1. (a) $\int_9^{17} E(t)\,dt = 6004.270$

6004 people entered the park by 5 pm.

$3 \begin{cases} 1: \text{ limits} \\ 1: \text{ integrand} \\ 1: \text{ answer} \end{cases}$

(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$

The amount collected was $104,048.

or

$\int_{17}^{23} E(t)\,dt = 1271.283$

1271 people entered the park between 5 pm and 11 pm, so the amount collected was $\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$

$1:$ setup

(c) Write a function, $H(t)$, that would find the number of people that are currently in the park at a given time t.

$H(t) = \int_9^t E(x) - L(x)\,dx$

(d) Use your function from part (c) to find the number of people that are currently in the park at 4:00pm.

$H(16) = \int_9^{16} E(x) - L(x)\,dx \approx$ 3,944 people

(e) Find the average number of people that enter the amusement park per hour on a given day.

$\frac{1}{23-9}\int_9^{23} E(x)\,dx =$ 520 people

(f) $H'(t) = E(t) - L(t) = 0$

$t = 15.794$ or 15.795

$2 \begin{cases} 1: \; E(t) - L(t) = 0 \\ 1: \text{ answer} \end{cases}$

2.

(a) $\int_0^{12} H(t)\,dt = 70.570$ or 70.571

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(b) $H(6) - R(6) = -2.924$,
so the level of heating oil is falling at $t = 6$.

1 : answer with reason

(c) $125 + \int_0^{12} (H(t) - R(t))\,dt = 122.025$ or 122.026

$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(d) The absolute minimum occurs at a critical point or an endpoint.
$H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$.

The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$.

$$125 + \int_0^{11.318} (H(t) - R(t))\,dt = 120.738$$

Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.

$3: \begin{cases} 1: \text{sets } H(t) - R(t) = 0 \\ 1: \text{volume is least at } t = 11.318 \\ 1: \text{analysis for absolute minimum} \end{cases}$

3.

(a) $\int_0^{18} L(t)\,dt \approx 1658$ cars

$2: \begin{cases} 1: \text{setup} \\ 1: \text{answer} \end{cases}$

(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$
Let $R = 12.42831$ and $S = 16.12166$
$L(t) \geq 150$ for t in the interval $[R, S]$

$$\frac{1}{S - R}\int_R^S L(t)\,dt = 199.426 \text{ cars per hour}$$

$3: \begin{cases} 1: t\text{-interval when } L(t) \geq 150 \\ 1: \text{average value integral} \\ 1: \text{answer with units} \end{cases}$

(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$$\int_{13}^{15} L(t)\,dt = 431.931 > 400$$

$4: \begin{cases} 1: \text{considers 400 cars} \\ 1: \text{valid interval } [h, h+2] \\ 1: \text{value of } \int_h^{h+2} L(t)\,dt \\ 1: \text{answer and explanation} \end{cases}$

OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \geq 200$ on that interval.
$L(t) \geq 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

OR

$4: \begin{cases} 1: \text{considers 200 cars per hour} \\ 1: \text{solves } L(t) \geq 200 \\ 1: \text{discusses 2 hour interval} \\ 1: \text{answer and explanation} \end{cases}$

Yes, a traffic signal is required.

4. (a) $\int_0^7 f(t)\,dt \approx 8264$ gallons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) The amount of water in the tank is decreasing on the intervals $0 \le t \le 1.617$ and $3 \le t \le 5.076$ because $f(t) < g(t)$ for $0 \le t < 1.617$ and $3 < t < 5.076$.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0$, 3, and 7.

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t)\,dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t)\,dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

$5 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

5. (a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.

1 : shows that $R(6) > 0$

(b) $R'(6) = -1.913$
Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

$2 : \begin{cases} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{cases}$

(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$

To the nearest whole number, there are 964 mosquitoes.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$
$R(t) > 0$ on $0 < t < 2.5\pi$
$R(t) < 0$ on $2.5\pi < t < 7.5\pi$
$R(t) > 0$ on $7.5\pi < t < 31$
The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.

$1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357$,

There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.

