Name Answer Key **Date**_______________

Calc I H – Midterm Review #3 – OE **Period**_______

*****All Problems must be complete in your notebooks*****

1. Find any horizontal and vertical asymptotes for each function.

a) $f(x)=\dfrac{x^2-x-12}{2x-5}$

VA: $2x-5=0$ $\boxed{x=5/2}$

HA: $\lim\limits_{x\to\infty}\dfrac{\frac{x^2}{x}-\frac{x}{x}-\frac{12}{x}}{\frac{2x}{x}-\frac{5}{x}}=\lim\limits_{x\to\infty}\dfrac{x-1-\frac{12}{x}}{2-\frac{5}{x}}$

$=\infty$ $\boxed{\text{No HA}}$

b) $f(x)=\dfrac{x^2-4}{x^2+x-6}=\dfrac{(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$

$=\dfrac{x+2}{x+3},\ x\neq 2$

$\boxed{\text{VA: } x=-3}$

HA: $\lim\limits_{x\to\infty}\dfrac{x+2}{x+3}=\lim\limits_{x\to\infty}\dfrac{1+\frac{2}{x}}{1+\frac{3}{x}}$

$=1$ $\boxed{y=1}$

c) $f(x)=\dfrac{3x}{x^2-9}$

$(x+3)(x-3)=0$

$\boxed{\text{VA: } x=\pm 3}$

HA: $\lim\limits_{x\to\infty}\dfrac{3x/x^2}{\frac{x^2}{x^2}-\frac{9}{x^2}}$ $\boxed{\text{HA: } y=0}$

$=\lim\limits_{x\to\infty}\dfrac{\frac{3}{x}}{1-\frac{9}{x^2}}=\dfrac{0}{1}=0$

2. Find the following limits.

a) $\lim\limits_{x\to 2}\dfrac{x^2-x-12}{2x-5}=\dfrac{4-2-12}{4-5}$

$=\dfrac{-10}{-1}=\boxed{10}$

b) $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^2+x-6}$

$=\lim\limits_{x\to 2}\dfrac{(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$

$=\lim\limits_{x\to 2}\dfrac{x+2}{x+3}=\boxed{\dfrac{4}{5}}$

c) $\lim\limits_{x\to 3}\dfrac{3x^2}{x^2-9}$

DNE (VA at $x=3$)

d) $\lim\limits_{x\to\infty}\dfrac{2x^3/x^4-4x^2/x^4-1/x^4}{5x^4/x^4}$

$=\lim\limits_{x\to\infty}\dfrac{\cancel{\frac{2}{x}}\to 0-\cancel{\frac{4}{x^2}}\to 0-\cancel{\frac{1}{x^4}}\to 0}{5}=\boxed{0}$

e) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}\cdot\dfrac{3}{3}$

$=3\lim\limits_{x\to 0}\dfrac{\sin(3x)}{3x}$

$=3\cdot 1=\boxed{3}$

f) $\lim\limits_{x\to 0}\dfrac{4\tan x}{3x}$

$=\dfrac{4}{3}\lim\limits_{x\to 0}\dfrac{\sin x}{x}\cdot\dfrac{1}{\cos x}$

$=\dfrac{4}{3}\cdot 1\cdot\dfrac{1}{\cos(0)}=\boxed{\dfrac{4}{3}}$

3. Given $f(x)=\begin{cases}3x-4, & x<-2\\ 2x^2-5, & -2\le x<5\\ -2x, & x\ge 5\end{cases}$. Find the following limits.

a) $\lim\limits_{x\to -2^-}f(x)=3(-2)-4$ $=\boxed{-10}$

b) $\lim\limits_{x\to -2^+}f(x)=2(4)-5$ $=\boxed{3}$

c) $\lim\limits_{x\to -2}f(x)$ DNE

d) $\lim\limits_{x\to 5}f(x)$ DNE

$\lim\limits_{x\to 5^-}f(x)\overset{?}{=}\lim\limits_{x\to 5^+}f(x)$

$2(25)-5\neq -2(5)$

e) $f(-2)=\boxed{3}$

f) $f(5)=\boxed{-10}$

4. Find $\dfrac{dy}{dx}$ by limit definition if $y=x^3-4$.

$\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{(x+h)^3-4-(x^3-4)}{h}=\lim\limits_{h\to 0}\dfrac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-4}\cancel{-x^3}\cancel{+4}}{h}$

