Name Answer Key **Date**__________

Calc I H – Midterm Review #2 - MC **Period**______

1. Let $f(x) = 3x + 1$ and $g(x) = x^2$. Find the limits:

   (a) $\lim_{x\to 3} f(x)$ $= 3(3)+1 = 10$ (b) $\lim_{x\to 5} g(x)$ $= 5^2 = 25$ (c) $\lim_{x\to -3} g(f(x))$ $= ((3(-3)+1))^2 = (-8)^2 = 64$

   A) 9, 25, 28 B) 3, 5, 9 C) 4, 2, 28 D) 9, 5, 1 $\boxed{\text{E) 10, 25, 64}}$

2. Find the limit: $\lim_{x\to \frac{5\pi}{6}} \sin x$ $= \sin\frac{5\pi}{6} = \frac{1}{2}$

   A) $-\frac{1}{2}$ $\boxed{\text{B) } \frac{1}{2}}$ C) $\frac{1}{4}$ D) $-\frac{1}{4}$ E) Does not exist

3. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point. $\lim_{x\to 1} \frac{x^3 - 1}{x - 1}$ $= \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}} = 1+1+1 = 3$

   $\boxed{\text{A) 3, } x^2 + x + 1}$ B) 1, $x^2 - x + 1$ C) 1, $x^2 + x - 1$ D) -3, $x^2 + x + 1$ E) Does not exist

4. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point. $\lim_{x\to -9} \frac{10x^2 + 105x + 135}{x + 9}$ $= \lim_{x\to -9} \frac{5(2x^2+21x+27)}{x+9} = \lim_{x\to -9} \frac{5(2x+3)\cancel{(x+9)}}{\cancel{x+9}} = \lim_{x\to -9} 5(2x+3) = 5(-18+3) = -75$

   A) Does not exist B) -105, $10x - 15$ C) 105, $-10x + 15$ D) 75, $-10x - 15$ $\boxed{\text{E) -75, } 10x + 15}$

5. Find the limit (if it exists): $\lim_{x\to 11} \frac{-x + 11}{x^2 - 121}$ $= \lim_{x\to 11} \frac{-1\cancel{(x-11)}}{(x+11)\cancel{(x-11)}} = \lim_{x\to 11} \frac{-1}{x+11} = \frac{-1}{22}$

   A) -44 B) $\frac{1}{22}$ $\boxed{\text{C) } -\frac{1}{22}}$ D) 11 E) $-\frac{1}{44}$

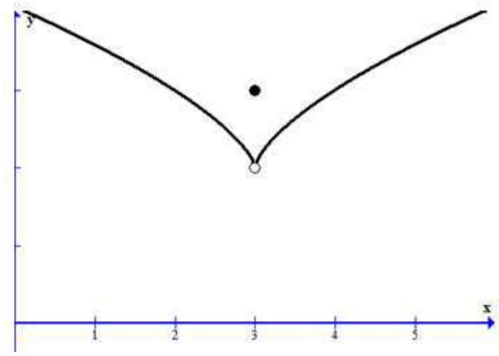
6. Determine the limit (if it exists): $\lim_{x\to 0} \frac{-4(1 - \cos x)}{x}$ $= -4 \lim_{x\to 0} \frac{1-\cos x}{x} = -4(0) = 0$

   $\boxed{\text{A) 0}}$ B) -8 C) -16 D) Does not exist E) 4

7. Use the graph as shown to determine the following limits, and discuss the continuity of the function at $x=3$.

(i) $\lim\limits_{x\to 3^+} f(x)$ $=2$ (ii) $\lim\limits_{x\to 3^-} f(x)$ $=2$ (iii) $\lim\limits_{x\to 3} f(x)$ $=2$

Hole discontinuity at $x=3$



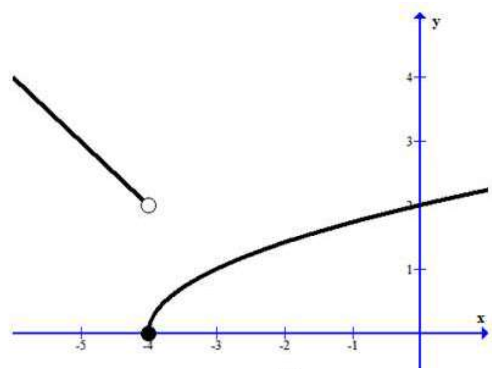
A) 4, 4, 4, Not continuous B) 3, 3, 3, Not continuous C) 2, 2, 2, Continuous

D) 3, 3, 3, Continuous E) 2, 2, 2, Not continuous

8. Use the graph to determine the following limits, and discuss the continuity of the function at $x=-4$.

(i) $\lim\limits_{x\to -4^+} f(x)$ $=0$ (ii) $\lim\limits_{x\to -4^-} f(x)$ $=2$ (iii) $\lim\limits_{x\to -4} f(x)$ DNE

