Name <u>Answer key</u> Date______

Calc I H – Midterm Review #1 - Limits Period______

Directions: Evaluate each of the following limits.

1.) $\lim_{x\to 8}\left(x^2-5x-11\right) = 8^2-5(8)-11 = \boxed{13}$

2.) $\lim_{x\to 5}\left(\frac{x+3}{x^2-15}\right) = \frac{5+3}{25-15} = \frac{8}{10} = \boxed{\frac{4}{5}}$

3.) $\lim_{x\to 0}\pi^2 = \boxed{\pi^2}$

4.) $\lim_{x\to 3}\left(\frac{x^2-2x-3}{x-3}\right) = \lim_{x\to 3}\frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}} = 3+1 = \boxed{4}$

5.) $\lim_{x\to\infty}\left(\frac{10x^2+25x+1}{x^4-8}\right) = \lim_{x\to\infty}\frac{\frac{10x^2}{x^4}+\frac{25x}{x^4}+\frac{1}{x^4}}{1-\frac{8}{x^4}} = \lim_{x\to\infty}\frac{\frac{10}{x^2}+\frac{25}{x^3}+\frac{1}{x^4}}{1-8/x^4} = \frac{0}{1} = \boxed{0}$

6.) $\lim_{x\to\infty}\left(\frac{x^4-8}{10x^2+25x+1}\right) = \lim_{x\to\infty}\frac{\frac{x^4}{x^2}-\frac{8}{x^2}}{10+\frac{25x}{x^2}+\frac{1}{x^2}} = \lim_{x\to\infty}\frac{x^2-8/x^2}{10+25/x+1/x^2} = \lim_{x\to\infty}\frac{x^2}{10} = \boxed{\infty}$

7.) $\lim_{x\to\infty}\left(\frac{(x^4-8)/x^4}{(10x^4+25x+1)/x^4}\right) = \lim_{x\to\infty}\frac{1-\frac{8}{x^4}}{10+\frac{25}{x^3}+\frac{1}{x^4}} = \boxed{\frac{1}{10}}$

8.) $\lim_{x\to\infty}\left(\frac{\sqrt{5x^4+2x}}{x^2}\right) \Rightarrow \lim_{x\to\infty}\frac{\sqrt{5}x^2}{x^2} = \boxed{\sqrt{5}}$

9.) $\lim_{x\to 6^+}\left(\frac{x+2}{x^2-4x-12}\right) = \boxed{\infty}$ VA at $x=6$

$(x-6)(x+2)$

10.) $\lim_{x\to 6^-}\left(\frac{x+2}{x^2-4x-12}\right) = \boxed{-\infty}$

11.) $\lim_{x\to 6}\left(\frac{x+2}{x^2-4x-12}\right)$ DNE

12.) $\lim_{x\to 0^+}\left(\frac{x}{|x|}\right) = \boxed{1}$

13.) $\lim_{x\to 0^-}\left(\frac{x}{|x|}\right) = \boxed{-1}$

14.) $\lim_{x\to 7^+}\left(\frac{x}{x^2-49}\right) = \boxed{\infty}$ VA at $x=7$

$(x+7)(x-7)$

15.) $\lim_{x\to 7}\left(\frac{x}{x^2-49}\right)$ DNE

16.) $\lim_{x\to 7}\frac{x}{(x-7)^2} = \infty$ or DNE

17.) Let $f(x)=\begin{cases}x^2-5 & x\le 3\\ x+2 & x>3\end{cases}$

Find: (a) $\lim_{x\to 3^-}f(x) = 3^2-5 = \boxed{4}$ (b) $\lim_{x\to 3^+}f(x)$ $3+2 = \boxed{5}$ (c) $\lim_{x\to 3}f(x)$ DNE

