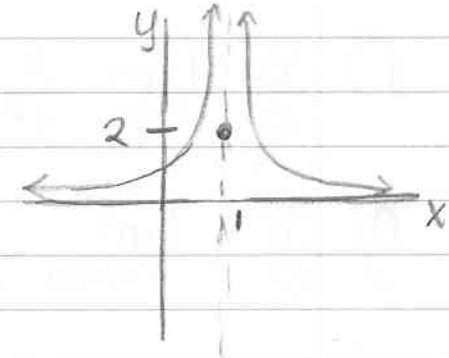
AP Review #1 - OE

1) $g(x) = \begin{cases} \frac{1}{(x-1)^2}, & x \neq 1 \\ 2, & x = 1 \end{cases}$



a) $\lim_{x \to 1^-} g(x) = \infty$

b) $\lim_{x \to 1^+} g(x) = \infty$

c) $\lim_{x \to 1} g(x) = \infty$

g(x) is NOT continuous, Since $\lim_{x \to 1} g(x) = 1 \neq g(1) = 2$.

There is nonremovable, infinite discontinuity.

2) $f(x) = 1 + \frac{3}{x-2} = \frac{x+1}{x-2}$

$\lim_{x \to 2^+} f(x) = \infty \neq \lim_{x \to 2^-} f(x) = -\infty \therefore \lim_{x \to 2} f(x)$ DNE

3) a) $\lim_{x \to 0} \frac{\sin(3x)}{x}$ "$\frac{0}{0}$" apply L.R.

$= \lim_{x \to 0} \frac{3\cos(3x)}{1} = \boxed{3}$

b) "$\frac{0}{0}$" $\lim_{x \to 0} \frac{\cos x - 1}{2x^2} = \lim_{x \to 0} \frac{-\sin x}{4x}$ ("$\frac{0}{0}$")

$= \lim_{x \to 0} \frac{-\cos x}{4} = \boxed{\frac{-1}{4}}$

c) "$\frac{0}{0}$" $\lim_{x \to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} = \lim_{x \to 0} \frac{\frac{-1}{(2+x)^2}}{1} = \boxed{\frac{-1}{4}}$

d) "$\frac{0}{0}$" $\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\sec^2 x}{1} = \boxed{1}$

e) $\lim_{x \to 1} \frac{x^2-1}{x^2-1} = \lim_{x \to 1} 1 = \boxed{1}$

f) $\frac{"0"}{0}$ $\lim_{x\to 0} \frac{x}{x^3} = \lim_{x\to 0} \frac{1}{3x^2} = \boxed{\infty}$

g) $\frac{"0"}{0}$ $\lim_{x\to \pi/4} \frac{1-\tan x}{\sin x - \cos x} = \lim_{x\to \pi/4} \frac{-\sec^2 x}{\cos x + \sin x} = \frac{-(\sqrt{2})^2}{\sqrt{2}}$

$= \boxed{-\sqrt{2}}$

h) $\lim_{x\to 1} \frac{\tan^{-1} x}{\sin^{-1} x + 1} = \frac{(\pi/4)\,4}{(\pi/2+1)\,4} = \boxed{\frac{\pi}{2\pi+4}}$

i) $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2 - 2(x+\Delta x) - (x^2-2x)}{\Delta x}$

Limit definition of derivative of $y = x^2 - 2x$

$\boxed{y' = 2x - 2}$

4) $f(x) = \begin{cases} x^2, & x < C \\ x+1, & x \geq C \end{cases}$

$\lim_{x\to C^-} f(x) = \lim_{x\to C^+} f(x)$

$C^2 = C + 1$

$C^2 - C - 1 = 0$ *NOT factorable

$C^2 - C + \frac{1}{4} = 1 + \frac{1}{4}$ or use Quad form!

$(C - \frac{1}{2})^2 = \frac{5}{4}$

$C - \frac{1}{2} = \pm\frac{\sqrt{5}}{2}$

$\boxed{C = \frac{1}{2} \pm \frac{\sqrt{5}}{2}}$

5) $f(x) = \begin{cases} 3x+2, & x < -1 \\ 2x^2 - 3x + 6, & x \geq -1 \end{cases}$

$\lim_{x\to -1^-} f(x) = 3(-1) + 2 = -1$

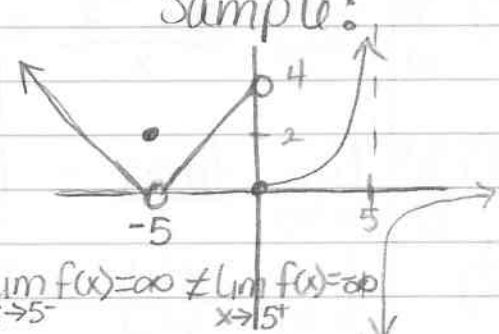
$\lim_{x\to -1^+} f(x) = 2(-1)^2 - 3(-1) + 6 = 11$

Since $\lim_{x\to -1} f(x)$ DNE,

$f(x)$ not cont at $x = -1$.

6) Answers will vary

sample:



$\lim_{x\to 5^-} f(x) = \infty \neq \lim_{x\to 5^+} f(x) = -\infty$

$\lim_{x\to -5} f(x) = 0,\ f(-5) = 2$ | $\lim_{x\to -5} \neq f(-5)$

$\lim_{x\to 0^-} f(x) = 4 \neq \lim_{x\to 0^+} f(x) = 0$

AP Review #1 - MC

1) B
2) A
3) C
4) D
5) C
6) E
7) B
8) C
9) C
10) A
11) C
12) A
13) B
14) E
15) E
16) B
17) B
18) C
19) D
20) B