$=\lim\limits_{h\to 0}\dfrac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}=\boxed{3x^2}$

5. Find the value of $\boldsymbol{a}$ for which $f(x)$ is continuous.

$$f(x)=\begin{cases}3x-5, & x<2\\ 5x+\boldsymbol{a}, & x\geq 2\end{cases}$$

① $f(2) = 10+a$

② $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$3(2)-5 = 5(2)+a$

$1 = 10+a$

$\boxed{a=-9}$

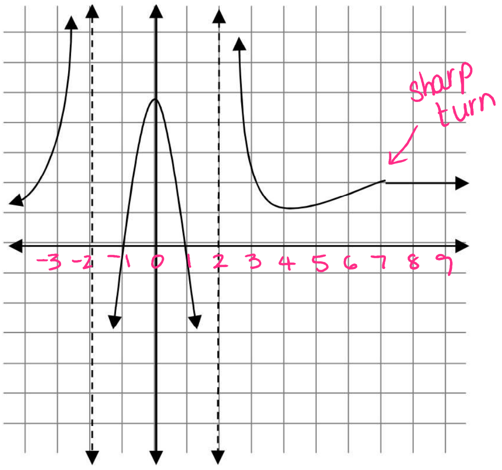
Then: $\lim_{x\to 2^+} f(x) = 5(2)-9 = 1$

And $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 1$

③ $f(2) = 10-9 = 1 = \lim_{x\to 2} f(x)$ ✓

Continuous

6. Use interval notation to answer the following questions.



a) Where $f(x)$ is differentiable $(-\infty,-2)\cup(-2,2)\cup(2,7)\cup(7,\infty)$

