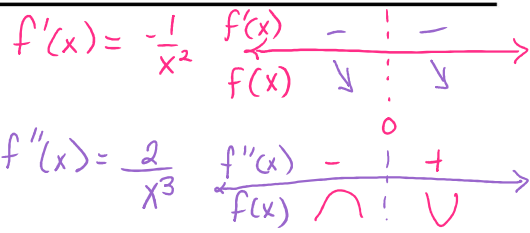


1. The function $f(x) = \frac{1}{x}$ has
- (a) one point of inflection and one extrema
 - (b) one point of inflection
 - (c) Neither a point of inflection nor an extrema
 - (d) a relative min and a relative max
 - (e) one extrema

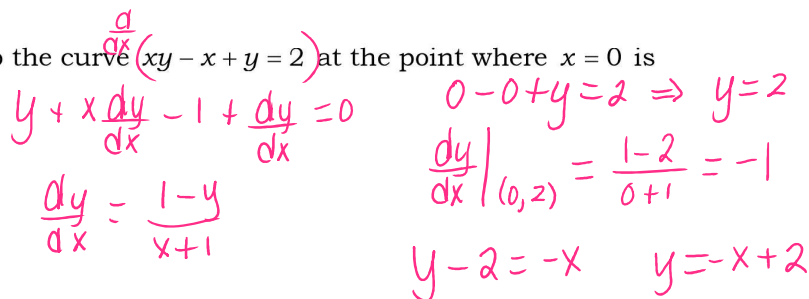


2. $\lim_{x \rightarrow 9} \frac{\sqrt{x-5}-2}{x-9} \cdot \frac{\sqrt{x-5}+2}{\sqrt{x-5}+2} = \lim_{x \rightarrow 9} \frac{x-5-4}{(x-9)(\sqrt{x-5}+2)} = \lim_{x \rightarrow 9} \frac{1}{\sqrt{x-5}+2} = \frac{1}{4}$
- (a) $\frac{1}{4}$
 - (b) $-\frac{1}{4}$
 - (c) 1
 - (d) 0
 - (e) DNE

3. If $f'(x)$ exists on the closed interval $[a, b]$, then it follows that: *If differentiable on $[a, b]$, continuous on $[a, b]$*
- (a) $f(x)$ is constant on $[a, b]$
 - (b) Rolle's Theorem applies *only if $f(a) = f(b)$*
 - (c) The function has a maximum value on the open interval (a, b) *true on $[a, b]$*
 - (d) Mean Value Theorem applies \checkmark
 - (e) $f''(x) > 0$ on $[a, b]$

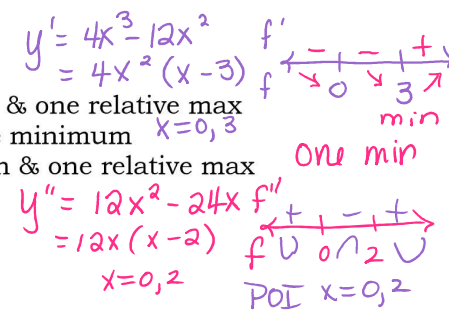
4. The equation of the tangent to the curve $\frac{d}{dx}(xy - x + y = 2)$ at the point where $x = 0$ is

- (a) $y = -x$
- (b) $y = \frac{x}{2} + 2$
- (c) $y = x + 2$
- (d) $y = 2$
- (e) $y = 2 - x$



5. Which of the following is true of the graph of $y = x^4 - 4x^3$?

- (a) The graph has no relative extreme point.
- (b) The graph has one point of inflection, one relative min & one relative max
- (c) The graph has two points of inflection and one relative minimum
- (d) The graph has two points of inflection, one relative min & one relative max
- (e) None of the above



6. If $y = \sin^3(1-2x)$, then $\frac{dy}{dx} =$

- (a) $3 \sin^2(1-2x)$
- (b) $-2 \cos^3(1-2x)$
- (c) $-6 \sin^2(1-2x)$
- (d) $-6 \sin^2(1-2x) \cos(1-2x)$
- (e) $-6 \cos^2(1-2x)$

$\frac{dy}{dx} = 3 \sin^2(1-2x) \cdot \cos(1-2x) \cdot -2$

7. $\lim_{x \rightarrow \infty} \frac{\sqrt{x}-4}{4-3\sqrt{x}} = \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}} = \lim_{x \rightarrow \infty} \frac{1 - \frac{4}{\sqrt{x}}}{\frac{4}{\sqrt{x}} - 3} = -\frac{1}{3}$

- (a) $-\frac{1}{3}$
- (b) -1
- (c) ∞
- (d) 0
- (e) $\frac{1}{3}$

8. The equation of the tangent to the curve of $y = x^2 - 4x$ at the point where the curve crosses the y-axis is: $x=0, y=0 (0,0)$

- (a) $y = 8x - 4$
- (b) $y = -4x$
- (c) $y = -4$
- (d) $y = 4x$
- (e) $y = 4x - 8$

