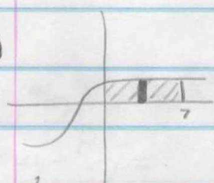
AP Review Integration #4

A ① $u = x^3+1$ $\qquad \int \frac{3x^2}{u^{1/2}}\,\frac{du}{3x^2} = \int u^{-1/2}\,du = 2u^{1/2}+C$

$\frac{du}{dx} = 3x^2 \to dx = \frac{du}{3x^2}$ $\qquad = 2\sqrt{x^3+1}+C$

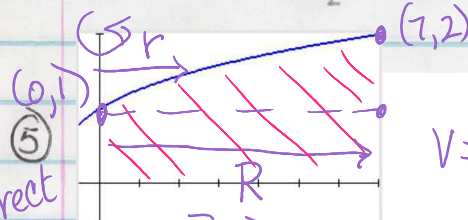
E ② $\int x^4+2x^2+1\,dx = \frac{x^5}{5}+\frac{2}{3}x^3+x+C$

B ③ 

Shell $\quad 2\pi\int_0^7 x(x+1)^{1/3}\,dx$

B ④ $\int_1^2 13^x-11^x\,dx + \int_2^{500} 13^x-11^x\,dx + \int_2^{500} 11^x-13^x\,dx = \int_1^2 13^x-11^x\,dx + \int_2^{500} 0\,dx$

$= 14.946$

C ⑤ 

$R=7$

$V=\pi\int_0^1 7^2\,dy + \pi\int_1^2 7^2-(y^3-1)^2\,dy$

$y=(x+1)^3$

$x=y^3-1$

Also correct using shell method

A ⑥ $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} = \arcsin\left(\frac{x}{2}\right)\Big]_0^{\sqrt{3}}$

$= \arcsin\frac{\sqrt{3}}{2} - \arcsin 0 = \frac{\pi}{3}$

A ⑦ $\frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\,dx = \frac{1}{2}\cdot\frac{1}{3}\int_1^9 u^{1/2}\,du = \frac{1}{6}\cdot\frac{2}{3}u^{3/2}\Big]_1^9 = \frac{1}{9}(27-1) = \frac{26}{9}$

$u=x^3+1 \quad \frac{du}{dx}=3x^2 \quad dx=\frac{du}{3x^2}$

C ⑧ $\int_1^2 4x^3-6x\,dx = x^4-3x^2\Big]_1^2 = 16-12-1+3 = 6$

C (9) $\int_a^b f(x)\,dx + \int_a^b 5\,dx = a + 2b + 5x\Big]_a^b = a + 2b + 5b - 5a$

$= 7b - 4a$

C (10) $u = \frac{1}{2}t$ $\qquad \frac{1}{2}\int e^u\, 2du = \int e^u\,du$

$\frac{du}{dt} = \frac{1}{2} \rightarrow 2du = dt$ $\qquad = e^u + c$

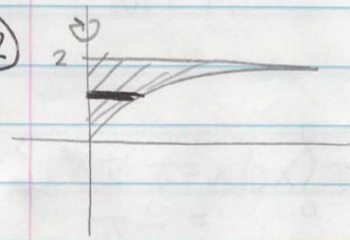
$= e^{\frac{t}{2}} + C$

C (11) $u = \tan x$ $\qquad \int_0^1 \frac{e^u}{\cos^2 x} \cos^2 x\,du = \int_0^1 e^u\,du$

$\frac{du}{dx} = \sec^2 x \rightarrow dx = \frac{1}{\sec^2 x}du$ $\qquad = e^1 - e^0$

$dx = \cos^2 x\,du$ $\qquad = e - 1$

A (12)



$V = \pi \int_0^2 (y^2)^2\,dy = \pi \int_0^2 y^4\,dy$

$= \frac{\pi}{5} y^5\Big]_0^2 = \frac{32\pi}{5}$

B (13) $\underbrace{\boxed{\frac{1}{50}}}_{\text{width}} = \frac{b-a}{n} \Rightarrow 1 = b - a$

$\int_0^1 \sqrt{x}\,dx$

height $\sqrt{0 + \frac{1}{50}}$

↑ start ↑ Δx

A (14) $\frac{d}{dx}\left(-\frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) + c\right)$ *work backwards*

$-\frac{1}{2}\cos(2x) + \frac{x}{2}\sin(2x)(2) + \frac{1}{4}\cos(2x)(2)$

$-\frac{1}{2}\cos(2x) + x\sin(2x) + \frac{1}{2}\cos(2x)$

$x\sin 2x$

$\int x\sin(2x)\,dx$

C (15) $\int_1^2 x^{-2}\,dx = -\frac{1}{x}\Big]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

E (16) $\int_1^e x - \frac{1}{x}\,dx = \frac{1}{2}x^2 - \ln x\Big]_1^e = \frac{1}{2}e^2 - \ln e - \frac{1}{2} + \ln 1$

$= \frac{1}{2}e^2 - 1 - \frac{1}{2}$

$= \frac{e^2}{2} - \frac{3}{2}$

A (17) $\int_{-3}^{k} x^2\,dx = \frac{1}{3}x^3\Big]_{-3}^{k} = \frac{k^3}{3} + 9 = 0$

$k = -3$

B (18) $\int \frac{1}{y}\,dy = \int k\,dt$

$\ln y = kt + c$

$y = e^{kt+c} = e^{kt}e^{c} = Ce^{kt}$

D (19) $\int_0^2 x^2 - (-x)\,dx = \int_0^2 x^2 + x\,dx = \frac{1}{3}x^3 + \frac{1}{2}x^2\Big]_0^2$

$= \frac{8}{3} + 2 = \frac{14}{3}$

D (20) $\int_{-4}^{-1} f'(x)\,dx = f(x)\Big]_{-4}^{-1} = f(-1) - f(-4)$

$= -1.5 - \frac{3}{4}$

$= 1 - \frac{9}{4}$