b) Where $f(x)$ is not continuous At $x=\pm 2$,

c) $\lim_{x\to 7} f(x) = 2$

d) Type of discontinuity at $x=2$

Infinite discontinuity

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = \infty$

7. Describe the graph of the following functions. Tell which type of function the derivative will be.

a) $y=4x-5$

y linear function

$y' = 4$

$y' \to$ constant function

b) $y=4x^2+3x-6$

y quadratic function

$y' = 8x+3$

$y' \to$ linear function

c) $y=x^3-x^2$

y cubic function

$y' = 3x^2-2x$

$y' \to$ quadractic function

8. Find a point where the slope of the tangent line to the given function $f(x)=3x^2-4$ is

$f'(x) = 6x$

a) 5 $= f'(x)$

$f'(x) = 6x = 5$

$x = \frac{5}{6}$

$f\left(\frac{5}{6}\right) = 3\left(\frac{5}{6}\right)^2 - 4$

$= -\frac{23}{12}$

$\boxed{\left(\frac{5}{6}, -\frac{23}{12}\right)}$

b) 0

$f'(x) = 6x = 0$

$x = 0$

$f(0) = -4$

$\boxed{(0,-4)}$

9. Find $\frac{dy}{dx}$.

a) $y=\cos^3(2x+4)$

$\frac{dy}{dx}=-3\cos^2(2x+4)\sin(2x+4)\cdot 2$

$=\boxed{-6\cos^2(2x+4)\sin(2x+4)}$

b) $y=(3x+4)\sin 5x$

$\frac{dy}{dx}=3\sin 5x+(3x+4)\cos(5x)\cdot 5$

$=\boxed{3\sin(5x)+5(3x+4)\cos(5x)}$

c) $y=5(3x-5)^4+\sqrt{2-5x}$

$y'=20(3x-5)^3\cdot 3+\frac{1}{2}(2-5x)^{-\frac{1}{2}}\cdot -5$

$\boxed{y'=60(3x-5)^3-\frac{5}{2\sqrt{2-5x}}}$

d) $y=\sec x-\tan^2 x$

$\frac{dy}{dx}=\sec x\tan x-2\tan x\sec^2 x$

$\frac{dy}{dx}=\boxed{\sec\tan x\left(1-2\sec x\right)}$

e) $y=\frac{\sin x}{3x-5}$

$\frac{dy}{dx}=\frac{\cos x(3x-5)-3\sin x}{(3x-5)^2}$

$=\boxed{\frac{(3x-5)\cos x-3\sin x}{(3x-5)^2}}$

f) $y=\sqrt[3]{(6-5x)^2}=(6-5x)^{2/3}$

$\frac{dy}{dx}=\frac{2}{3}(6-5x)^{-1/3}\cdot -5$

$=\boxed{\frac{-10}{3\sqrt[3]{6-5x}}}$

10. Find $\frac{d^2y}{dx^2}$.

a) $\frac{d}{dx}\left(y=\frac{2x-4}{x-5}\right)$

$2x-10 \quad -2x+4$

$\frac{dy}{dx}=\frac{2(x-5)-1(2x-4)}{(x-5)^2}$

$\frac{d}{dx}\left(\frac{dy}{dx}=\frac{-6}{(x-5)^2}\right)$

$\boxed{\frac{d^2y}{dx^2}=\frac{12}{(x-5)^3}}$

b) $\frac{d}{dx}\left(y=(3x-4)\sin x\right)$

$\frac{d}{dx}\left(\frac{dy}{dx}=3\sin x+(3x-4)\cos x\right)$

$\frac{d^2y}{dx^2}=3\cos x+3\cos x-(3x-4)\sin x$

$\boxed{\frac{d^2y}{dx^2}=6\cos x-(3x-4)\sin x}$

c) $\frac{d}{dx}\left(3x-4y^2-y=5\right)$

$3-8y\frac{dy}{dx}-\frac{dy}{dx}=0$

$3=\frac{dy}{dx}(8y+1)$

$\frac{d}{dx}\left(\frac{dy}{dx}=\frac{3}{8y+1}\right)$

$\frac{d^2y}{dx^2}=\frac{-3}{(8y+1)^2}\cdot 8\frac{dy}{dx}$

$\frac{d^2y}{dx^2}=\frac{-24}{(8y+1)^2}\cdot\frac{3}{8y+1}$

$\boxed{\frac{d^2y}{dx^2}=\frac{-72}{(8y+1)^3}}$

11. Find y'''.

a) $y = \frac{5}{x^4} - \sin x$ $= 5x^{-4} - \sin x$

$y' = -20x^{-5} - \cos x$

$y'' = 100x^{-6} + \sin x$

$y''' = -600x^{-7} + \cos x$

$\boxed{y''' = \frac{-600}{x^7} + \cos x}$

b) $y = 3\sqrt{x} - 5$ $= 3x^{1/2} - 5$

$y' = \frac{3}{2}x^{-1/2}$

$y'' = -\frac{3}{4}x^{-3/2}$

$y''' = \frac{9}{8}x^{-3/2}$

$y''' = \frac{9}{8x^{3/2}}$

$\boxed{y''' = \frac{9}{8\sqrt{x^3}}}$

12. Find $\frac{dy}{dx}$.

a) $\frac{d}{dx}\left(\sin(xy) = x - y\right)$

$\cos(xy)\left(y + x\frac{dy}{dx}\right) = 1 - \frac{dy}{dx}$

$y\cos(xy) + x\cos(xy)\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\frac{dy}{dx}\left(1 + x\cos(xy)\right) = 1 - y\cos(xy)$

$\boxed{\frac{dy}{dx} = \frac{1 - y\cos(xy)}{1 + x\cos(xy)}}$

b) $\frac{d}{dx}\left(2xy - y^2 = 5x\right)$ at $(0,0)$

$2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 5$

$\frac{dy}{dx}(2x - 2y) = 5 - 2y$

$\boxed{\frac{dy}{dx} = \frac{5-2y}{2x-2y}}$

$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{5-2(0)}{0}$

und

⇒ Non differentiable at that point

13. Find the equations of the tangent and normal lines for each of the given function.

a) $y = (4x-3)^4$, $(0,81)$

$y' = 4(4x-3)^3 \cdot 4$

$y' = 16(4x-3)^3$

$y'(0) = 16(-3)^3$

$= -432$

tan:

$y - 81 = -432x$

normal:

$y - 81 = \frac{1}{432}x$

b) $\frac{d}{dx}\left(xy - y^2 - 4y = 10\right)$, $(-3,-2)$

$y + x\frac{dy}{dx} - 2y\frac{dy}{dx} - 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x - 2y - 4) = -y$

$\frac{dy}{dx} = \frac{-y}{x - 2y - 4}$

$\left.\frac{dy}{dx}\right|_{(-3,-2)} = \frac{2}{-3+4-4} = \frac{-2}{3}$

tan:

$y + 2 = \frac{-2}{3}(x+3)$

normal:

$y + 2 = \frac{3}{2}(x+3)$

14. $\frac{d}{dt}\left(y = x^3 - 2x\right)$

a) Find $\frac{dy}{dt}$, when $x = 3$, $\frac{dx}{dt} = 2$.

