1. The function $f(x)=\dfrac{1}{x}$ has

$f'(x) = -\dfrac{1}{x^2}$ $\quad f'(x)\ -\ \vdots\ -$; $f(x)\ \searrow\ \vdots\ \searrow$; 0

$f''(x) = \dfrac{2}{x^3}$ $\quad f''(x)\ -\ \vdots\ +$; $f(x)\ \cap\ \vdots\ \cup$; 0

(a) one point of inflection and one extrema
(b) one point of inflection
$\boxed{\text{(c)}}$ Neither a point of inflection nor an extrema
(d) a relative min and a relative max
(e) one extrema

2. $\displaystyle\lim_{x\to 9}\frac{\sqrt{x-5}-2}{x-9}\cdot\frac{\sqrt{x-5}+2}{\sqrt{x-5}+2} = \lim_{x\to 9}\frac{\overset{(x-9)}{\cancel{x-5-4}}}{\cancel{(x-9)}(\sqrt{x-5}+2)} = \lim_{x\to 9}\frac{1}{\sqrt{x-5}+2} = \frac{1}{4}$

$\boxed{\text{(a)}\ \frac{1}{4}}$ (b) $-\frac{1}{4}$ (c) 1 (d) 0 (e) *DNE*

3. If $f'(x)$ exists on the closed interval $[a,b]$, then it follows that: If differentiable on $[a,b]$, Continuous on $[a,b]$

~~(a)~~ $f(x)$ is constant on $[a,b]$
~~(b)~~ Rolle's Theorem applies Only if $f(a)=f(b)$
~~(c)~~ The function has a maximum value on the open interval (a,b) true on $[a,b]$
$\boxed{\text{(d)}}$ Mean Value Theorem applies ✓
~~(e)~~ $f''(x)>0$ on $[a,b]$

4. The equation of the tangent to the curve $\dfrac{d}{dx}\left(xy-x+y=2\right)$ at the point where $x=0$ is

$y + x\dfrac{dy}{dx} - 1 + \dfrac{dy}{dx} = 0$ $\qquad 0-0+y=2 \Rightarrow y=2$

$\dfrac{dy}{dx} = \dfrac{1-y}{x+1}$ $\qquad \left.\dfrac{dy}{dx}\right|_{(0,2)} = \dfrac{1-2}{0+1} = -1$

$y-2=-x \qquad y=-x+2$

(a) $y=-x$
(b) $y=\dfrac{x}{2}+2$
(c) $y=x+2$
(d) $y=2$
$\boxed{\text{(e)}}$ $y=2-x$

5. Which of the following is true of the graph of $y=x^4-4x^3$?

$y' = 4x^3 - 12x^2 = 4x^2(x-3)$; $x=0,3$; $f'\ -\ -\ +$; $f\ \searrow\ 0\ \searrow\ 3\ \nearrow$; min ; one min

$y'' = 12x^2 - 24x = 12x(x-2)$; $x=0,2$; $f''\ +\ -\ +$; $f\ \cup\ 0\ \cap\ 2\ \cup$; POI $x=0,2$

(a) The graph has no relative extreme point.
~~(b)~~ The graph has one point of inflection, one relative min & one relative max
$\boxed{\text{(c)}}$ The graph has two points of inflection and one relative minimum
~~(d)~~ The graph has two points of inflection, one relative min & one relative max
(e) None of the above

6. If $y=\sin^3(1-2x)$, then $\dfrac{dy}{dx}=$

$\dfrac{dy}{dx} = 3\sin^2(1-2x)\cdot\cos(1-2x)\cdot -2$

(a) $3\sin^2(1-2x)$
(b) $-2\cos^3(1-2x)$
(c) $-6\sin^2(1-2x)$
$\boxed{\text{(d)}}$ $-6\sin^2(1-2x)\cos(1-2x)$
(e) $-6\cos^2(1-2x)$

