c. $x = -\pi$



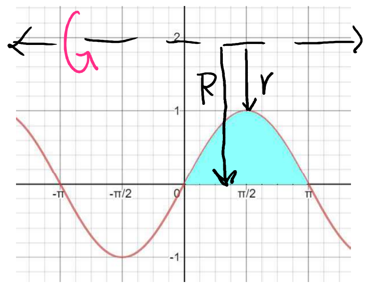
Shell (dx)

$r = x - (-\pi) = x + \pi$

$h = \sin x$

$V = 2\pi \int_0^{\pi} (x+\pi) \sin x \, dx$

d. $y = 2$



washer (dx)

$R = 2,\ r = 2 - \sin x$

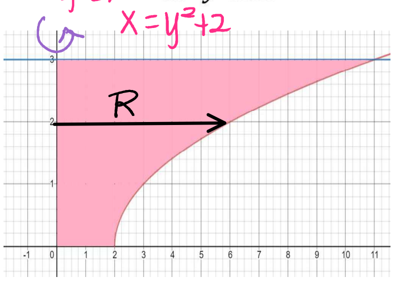
$V = \pi \int_0^{\pi} 2^2 - (2 - \sin x)^2 \, dx$

15. Set-up, but do not solve, an integral to find the volume obtained by rotating the region bounded by $y = \sqrt{x-2}$ and $x = 0$ from $y = 0$ to $y = 3$ around each line:

$y^2 = x - 2$

$x = y^2 + 2$

b. y-axis

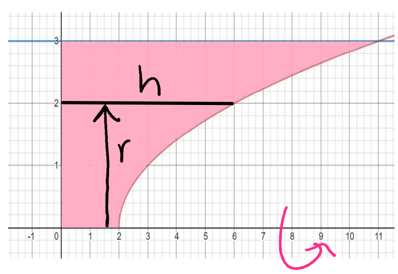


Disk (dy)

$R = y^2 + 2$

$V = \pi \int_0^3 (y^2+2)^2 \, dy$

b. x-axis



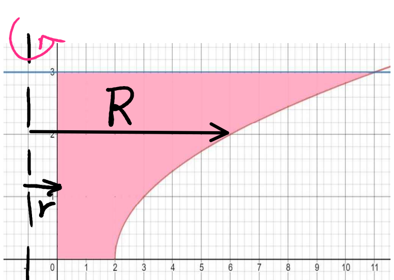
Shell (dy)

$r = y,\ h = y^2 + 2$

$V = 2\pi \int_0^3 y(y^2+2) \, dy$

Note: Possible with disk / washer → 2 integrals

d. $x = -1$



washer (dy)

$R = y^2 + 2 - (-1)$

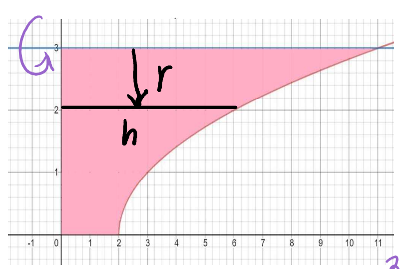
$= y^2 + 3$

$r = 0 - (-1)$

$= 1$

$V = \pi \int_0^3 \left[(y^2+3)^2 - 1^2\right] dy$

d. $y = 3$



Shell (dy)

$r = 3 - y$

$h = y^2 + 2$

$V = 2\pi \int_0^3 (3-y)(y^2+2) \, dy$

NOTE: Possible with disk ⇒ 2 integrals