$4 : \begin{cases} 2 : \text{absolute maximum value} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{answer} \\ 2 : \text{analysis} \\ \quad 1 : \text{computes interior critical points} \\ \quad 1 : \text{completes analysis} \end{cases}$

6.

(a) Method 1: $\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2}\Big\vert_0^3 = \frac{14}{3}$
 	
 – or –
 	
 Method 2: $L(t)$ = gallons leaked in first t minutes
 $\frac{dL}{dt} = \sqrt{t+1}; \quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$
 $L(0) = 0; \quad C = -\frac{2}{3}$
 $L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; \quad L(3) = \frac{14}{3}$
	Method 1:
	 3 { 2 : definite integral
	     1 : limits
	     1 : integrand
	 1 : answer }
	
	 – or –
	
	 Method 2:
	 3 { 1 : antiderivative with C
	 1 : solves for C using $L(0) = 0$
	 1 : answer }

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$	1 : answer
(c) Method 1: $A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right) dx$ $= 30 + 8t - \int_0^t \sqrt{x+1}\, dx$ – or – Method 2: $\frac{dA}{dt} = 8 - \sqrt{t+1}$ $A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$ $30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$ $A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$	Method 1: 2 { 1 : $30 + 8t$ 1 : $-\int_0^t \sqrt{x+1}\,dx$ } – or – Method 2: 2 { 1 : antiderivative with C 1 : answer }
(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$ $A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.	3 { 1 : sets $A'(t) = 0$ 1 : solves for t 1 : justification }

7. (a) $\int_0^6 f(t)\,dt = 142.274$ or 142.275 cubic feet

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.

$1 : \text{answer}$

(c) $h(0) = 0$

For $0 < t \le 6$, $h(t) = h(0) + \int_0^t g(s)\,ds = 0 + \int_0^t 0\,ds = 0$.

For $6 < t \le 7$, $h(t) = h(6) + \int_6^t g(s)\,ds = 0 + \int_6^t 125\,ds = 125(t-6)$.

For $7 < t \le 9$, $h(t) = h(7) + \int_7^t g(s)\,ds = 125 + \int_7^t 108\,ds = 125 + 108(t-7)$.

Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 125 + 108(t-7) & \text{for } 7 < t \le 9 \end{cases}$

$3 : \begin{cases} 1 : h(t) \text{ for } 0 \le t \le 6 \\ 1 : h(t) \text{ for } 6 < t \le 7 \\ 1 : h(t) \text{ for } 7 < t \le 9 \end{cases}$

(d) Amount of snow is $\int_0^9 f(t)\,dt - h(9) = 26.334$ or 26.335 cubic feet.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$

8.

(a) $\int_0^{30} F(t)\,dt = 2474$ cars

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $F'(7) = -1.872$ or -1.873
Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.

$1 : \text{answer with reason}$

(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2

$1 : \text{answer}$

Units of cars/min in (c) and cars/min^2 in (d)

$1 : \text{units in (c) and (d)}$

9. (a) $H'(3.5) \approx \dfrac{H(5) - H(2)}{5 - 2}$

$= \dfrac{52 - 60}{3} = -2.666$ or -2.667 degrees Celsius per minute

1 : answer

(b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{1}{10}\left(2\cdot\frac{66+60}{2} + 3\cdot\frac{60+52}{2} + 4\cdot\frac{52+44}{2} + 1\cdot\frac{44+43}{2}\right)$$
$$= 52.95$$

3 : { 1 : meaning of expression; 1 : trapezoidal sum; 1 : estimate }

(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$

The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : { 1 : value of integral; 1 : meaning of expression }

(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275;\quad H(10) - B(10) = 8.817$

The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : { 1 : integrand; 1 : uses $B(0) = 100$; 1 : answer }

10.

(a) No; the amount of water is not increasing at $t = 15$ since $W(15) - R(15) = -121.09 < 0$.

1 : answer with reason

(b) $1200 + \int_0^{18} (W(t) - R(t))\,dt = 1309.788$

1310 gallons

3 : { 1 : limits; 1 : integrand; 1 : answer }

(c) $W(t) - R(t) = 0$

$t = 0, 6.4948, 12.9748$

t (hours)	gallons of water
0	1200
6.495	525
12.975	1697
18	1310

The values at the endpoints and the critical points show that the absolute minimum occurs when $t = 6.494$ or 6.495.

