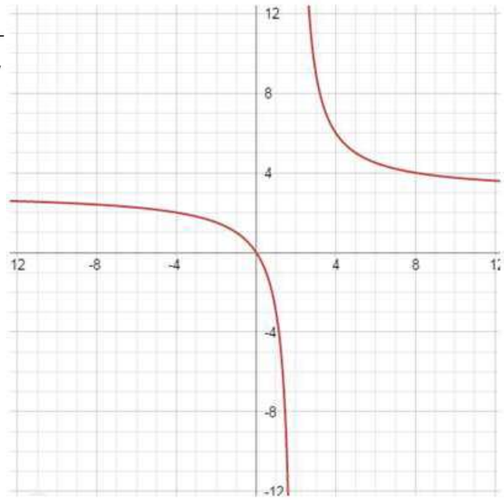
Name Answer Key   Date

Calc I H – Ch 1 Review 2   Period

Use the graph to determine the limit (if it exists).

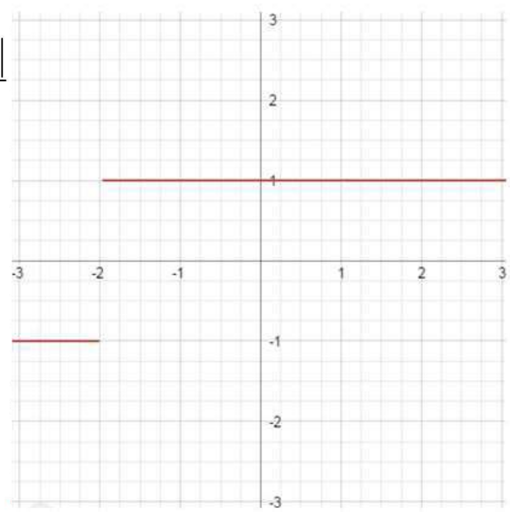
1) $g(x)=\dfrac{3x}{x-2}$



(a) $\lim\limits_{x\to 2^+} g(x) = \infty$

(b) $\lim\limits_{x\to 0} g(x) = 0$

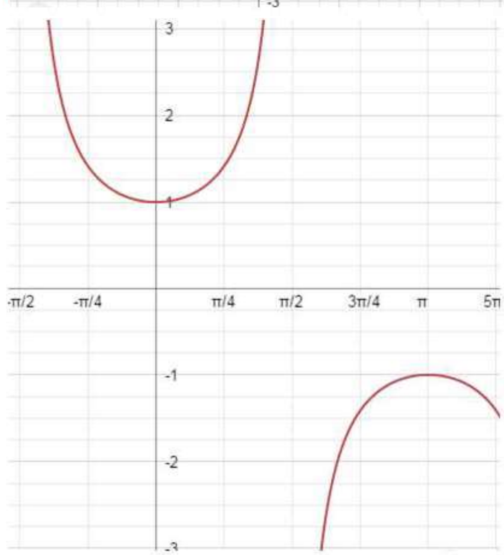
2) $h(x)=\dfrac{|x+2|}{x+2}$



(a) $\lim\limits_{x\to -3} h(x) = -1$

(b) $\lim\limits_{x\to -2} h(x)$ DNE

3) $f(x)=\sec x$



(a) $\lim\limits_{x\to 0} f(x) = 1$

(b) $\lim\limits_{x\to \frac{\pi}{2}} f(x)$ DNE

Find the limit (if it exists)

4) $\lim_{x\to 2}\frac{3x+5}{5x-3}$

$= \frac{3(2)+5}{5(2)-3} = \boxed{\frac{11}{7}}$

5) $\lim_{x\to -3}\frac{x+3}{x^2-9}$

$= \lim_{x\to -3}\frac{\cancel{x+3}}{(\cancel{x+3})(x-3)}$

$= \lim_{x\to -3}\frac{1}{x-3}$

$= \boxed{-\frac{1}{6}}$

6) $\lim_{x\to 0}\frac{(\sqrt{4+x}-2)}{x}\frac{(\sqrt{4+x}+2)}{(\sqrt{4+x}+2)}$

$= \lim_{x\to 0}\frac{\cancel{4}+x-\cancel{4}}{x(\sqrt{4+x}+2)}$

$= \lim_{x\to 0}\frac{\cancel{x}^{1}}{\cancel{x}(\sqrt{4+x}+2)}$

$= \lim_{x\to 0}\frac{1}{\sqrt{4+x}+2} = \boxed{\frac{1}{4}}$

7) $\lim_{x\to 0}\frac{\frac{1}{x+1}-1\left(\frac{x+1}{x+1}\right)}{x}$

$= \lim_{x\to 0}\frac{\frac{1-(x+1)}{x+1}}{x}$

$= \lim_{x\to 0}\frac{\cancel{1}-x-\cancel{1}}{x+1}\cdot\frac{1}{x}$

$= \lim_{x\to 0}\frac{-\cancel{x}}{x+1}\cdot\frac{1}{\cancel{x}} = \lim_{x\to 0}\frac{-1}{x+1} = \boxed{-1}$