$\lim_{x\to 3^-}f(x) \ne \lim_{x\to 3^+}f(x)$

18.) $f(x)=\begin{cases}x^2-5 & x\le 3\\ x+1 & x>3\end{cases}$

Find: (a) $\lim_{x\to 3^-}f(x) = 3^2-5 = \boxed{4}$ (b) $\lim_{x\to 3^+}f(x)$ $3+1 = \boxed{4}$ (c) $\lim_{x\to 3}f(x) = \boxed{4}$

19.) $\lim_{x\to\frac{\pi}{4}}3\cos x = 3\cos(\pi/4) = \frac{3\sqrt{2}}{2}$ or $\boxed{\frac{3}{\sqrt{2}}}$

20.) $\lim_{x\to 0}3\frac{x}{\cos x} = 3\left(\frac{0}{\cos 0}\right) = 3\left(\frac{0}{1}\right) = \boxed{0}$

21.) $\lim_{x\to 0}3\frac{x}{\sin x} = 3\underbrace{\lim_{x\to 0}\frac{x}{\sin x}}_{1} = 3\cdot 1 = \boxed{3}$

22.) $\lim_{x\to 0}\frac{\sin 3x}{\sin 8x} = \lim_{x\to 0}\frac{3\sin 3x}{3x}\cdot\frac{8x}{8\sin 8x} = \frac{3}{8}\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\frac{8x}{\sin 8x} = \frac{3}{8}\cdot 1\cdot 1 = \boxed{\frac{3}{8}}$

23.) $\lim_{x\to 0}\frac{\tan 7x}{\sin 5x} = \lim_{x\to 0}\frac{\sin(7x)}{\cos(7x)}\cdot\frac{1}{\sin(5x)}$

$= \lim_{x\to 0}\frac{1}{\cos(7x)}\cdot\frac{7\sin(7x)}{7x}\cdot\frac{5x}{5\sin(5x)} = \frac{7}{5}\lim_{x\to 0}\frac{1}{\cos 7x}\cdot\frac{\sin(7x)}{7x}\cdot\frac{5x}{\sin 5x} = \frac{7}{5}\cdot\frac{1}{\cos(0)}\cdot 1\cdot 1 = \boxed{\frac{7}{5}}$

24.) $\lim_{x\to\infty}\sin x$ DNE (oscillates)

25.) $\lim_{x\to\infty} \sin \frac{1}{x} \xrightarrow{} 0 = \boxed{0}$

26.) $\lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)} = \lim_{h\to 0} \frac{\frac{x-x-h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-h}{x(x+h)} \cdot \frac{1}{h} = \boxed{\frac{-1}{x^2}}$

27.) $\lim_{x\to 0} \frac{\sin^2 7x}{\sin^2 11x} = \lim_{x\to 0} \frac{7^2 \sin^2 7x}{(7x)^2} \cdot \frac{(11x)^2}{11^2 \sin^2 11x}$

$= \frac{49}{121} \lim_{x\to 0} \frac{\sin^2 7x}{(7x)^2} \cdot \frac{(11x)^2}{\sin(11x)}$

$= \frac{49}{121} \cdot 1 \cdot 1 = \boxed{\frac{49}{121}}$

28.) $\lim_{h\to 0} \frac{(3+h)^2 - 9}{h}$

$= \lim_{h\to 0} \frac{\cancel{9} + 6h + h^2 - \cancel{9}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(6+h)}{\cancel{h}} = \boxed{6}$

ANSWERS:

1.) 13
2.) $\frac{4}{5}$
3.) π^2
4.) 4
5.) 0
6.) ∞
7.) $\frac{1}{10}$
8.) $\sqrt{5}$
9.) ∞
10.) $-\infty$
11.) DNE
12.) 1
13.) -1
14.) ∞
15.) DNE
16.) ∞
17.) (a) 4; (b) 5; (c) DNE
18.) (a) 4; (b) 4; (c) 4
19.) $\frac{3}{\sqrt{2}}$
20.) 0
21.) 3
22.) $\frac{3}{8}$
23.) $\frac{7}{5}$
24.) DNE
25.) 0
26.) $-\frac{1}{x^2}$
27.) $\frac{49}{121}$
28.) 6