7. $\displaystyle\lim_{x\to\infty}\frac{\sqrt{x}-4}{4-3\sqrt{x}} = \cdot\frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}} = \lim_{x\to\infty}\frac{1-\frac{4}{\sqrt{x}}\to 0}{\frac{4}{\sqrt{x}}\to 0 - 3} = -\frac{1}{3}$

$\boxed{\text{(a)}}$ $-\dfrac{1}{3}$ (b) - 1 (c) ∞ (d) 0 (e) $\dfrac{1}{3}$

8. The equation of the tangent to the curve of $y = x^2 - 4x$ at the point where the curve crosses the y-axis is: $x=0$, $y=0$ $(0,0)$ $y' = 2x-4$ $y'(0) = -4$

(a) $y = 8x - 4$
$\boxed{\text{(b)}}$ $y = -4x$
(c) $y = -4$
(d) $y = 4x$
(e) $y = 4x - 8$

$y = -4x$

9. If $f(x)$ is continuous at the point where $x = a$, which of the following statements may be false?

(a) $\lim_{x \to a} f(x)$ exists true (b) $\lim_{x \to a} f(x) = f(a)$ true $\boxed{\text{(c)}}$ $f'(a)$ exists

(d) $f(a)$ is defined true (e) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ true

Not always true – Cusp, sharp turn

10. Let $f(x) = \begin{cases} c & \text{if } x = 5 \\ \dfrac{x^2 - 25}{x - 5} & \text{if } x \neq 5 \end{cases}$ and let $f(x)$ be continuous at $x = 5$. Then $c =$

$\lim_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}} = \lim_{x \to 5} (x+5) = 10$ $\lim_{x \to 5} f(x) = f(5)$

$10 = c$

(a) 0 (b) $\frac{1}{2}$ $\boxed{\text{(c)}}$ 10 (d) ∞ (e) $\frac{1}{10}$

11. The value of c for which $f(x) = x + \frac{c}{x}$ has a relative minimum at $x = 3$ is

(a) - 9 (b) - 6 (c) - 3 (d) 6 $\boxed{\text{(e)}}$ 9

f': $+$ $-$ $-$ $+$; f: $-\sqrt{c}$ c $\sqrt{c}$

$f'(x) = 1 - \frac{c}{x^2} = 0$

$x^2 = c$

$x = \pm\sqrt{c}$

min at $\sqrt{c}$

$x = \sqrt{c}$

$3 = \sqrt{c}$

$c = 9$

12. Differentiate: $\frac{d}{dx}(3x^2y - 5y^3 = 1)$

(a) $\frac{2xy}{x - y^2}$ (b) $\frac{5x}{2y^2 - x}$ (c) $\frac{x}{y^2 - 5x}$ (d) $\frac{xy}{x^2 - 5y^2}$ $\boxed{\text{(e)}}$ $\frac{-2xy}{x^2 - 5y^2}$

$6xy + 3x^2\frac{dy}{dx} - 15y^2\frac{dy}{dx} = 0$

$2xy + x^2\frac{dy}{dx} - 5y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2xy}{x^2 - 5y^2}$

13. If $y = \ln(3x+5)$, then $\frac{d^2y}{dx^2} =$

$\frac{dy}{dx} = \frac{1}{3x+5} \cdot 3$

(a) $\frac{3}{3x+5}$ (b) $\frac{3}{(3x+5)^2}$ (c) $\frac{9}{(3x+5)^2}$ $\boxed{\text{(d)}}$ $\frac{-9}{(3x+5)^2}$ (e) $\frac{-3}{(3x+5)^2}$

$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{3}{3x+5}\right)$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{-3 \cdot 3}{(3x+5)^2} = \frac{-9}{(3x+5)^2}$

14. If the slope of a strictly monotonic function f is $\frac{4}{9}$ at a particular point (a, b), what is the slope of f^{-1} at the point (b, a)?

entirely incr or decr, ∴ one-to-one and has inverse function.