Jump discontinuity at $x=-4$



A) 2, 0, Does not exist, Not continuous B) 0, 2, Does not exist, Not continuous

C) 0, 2, 0, Continuous D) -4, 0, Does not exist, Not continuous

E) 2, -2, Does not exist, Not continuous

9. Find the limit: $\lim\limits_{x\to 8^-} \dfrac{x-5}{-x+8}$ VA at $x=8$

A) ∞ B) $-\infty$ C) 0 D) -1 E) 1

10. Find the limit: $\lim\limits_{x\to -7} \dfrac{x^2+7x}{(x^2+49)(x+7)}$ $= \lim\limits_{x\to -7} \dfrac{x\cancel{(x+7)}}{(x^2+49)\cancel{(x+7)}} = \dfrac{-7}{98} = \dfrac{-1}{14}$

A) 14 B) $\dfrac{1}{14}$ C) -14 D) $-\dfrac{1}{14}$ E) -7

11. Find $\frac{dy}{dx}$ by implicit differentiation. $\frac{d}{dx}\left(\sin x + 7\cos 7y = 5\right)$ $\cos x - 7\sin(7y)\cdot 7\frac{dy}{dx} = 0$

A) $\boxed{\frac{dy}{dx} = \frac{\cos x}{49\sin 7y}}$ B) $\frac{dy}{dx} = -\frac{\cos x}{49\sin 7y}$ C) $\frac{dy}{dx} = \frac{\cos x}{7\sin 7y}$ $\frac{dy}{dx} = \frac{\cos x}{49\sin(7y)}$

D) $\frac{dy}{dx} = \frac{\cos x}{49\cos 7y}$ E) $\frac{dy}{dx} = \frac{\cos x}{49\sin y}$

12. Find the second derivative of the function. $f(x) = \sin(3x^4)$ $f'(x) = \cos(3x^4)\cdot 12x = 12x^3\cos(3x^4)$

A) $f''(x) = 48x^2\cos 3x^4 - 144x^6\sin 3x^4$ B) $f''(x) = 36x^2\cos 3x^4 + 144x^6\sin 3x^4$

C) $\boxed{f''(x) = 36x^2\cos 3x^4 - 144x^6\sin 3x^4}$ D) $f''(x) = 12x^2\cos 3x^4 - 144x^6\sin 3x^4$

E) $f''(x) = 12x^2\cos 3x^4 + 12x^6\sin 3x^4$

$f''(x) = 36x^2\cos(3x^4) - 12x^3\sin(3x^4)\cdot 12x^3$
$= 36x^2\cos(3x^4) - 144x^6\sin(3x^4)$

13. Find an equation of the tangent line for the graph of $f(x) = \left(2x^3+6\right)^2$ at the point $(-1,16)$.

$f'(x) = 2(2x^3+6)\cdot 6x^2$
$f'(x) = 12x^2(2x^3+6)$

A) $\boxed{y-16 = 48(x+1)}$ B) $y+16 = 48(x-1)$ C) $y-16 = -48(x+1)$

D) $y-16 = 24(x+1)$ E) $y+16 = 24(x-1)$

$y-16 = 48(x+1)$ $f'(-1) = 12(-1)^2(2(-1)^3+6)$
$= 12(4) = 48$

14. Evaluate the derivative of the function $f(t) = \frac{2t^2+4}{4t-1}$ at the point $\left(2, \frac{12}{7}\right)$. $f'(t) = \frac{4t(4t-1) - 4(2t^2+4)}{(4t-1)^2}$

A) $f'(2) = \frac{8}{7}$ B) $f'(2) = -\frac{8}{7}$ C) $\boxed{f'(2) = \frac{8}{49}}$ D) $f'(2) = -\frac{8}{49}$ E) $f'(2) = \frac{8}{343}$

$f'(2) = \frac{8(4) - 4(2) - 16}{7^2} = \frac{8}{49}$

$f'(t) = \frac{16t^2 - 4t - 8t^2 - 16}{(4t-1)^2}$

$f'(t) = \frac{8t^2 - 4t - 16}{(4t-1)^2}$

15. Evaluate the derivative of the function $y = \sqrt[5]{7x^5+3x}$ at $x = 3$.

A) $y'(3) = \frac{2838}{5(1710)^{\frac{6}{5}}}$ B) $y'(3) = \frac{2838}{5(1710)^{\frac{3}{5}}}$ C) $y'(3) = \frac{1419}{5(1710)^{\frac{4}{5}}}$

D) $\boxed{y'(3) = \frac{2838}{5(1710)^{\frac{4}{5}}}}$ E) $y'(3) = \frac{2838}{(1710)^{\frac{4}{5}}}$