$\frac{dy}{dt} = 3x^2\frac{dx}{dt} - 2\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{dx}{dt}(3x^2 - 2)$

$\frac{dy}{dt} = 2(3(3)^2 - 2)$

$\boxed{\frac{dy}{dt} = 50}$

b) Find $\frac{dx}{dt}$, when $x = 1$, $\frac{dy}{dt} = -4$.

$\frac{dx}{dt} = \frac{dy/dt}{3x^2 - 2}$

$\frac{dx}{dt} = \frac{-4}{3(1)^2 - 2}$

$\boxed{\frac{dx}{dt} = -4}$

15. A ball is tossed in the air from a bridge and its height (y, in feet) above the ground, *t* seconds after it is thrown is given by $y=-16t^2+45t+24$.

(a) What is the initial height of the bridge? What is the initial velocity of the ball?

$t=0 \quad y(0)=24 \quad \boxed{24\text{ ft tall}}$ $\qquad v(t)=y'=-32t+45 \quad \boxed{v(0)=45\ \text{ft}/\text{sec}}$

(b) When does the ball reach the ground? What is the speed of the ball when it hits the ground?

$y=0=-16t^2+45t+24$

$t=\dfrac{-45\pm\sqrt{45^2-4(-16)(24)}}{2(-16)}$

$t=\dfrac{-45\pm\sqrt{3561}}{-32}$

$t=3.271\text{ sec}, -.\cancel{459}$

$v(3.271)=-59.674\,\frac{\text{ft}}{\text{sec}}$

$\boxed{\text{Speed}=59.674\,\frac{\text{ft}}{\text{sec}}}$

(c) How long does it take the ball to reach its maximum height?

$y'=0=-32t+45$

$t=45/32$

$\boxed{t=1.406\text{ sec}}$

(d) What is the maximum height the ball reaches?

$\boxed{y(1.406)=55.641\text{ ft}}$

(e) What is the velocity function for the ball? When is its velocity -10 ft/sec?

$v(t)=-32t+45=-10 \qquad -32t=-55$

$\boxed{t=1.719\text{ sec}}$

(f) What is the velocity 2 seconds after it's thrown?

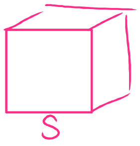
$\boxed{V(2)=-19\ \text{ft}/\text{sec}}$

(g) What is the acceleration function for the ball? What is the acceleration 1 second after it's thrown?

$a(t)=-32 \qquad \boxed{a(1)=-32\ \text{ft}/\text{s}^2}$

16. A cube of ice is melting. The sides of the cube are decreasing at the constant rate of 3 inches per hour. How fast is the volume of the cube decreasing when each side is 6 inches?

$\frac{ds}{dt}=-3\,\frac{\text{in}}{\text{hr}}$ $\qquad$ Find $\frac{dV}{dt}$ when $s=6$ in.



$\frac{d}{dt}(V=s^3)$

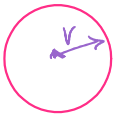
$\frac{dV}{dt}=3s^2\frac{ds}{dt}$

$\frac{dV}{dt}=3(6)^2(-3)$

$\boxed{\frac{dV}{dt}=-324\,\frac{\text{in}^3}{\text{hr}}}$

17. Air is filling a spherical balloon at the rate of 7 cubic inches per minute. When the radius is 10 inches, how fast is the radius increasing?

$\frac{dV}{dt} = 7 \frac{in^3}{min}$ Find $\frac{dr}{dt}$ when $r = 10 in$



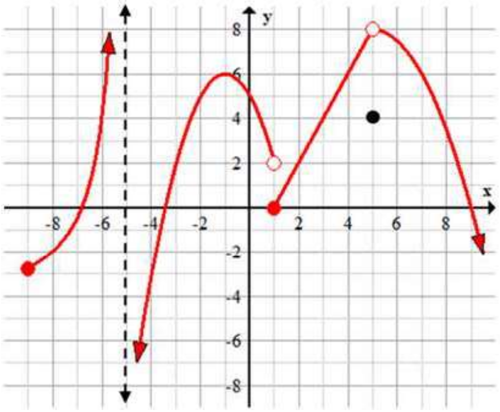
$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2}$

$\frac{dr}{dt} = \frac{7}{4\pi(10)^2}$

$$\boxed{\frac{dr}{dt} = \frac{7}{400\pi} \frac{in}{min} \approx .006 \frac{in}{min}}$$



18. For the function above, find:

(a) the *x*-coordinate of EACH point of discontinuity

(b) the type of EACH discontinuity

(c) the conditions of continuity that are not satisfied for EACH discontinuity.