(a) $\frac{4}{9}$ $\boxed{\text{(b)}}$ $\frac{9}{4}$ (c) -5 (d) 5 (e) Cannot be determined

$(f^{-1})'(b) = \frac{1}{f'(a)} = \frac{1}{4/9} = \frac{9}{4}$

15. A LRAM sum with 4 equal subdivision is used to approximate the area under the sine curve from $x=0$ to $x=\pi$. What is the approximation? $\Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$

(a) $\frac{\pi}{4}\left(\frac{1}{2}+\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}+1\right)$ (b) $\frac{\pi}{4}\left(0+\frac{1}{2}+\frac{\sqrt{3}}{2}+1\right)$ $\boxed{\text{(c)}}$ $\frac{\pi}{4}\left(0+\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2}\right)$ (d) $\frac{\pi}{4}\left(0+\frac{1}{2}+\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\right)$

$A = \frac{\pi}{4}\left(f(0)+f\left(\frac{\pi}{4}\right)+f\left(\frac{\pi}{2}\right)+f\left(\frac{3}{4}\right)\right)$

$= \frac{\pi}{4}\left(0+\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2}\right)$

16. The graph of $f(x)=\dfrac{3x^2}{x^2-4}$ has

(a) one horizontal and one vertical asymptote
(b) two horizontal and no vertical asymptotes
(c) two horizontal and one vertical asymptote
$\boxed{\text{(d)}}$ one horizontal and two vertical asymptotes
(e) two vertical and no horizontal asymptotes

VA $x^2-4=0$
$x=\pm 2$

HA $\lim_{x\to\infty} \frac{3x^2 \cdot \frac{1}{x^2}}{(x^2-4)\cdot\frac{1}{x^2}}$

$=\lim_{x\to\infty} \frac{3}{1-\frac{4}{x^2}\to 0} = 3$

Same for $\lim_{x\to -\infty} f(x)$.

17. The maximum value of $f(x)=x^4-4x^3+6$ on the interval $[1,4]$ is

(a) 1
(b) 0
(c) 3
$\boxed{\text{(d)}}$ 6
(e) None of these

$f'(x)=4x^3-12x^2$
$=4x^2(x-3)=0$
$x=0,3$

$f'(x)$: $[\ -\ \ 3\ \ +\ ]$ on $[1,4]$
$f(x)$: $\searrow$ then $\nearrow$

$f(1)=3$
$\boxed{f(4)=6}$ abs max!

18. If $f(x)=-\cos(x)$, then $f''(x)=$

$\boxed{\text{(a)}}$ $\cos x$ (b) $\sin x$ (c) $-\cos x$ (d) $-\sin x$ (e) none of these

$f'(x)=\sin x$, $f''(x)=\cos x$

19. The $\lim\limits_{h\to 0}\dfrac{\tan 3(x+h)-\tan(3x)}{h}$ is $=f'(x)$ when $f(x)=\tan(3x)$

$f'(x)=3\sec^2(3x)$

(a) 0 $\boxed{\text{(b)}}$ $3\sec^2(3x)$ (c) $\sec^2(3x)$ (d) $3\tan(3x)\sec(3x)$ (e) nonexistent

20. Which of the following does not correctly match a function and its derivative?

(a) $\frac{d}{dx}(\text{arc}\cot(x))=\frac{-1}{1+x^2}$ ✓ (b) $\frac{d}{dx}(\ln(u))=\frac{u'}{u}$ ✓ $\boxed{\text{(c)}}$ $\frac{d}{dx}(a^x)=\cancel{\frac{a^x}{\ln(a)}}$

(d) $\frac{d}{dx}(\arccos(x))=\frac{-1}{\sqrt{1-x^2}}$ ✓ (e) $\frac{d}{dx}(\log_a(x))=\frac{1}{\ln(a)x}$ ✓ $=a^x\ln(a)$