$y' = \frac{1}{5}(7x^5+3x)^{-4/5}(35x^4+3)$

$y' = \frac{35x^4+3}{5(7x^5+3x)^{4/5}}$

$y'(3) = \frac{2838}{5(1710)^{4/5}}$

16. Find the derivative of the function. $f(t) = 4\sec^2(5\pi t - 1)$

$f'(t) = 8\sec(5\pi t - 1) \cdot \sec(5\pi t - 1)\tan(5\pi t - 1) \cdot 5\pi$

$\boxed{\text{A) } f'(t) = 40\pi \sec^2(5\pi t - 1)\tan(5\pi t - 1)}$ B) $f'(t) = 40\sec^2(5\pi t - 1)\tan(5\pi t - 1)$

C) $f'(t) = 5\pi \sec^2(5\pi t - 1)\tan(5\pi t - 1)$ D) $f'(t) = 20\pi \sec^2(5\pi t - 1)\tan(5\pi t - 1)$

E) $f'(t) = 40\pi \sec^2(5\pi t - 1)\tan(1 - 5\pi t)$

$f'(t) = 40\pi \sec^2(5\pi t - 1)\tan(5\pi t - 1)$

17. Find the derivative of the function. $f(\theta) = \frac{5}{13}\sin^2 5\theta$

$f'(\theta) = \frac{10}{13}\sin(5\theta)\cos(5\theta) \cdot 5$

$= \frac{50}{13}\sin(5\theta)\cos(5\theta)$

A) $f'(\theta) = \frac{5\sin 5\theta \cos 5\theta}{13}$ $\boxed{\text{B) } f'(\theta) = \frac{50\sin 5\theta \cos 5\theta}{13}}$ C) $f'(\theta) = \frac{50\cos 5\theta}{13}$

D) $f'(\theta) = -\frac{50\sin 5\theta \cos 5\theta}{13}$ E) $f'(\theta) = \frac{50\sin 5\theta}{13}$

$x^2 + 2 - 2x^2 - 10x$

18. Find the derivative of the function. $g(x) = \left(\frac{x+5}{x^2+2}\right)^6$

$g'(x) = 6\left(\frac{x+5}{x^2+2}\right)^5 \left(\frac{1(x^2+2) - 2x(x+5)}{(x^2+2)^2}\right)$

$= \frac{6(x+5)^5(-x^2 - 10x + 2)}{(x^2+2)^7}$

A) $g'(x) = \frac{6(x^2 - 10x + 2)}{(x+5)(x^2+2)}\left(\frac{x+5}{(x^2+2)}\right)^6$ B) $g'(x) = \frac{6(-x^2 + 10x + 2)(x+5)^5}{(x^2+2)^7}$

C) $g'(x) = \frac{6(-x^2 - 10x + 2)(x+5)^7}{(x^2+2)^5}$ D) $g'(x) = -\frac{6(-x^2 - 10x + 2)(x+5)^5}{(x^2+2)^7}$

$\boxed{\text{E) } g'(x) = \frac{6(-x^2 - 10x + 2)(x+5)^5}{(x^2+2)^7}}$

19. Find the derivative of the function. $f(x) = x^3(5+8x)^6$

$f'(x) = 3x^2(5+8x)^6 + x^3 \cdot 6(5+8x)^5 \cdot 8$

$= 3x^2(5+8x)^6 + 48x^3(5+8x)^5$

A) $f'(x) = 3x^5(5+8x)^2(5+24x)$ B) $f'(x) = 24x^3(5+8x)^5(5+24x)$

C) $f'(x) = 3x^2(5+8x)^6(5+24x)$ $\boxed{\text{D) } f'(x) = 3x^2(5+8x)^5(5+24x)}$

E) $f'(x) = x^2(5+8x)^5(15+8x)$

GCF: $3x^2(5+8x)^5$

$f'(x) = 3x^2(5+8x)^5(5 + 8x + 16x)$

$= 3x^2(5+8x)^5(5+24x)$

20. Find the derivative of the function. $f(t) = (8+7t)^{\frac{2}{5}}$

$f'(t) = \frac{2}{5}(8+7t)^{-3/5}(7)$

$= \frac{14}{5(8+7t)^{3/5}}$

A) $f'(t) = \frac{8}{5(8+7t)^{\frac{3}{5}}}$ B) $f'(t) = \frac{7}{(8+7t)^{\frac{3}{5}}}$ C) $f'(t) = \frac{14}{5(8+7t)^{\frac{3}{2}}}$

D) $f'(t) = \frac{7}{5(8+7t)^{\frac{3}{5}}}$ $\boxed{\text{E) } f'(t) = \frac{14}{5(8+7t)^{\frac{3}{5}}}}$.