3 : { 1 : interior critical points; 1 : amount of water is least at $t = 6.494$ or 6.495; 1 : analysis for absolute minimum }

(d) $\int_{18}^{k} R(t)\,dt = 1310$

2 : { 1 : limits; 1 : equation }

11.

(a) $v'(16) \approx \dfrac{240-200}{20-12} = 5$ meters/min^2	1 : approximation
(b) $\int_0^{40} \lvert v(t)\rvert\, dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \le t \le 40$ minutes. $\int_0^{40} \lvert v(t)\rvert\, dt \approx 12\cdot\lvert v(12)\rvert + 8\cdot\lvert v(20)\rvert + 4\cdot\lvert v(24)\rvert + 16\cdot\lvert v(40)\rvert$ $= 12\cdot 200 + 8\cdot 240 + 4\cdot 220 + 16\cdot 150$ $= 2400 + 1920 + 880 + 2400$ $= 7600$ meters	3 : { 1 : explanation 1 : right Riemann sum 1 : approximation
(c) Bob's acceleration is $B'(t) = 3t^2 - 12t$. $B'(5) = 3(25) - 12(5) = 15$ meters/min^2	2 : { 1 : uses $B'(t)$ 1 : answer
(d) Avg vel $= \dfrac{1}{10}\int_0^{10}\left(t^3 - 6t^2 + 300\right) dt$ $= \dfrac{1}{10}\left[\dfrac{t^4}{4} - 2t^3 + 300t\right]_0^{10}$ $= \dfrac{1}{10}\left[\dfrac{10000}{4} - 2000 + 3000\right] = 350$ meters/min	3 : { 1 : integral 1 : antiderivative 1 : answer

12.

(a) $R'(2) \approx \dfrac{R(3)-R(1)}{3-1} = \dfrac{950-1190}{3-1} = -120$ liters/hr^2	2 : { 1 : estimate 1 : units
(b) The total amount of water removed is given by $\int_0^8 R(t)\, dt$. $\int_0^8 R(t)\, dt \approx 1\cdot R(0) + 2\cdot R(1) + 3\cdot R(3) + 2\cdot R(6)$ $= 1(1340) + 2(1190) + 3(950) + 2(740)$ $= 8050$ liters This is an overestimate since R is a decreasing function.	3 : { 1 : left Riemann sum 1 : estimate 1 : overestimate with reason
(c) Total $\approx 50000 + \int_0^8 W(t)\, dt - 8050$ $= 50000 + 7836.195325 - 8050 \approx 49786$ liters	2 : { 1 : integral 1 : estimate
(d) $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous. Therefore, the Intermediate Value Theorem guarantees at least one time t, $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$. For this value of t, the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.	2 : { 1 : considers $W(t) - R(t)$ 1 : answer with explanation

13. (a) $\dfrac{A(30)-A(0)}{30-0} = -0.197$ (or -0.196) lbs/day	1 : answer with units
(b) $A'(15) = -0.164$ (or -0.163) The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.	$2 : \begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$
(c) $A(t) = \dfrac{1}{30}\int_0^{30} A(t)\,dt \Rightarrow t = 12.415$ (or 12.414)	$2 : \begin{cases} 1 : \dfrac{1}{30}\int_0^{30} A(t)\,dt \\ 1 : \text{answer} \end{cases}$
(d) $L(t) = A(30) + A'(30)\cdot(t-30)$ $A'(30) = -0.055976$ $A(30) = 0.782928$ $L(t) = 0.5 \Rightarrow t = 35.054$	$4 : \begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

14. (a) $v(5.5) = -0.45337$, $a(5.5) = -1.35851$ The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.	2 : conclusion with reason
(b) Average velocity $= \dfrac{1}{6}\int_0^6 v(t)\,dt = 1.949$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) Distance $= \int_0^6 \lvert v(t)\rvert\,dt = 12.573$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$. $v(t)$ changes sign from positive to negative at time $t = b$. $x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135	$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$