21. Find the derivative of $f(\theta)=\cos^2(\theta)-\sin^2(\theta)$. $= \cos(2\theta)$

$f'(\theta) = -2\sin(2\theta)$

(a) $\cos(2\theta)$ $\quad$ $\boxed{\text{b)}}$ $-2\sin(2\theta)$ $\quad$ (c) $-\sin(2\theta)$ $\quad$ (d) $-2\sin(2\theta)\cos(2\theta)$ $\quad$ (e) $-2\cos(2\theta)$

22. Find the slope of the tangent line to $f(x)=x^2\sqrt{x^3+5}$ at the point $(1,\sqrt{6})$. $f'(x) = \frac{2\sqrt{x^3+5}}{2\sqrt{x^3+5}}\left(2x\sqrt{x^3+5}\right) + \frac{x^2 \cdot 3x^2}{2\sqrt{x^3+5}}$

$\boxed{\text{(a)}}$ $\frac{9\sqrt{6}}{4}$ $\quad$ (b) $\frac{3}{2\sqrt{6}}$ $\quad$ (c) $\frac{\sqrt{6}}{4}$ $\quad$ (d) $\sqrt{6}$ $\quad$ (e) 1

$f'(1) = \frac{27}{2\sqrt{6}} = \frac{27\sqrt{6}}{2\cdot 6} = \frac{9\sqrt{6}}{4} \leftarrow f'(x) = \frac{7x^4+20x}{2\sqrt{x^3+5}} \leftarrow f'(x) = \frac{4x(x^3+5)+3x^4}{2\sqrt{x^3+5}}$

23. A particle moves along the x-axis so that its position at time t is given by $x(t)=t^2-6t+5$. For what value of t is the velocity of the particle zero?

$v(t) = x'(t) = 2t-6 = 0$

(a) 1 $\quad$ (b) 2 $\quad$ $\boxed{\text{(c) 3}}$ $\quad$ (d) 4 $\quad$ (e) 5

$t=3$

24. Find the derivative of $f(x)=e^{\ln(\tan x)}$. $= \tan x$ $\quad$ $f'(x) = \sec^2 x$

(a) $\tan(x)$ $\quad$ $\boxed{\text{(b)}}$ $\sec^2(x)$ $\quad$ (c) $\ln(\tan(x))\sec^2(x)$ $\quad$ (d) $e^{\ln(\tan(x))}$ $\quad$ (e) $e^{\ln(\tan(x))}\frac{1}{\tan(x)}$

25. Let f be the function given by $f(x)=3e^{2x}$ and let g be the function given by $g(x)=6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(a) -0.701 $\quad$ (b) -0.567 $\quad$ $\boxed{\text{(c)}}$ -0.391 $\quad$ (d) -0.302 $\quad$ (e) -0.258

$f'(x) = 6e^{2x}$ $\quad$ $6e^{2x} = 18x^2$ $\quad$ $e^{2x} - 3x^2 = 0$ on GC

$g'(x) = 18x^2$ $\quad$ $e^{2x} = 3x^2$ $\quad$ $x = -.391$

26. Which of the following gives $\frac{dy}{dx}$ for $y=\log_{10}(2x-3)$? $= \frac{\ln(2x-3)}{\ln(10)}$

(a) $\frac{1}{2x}$ $\quad$ (b) $\frac{1}{2x-3}$ $\quad$ (c) $\frac{2}{2x-3}$ $\quad$ $\boxed{\text{(d)}}$ $\frac{2}{(2x-3)\ln 10}$ $\quad$ (e) $\frac{1}{(2x-3)\ln 10}$

$y' = \frac{1}{(2x+3)\ln 10} \cdot 2$

27. Which of the following gives the slope of the tangent line to the graph of $y=2^{1-x}$ at $x=2$?

(a) $\frac{1}{2}$ $\quad$ (b) $-\frac{1}{2}$ $\quad$ (c) -2 $\quad$ (d) 2 $\quad$ $\boxed{\text{(e)}}$ $-\frac{\ln 2}{2}$

$y' = \ln(2)\cdot 2^{1-x}\cdot -1$

$y'(2) = -\ln 2 \cdot 2^{-1} = -\frac{\ln 2}{2}$