Discontinuities	Type	Conditions
$x = -5$	Infinite, Non-Remov	$\lim_{x \to -5^-} f(x) = \infty \neq \lim_{x \to -5^+} f(x) = -\infty$ $f(-5)$ und
$x = 1$	Jump, Non-Remov	$\lim_{x \to 1^-} f(x) = 2 \neq \lim_{x \to 1^+} f(x) = 0$ $\therefore \lim_{x \to 1} f(x)$ DNE
$x = 5$	Hole, Remov	$f(5) = 4, \lim_{x \to 5} f(x) = 8$ $\therefore f(5) \neq \lim_{x \to 5} f(x)$

Cont on $(-\infty, -5) \cup (-5, 1) \cup (1, 5) \cup (5, \infty)$

19. Find $\frac{dy}{dx}$ for each of the following functions:

a) $\frac{d}{dx}\left(y = 3e^{4x^2+1}\right)$

$\frac{dy}{dx} = 3e^{4x^2+1}(8x)$

$\boxed{\frac{dy}{dx} = 24xe^{4x^2+1}}$

b) $y = \ln\left(\frac{4x^5}{3x^2+1}\right)$

$y = \ln 4 + 5\ln x - \ln(3x^2+1)$

$\boxed{y' = \frac{5}{x} - \frac{6x}{3x^2+1}}$

c) $\frac{d}{dx}\left(3xe^{5y} = 10x^2\right)$

$3e^{5y} + 3x \cdot 5e^{5y}\frac{dy}{dx} = 20x$

$15xe^{5y}\frac{dy}{dx} = 20x - 3e^{5y}$

$\boxed{\frac{dy}{dx} = \frac{20x - 3e^{5y}}{15xe^{5y}}}$

c) $4y = \ln(3xy)$

$\frac{d}{dx}\left(4y = \ln 3 + \ln x + \ln y\right)$

$4\frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}\frac{dy}{dx}$

$\left(4 - \frac{1}{y}\right)\frac{dy}{dx} = \frac{1}{x}$

$\frac{4y-1}{y}\frac{dy}{dx} = \frac{1}{x} \nearrow$ $\boxed{\frac{dy}{dx} = \frac{y}{x(4y-1)}}$

20. Find all extrema of $f(x) = 2x^3 - 6x$ on the interval $[0,3]$.

$f'(x) = 6x^2 - 6$

$f'(x) = 0$ $\qquad$ $f'(x)$ und

$6(x^2-1) = 0$ $\qquad$ none

$x = \pm 1$ $\qquad$ $\cancel{x=-1}$

not on interval

$f(0) = 0$

$f(1) = -4$ Abs min $(1,-4)$

$f(3) = 36$ Abs max $(3,36)$

21. Determine whether Rolle's Theorem can be applied to $h(x) = x^2 - 8x + 5$ on the interval $[2,6]$. If it can be applied, find the value of c that satisfies the theorem. If it cannot be applied state why.

$h(x)$ is a polynomial function, ∴ everywhere cont & diff. $h(x)$ is cont on $[2,6]$ & diff on $(2,6)$. $h(2) = -7 = h(6)$. Rolle's Thm applies.

$h'(x) = 2x - 8 = 0$

$x = 4$

$\boxed{c = 4}$

22. Determine whether Mean Value Theorem can be applied to $g(x) = x^4 - 8x$ on the interval $[0,2]$. If it can be applied, find the value of c that satisfies the theorem. If it cannot be applied state why.

$g(x)$ is a polynomial function, ∴ everywhere cont & diff. $g(x)$ is cont on $[0,2]$ & diff on $(0\ 2)$. MVT applies.

$g(0) = 0$, $g(2) = 0$

$g'(x) = 4x^3 - 8 = 0$

$4(x^3 - 2) = 0$

$x = \sqrt[3]{2}$

$\boxed{c = \sqrt[3]{2}}$