$y' = 2x - 4$
 $y'(0) = -4$

$y = -4x$

9. If $f(x)$ is continuous at the point where $x = a$, which of the following statements may be false?

- (a) $\lim_{x \rightarrow a} f(x)$ exists *true*
- (b) $\lim_{x \rightarrow a} f(x) = f(a)$ *true*
- (c) $f'(a)$ exists
- (d) $f(a)$ is defined *true*
- (e) $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a^-} f(x)$ *true*

Not always true - cusp, sharp turn

10. Let $f(x) = \begin{cases} c & \text{if } x = 5 \\ \frac{x^2 - 25}{x - 5} & \text{if } x \neq 5 \end{cases}$ and let $f(x)$ be continuous at $x = 5$. Then $c =$

$\lim_{x \rightarrow 5} \frac{(x+5)(x-5)}{x-5} = \lim_{x \rightarrow 5} (x+5) = 10$ $\lim_{x \rightarrow 5} f(x) = f(5)$

$10 = c$

- (a) 0
- (b) $\frac{1}{2}$
- (c) 10
- (d) ∞
- (e) $\frac{1}{10}$

11. The value of c for which $f(x) = x + \frac{c}{x}$ has a relative minimum at $x = 3$ is

- (a) -9
- (b) -6
- (c) -3
- (d) 6
- (e) 9

$f'(x) = 1 - \frac{c}{x^2} = 0$
 $x^2 = c$
 $x = \pm\sqrt{c}$

*min at \sqrt{c}
 $x = \sqrt{c}$
 $3 = \sqrt{c}$
 $c = 9$*

12. Differentiate: $\frac{d}{dx}(3x^2y - 5y^3 = 1)$

- (a) $\frac{2xy}{x - y^2}$
- (b) $\frac{5x}{2y^2 - x}$
- (c) $\frac{x}{y^2 - 5x}$
- (d) $\frac{xy}{x^2 - 5y^2}$
- (e) $\frac{-2xy}{x^2 - 5y^2}$

$6xy + 3x^2 \frac{dy}{dx} - 15y^2 \frac{dy}{dx} = 0$
 $2xy + x^2 \frac{dy}{dx} - 5y^2 \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2xy}{x^2 - 5y^2}$

13. If $y = \ln(3x + 5)$, then $\frac{d^2y}{dx^2} =$

- (a) $\frac{3}{3x + 5}$
- (b) $\frac{3}{(3x + 5)^2}$
- (c) $\frac{9}{(3x + 5)^2}$
- (d) $\frac{-9}{(3x + 5)^2}$
- (e) $\frac{-3}{(3x + 5)^2}$

$\frac{dy}{dx} = \frac{1}{3x+5} \cdot 3$
 $\frac{d}{dx} \left(\frac{dy}{dx} = \frac{3}{3x+5} \right) \Rightarrow \frac{d^2y}{dx^2} = \frac{-3 \cdot 3}{(3x+5)^2} = \frac{-9}{(3x+5)^2}$

14. If the slope of a strictly monotonic function f is $\frac{4}{9}$ at a particular point (a, b) , what is the slope of f^{-1} at the point (b, a) ?

- (a) $\frac{4}{9}$
- (b) $\frac{9}{4}$
- (c) -5
- (d) 5
- (e) Cannot be determined

entirely incr or decr, \therefore one-to-one and has inverse function.

$(f^{-1})'(b) = \frac{1}{f'(a)} = \frac{1}{\frac{4}{9}} = \frac{9}{4}$

15. A RRAM sum with 4 equal subdivision is used to approximate the area under the sine curve from $x=0$ to $x=\pi$. What is the approximation? $\Delta x = \frac{\pi-0}{4} = \frac{\pi}{4}$

- (a) $\frac{\pi}{4}(\frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + 1)$ (b) $\frac{\pi}{4}(0 + \frac{1}{2} + \frac{\sqrt{3}}{2} + 1)$ (c) $\frac{\pi}{4}(0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2})$ (d) $\frac{\pi}{4}(0 + \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2})$

$$A = \frac{\pi}{4} (f(0) + f(\frac{\pi}{4}) + f(\frac{\pi}{2}) + f(\frac{3\pi}{4}))$$

$$= \frac{\pi}{4} (0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2})$$

16. The graph of $f(x) = \frac{3x^2}{x^2-4}$ has

- (a) one horizontal and one vertical asymptote
 (b) two horizontal and no vertical asymptotes
 (c) two horizontal and one vertical asymptote
 (d) one horizontal and two vertical asymptotes
 (e) two vertical and no horizontal asymptotes

VA $x^2-4=0$
 $x = \pm 2$

HA $\lim_{x \rightarrow \infty} \frac{3x^2 \cdot \frac{1}{x^2}}{x^2-4 \cdot \frac{1}{x^2}}$
 $= \lim_{x \rightarrow \infty} \frac{3}{1 - \frac{4}{x^2}} = 3$
 Same for $\lim_{x \rightarrow -\infty} f(x)$.

17. The maximum value of $f(x) = x^4 - 4x^3 + 6$ on the interval $[1, 4]$ is

- (a) 1
 (b) 0
 (c) 3
 (d) 6
 (e) None of these

$$f'(x) = 4x^3 - 12x^2$$

$$= 4x^2(x-3) = 0$$

$$x = 0, 3$$

$f(1) = 3$
 $f(4) = 6$ abs max!

18. If $f(x) = -\cos(x)$, then $f''(x) =$

- (a) $\cos x$ (b) $\sin x$ (c) $-\cos x$ (d) $-\sin x$ (e) none of these

$$f'(x) = \sin x, f''(x) = \cos x$$

19. The $\lim_{h \rightarrow 0} \frac{\tan 3(x+h) - \tan 3x}{h}$ is $= f'(x)$ when $f(x) = \tan(3x)$

$$f'(x) = 3 \sec^2(3x)$$

- (a) 0 (b) $3 \sec^2(3x)$ (c) $\sec^2(3x)$ (d) $3 \tan(3x) \sec(3x)$ (e) nonexistent

20. Which of the following does not correctly match a function and its derivative?

(a) $\frac{d}{dx}(\text{arc cot}(x)) = \frac{-1}{1+x^2}$ ✓

(b) $\frac{d}{dx}(\ln(u)) = \frac{u'}{u}$ ✓

(c) $\frac{d}{dx}(a^x) = \frac{a^x}{\ln(a)}$

(d) $\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$ ✓

(e) $\frac{d}{dx}(\log_a(x)) = \frac{1}{\ln(a)x}$ ✓

$$= a^x \ln(a)$$

21. Find the derivative of $f(\theta) = \cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$

$$f'(\theta) = -2\sin(2\theta)$$

- (a) $\cos(2\theta)$ (b) $-2\sin(2\theta)$ (c) $-\sin(2\theta)$ (d) $-2\sin(2\theta)\cos(2\theta)$ (e) $-2\cos(2\theta)$

22. Find the slope of the tangent line to $f(x) = x^2\sqrt{x^3+5}$ at the point $(1, \sqrt{6})$. $f'(x) = \frac{2x\sqrt{x^3+5}}{2\sqrt{x^3+5}} + \frac{x^2 \cdot 3x^2}{2\sqrt{x^3+5}}$

(a) $\frac{9\sqrt{6}}{4}$ (b) $\frac{3}{2\sqrt{6}}$ (c) $\frac{\sqrt{6}}{4}$ (d) $\sqrt{6}$ (e) $\frac{1}{2\sqrt{x^3+5}}$

$$f'(1) = \frac{27}{2\sqrt{6}} = \frac{27\sqrt{6}}{2 \cdot 6} = \frac{9\sqrt{6}}{4}$$

$$f'(x) = \frac{7x^4 + 20x}{2\sqrt{x^3+5}}$$

$$f'(x) = \frac{4x(x^3+5) + 3x^4}{2\sqrt{x^3+5}}$$

23. A particle moves along the x -axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$.

For what value of t is the velocity of the particle zero?

$$v(t) = x'(t) = 2t - 6 = 0$$

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

$$t = 3$$

24. Find the derivative of $f(x) = e^{\ln(\tan x)} = \tan x$ $f'(x) = \sec^2 x$

- (a) $\tan(x)$ (b) $\sec^2(x)$ (c) $\ln(\tan(x))\sec^2(x)$ (d) $e^{\ln(\tan(x))}$ (e) $e^{\ln(\tan(x))} \frac{1}{\tan(x)}$

25. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

- (a) -0.701 (b) -0.567 (c) -0.391 (d) -0.302 (e) -0.258

$$f'(x) = 6e^{2x}$$

$$g'(x) = 18x^2$$

$$6e^{2x} = 18x^2$$

$$e^{2x} = 3x^2$$

$$e^{2x} - 3x^2 = 0 \text{ on TI-84}$$

$$x = -0.391$$

26. Which of the following gives $\frac{dy}{dx}$ for $y = \log_{10}(2x-3)$? $= \frac{\ln(2x-3)}{\ln(10)}$

- (a) $\frac{1}{2x}$ (b) $\frac{1}{2x-3}$ (c) $\frac{2}{2x-3}$ (d) $\frac{2}{(2x-3)\ln 10}$ (e) $\frac{1}{(2x-3)\ln 10}$

$$y' = \frac{1}{(2x-3)\ln 10} \cdot 2$$

27. Which of the following gives the slope of the tangent line to the graph of $y = 2^{1-x}$ at $x = 2$?

- (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) -2 (d) 2 (e) $-\frac{\ln 2}{2}$

$$y' = \ln(2) \cdot 2^{1-x} \cdot -1$$

$$y'(2) = -\ln 2 \cdot 2^{-1} = -\frac{\ln 2}{2}$$