8) $\lim_{x\to -2}\frac{x^2-4}{x^3+8}$

$= \lim_{x\to -2}\frac{(\cancel{x+2})(x-2)}{(\cancel{x+2})(x^2-2x+4)}$

$= \lim_{x\to -2}\frac{x-2}{x^2-2x+4}$

$= \frac{-4}{12} = \boxed{-\frac{1}{3}}$

9) $\lim_{x\to 0^+}\left(x-\frac{1}{x^3}\right) = -\infty$

10) $\lim_{x\to -2^-}\frac{2x^2+x+1}{x+2}$ $\boxed{=-\infty}$

11) $\lim_{x\to -1^+}\frac{x^2-2x+1}{x+1}$

$= \lim_{x\to -1^+}\frac{(x-1)^2}{x+1}$

$= \boxed{\infty}$

12) $\lim_{x\to 0^+}\frac{\sin 4x}{5x}\cdot\frac{4}{4}$

$= \lim_{x\to 0^+}\frac{4}{5}\cdot\frac{\sin 4x}{4x}$

$= \frac{4}{5}\cdot 1 = \boxed{\frac{4}{5}}$

13) $\lim_{x\to 0^+}\frac{\sec x}{x} = \lim_{x\to 0^+}\frac{1}{x\cos x} = \boxed{\infty}$

Find all points of discontinuity, identify each type of discontinuity, and identify the ~~first~~ condition*s* of continuity that ~~is~~ *are* not satisfied for each discontinuity.

14) $f(x)=\begin{cases}5-x & x\le 2\\ 2x-3 & x>2\end{cases}$

$\lim_{x\to 2^-} f(x) \overset{?}{=} \lim_{x\to 2^+} f(x)$

$5-2 = 2(2)-3$

$3 \neq 1$

$\lim_{x\to 2} f(x)$ DNE

At $x=2$, Nonremovable Jump

15) $f(x)=\dfrac{3}{x+1}$

$x=-1$ Non Remov Infinite

$\lim_{x\to -1^-} f(x) = -\infty$

$\lim_{x\to -1^+} f(x) = \infty$

$\lim_{x\to -1} f(x)$ DNE

$f(-1)$ und

16) $g(x)=\dfrac{x+1}{2x+2} = \dfrac{\cancel{x+1}}{2(\cancel{x+1})} = \dfrac{1}{2}$

$x=-1$ Removable hole

$f(-1)$ und

Note: $\lim_{x\to -1} g(x) = \frac{1}{2}$

Determine the value of *c* such that the function is continuous on the entire real number line.

17) $f(x)=\begin{cases}x+3 & x\le 2\\ cx+6 & x>2\end{cases}$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$2+3 = 2c+6$

$5 = 2c+6$

$-1 = 2c$

$\boxed{c = -1/2}$

Check: $f(2)=5$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$5 = -\frac{1}{2}(2)+6$

$5 = 5$ ✓

$f(2) = \lim_{x\to 2} f(x)$ ✓

Find the vertical asymptotes (if any) of the function.

18) $g(x)=1+\dfrac{2}{x}$

$x=0$

$\lim_{x\to 0} g(x)$ DNE

19) $h(x)=\dfrac{4x}{4-x^2}$

$x=\pm 2$

$\lim_{x\to 2} h(x)$ DNE

$\lim_{x\to -2} h(x)$ DNE

20) $f(x)=\dfrac{6\cancel{x}\cancel{(x-2)}\cancel{(x+7)}}{\cancel{x}\cancel{(x+7)}\cancel{(x-2)}} = 6$

None!

Find the limit.

21) $\lim_{x\to\infty}\dfrac{2x^2}{3x^2+5}$

$= \lim_{x\to\infty}\dfrac{2}{3+\cancel{\frac{5}{x^2}}\to 0}$

$= \boxed{\dfrac{2}{3}}$

22) $\lim_{x\to\infty}\dfrac{2x}{3x^2+5}$

$= \lim_{x\to\infty}\dfrac{\cancel{\frac{2}{x}}\to 0}{3+\cancel{\frac{5}{x^2}}\to 0}$

$= \boxed{0}$

23) $\lim_{x\to\infty}\dfrac{3x/|x|}{\sqrt{x^2+4}/\sqrt{x^2}}$

$= \lim_{x\to\infty}\dfrac{\frac{3x}{|x|}}{\sqrt{1+\cancel{\frac{4}{x^2}}\to 0}} = \boxed{3}$

Find any vertical and horizontal asymptotes of the graph of the function.

24) $f(x)=\dfrac{2x-3}{x-4}$

VA: $\lim_{x\to 4} f(x)$ DNE

$\boxed{x=4}$

HA: $\lim_{x\to\infty} \dfrac{2x-3}{x-4}$

$= \lim_{x\to\infty} \dfrac{2-\cancel{\frac{3}{x}}\to 0}{1-\cancel{\frac{4}{x}}\to} = 2$

$\boxed{y=2}$

25) $g(x)=\dfrac{5x^3}{x^2+2}$

VA: None $x^2+2\neq 0$

HA: $\lim_{x\to\infty} \dfrac{5x^3}{x^2+2} = \lim_{x\to\infty} \dfrac{5x}{1+\cancel{\frac{2}{x^2}}\to 0}$

$=\infty$

None.

Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

26) $\lim_{x\to 0}\dfrac{|x|}{x}=1$

False

$\lim_{x\to 0}$ DNE

27) $\lim_{x\to 0}x^3 = 0$

True

28) If $\lim_{x\to c}f(x)=L$, then $f(c)=L$

False

$\Rightarrow$ Not necessarily true –
Could be a hole
only true if continuous

Answer Key

1) a) ∞ b) 0   2) a)-1 b) DNE   3) a)1 b) DNE   4) $\dfrac{11}{7}$ 5) $-\dfrac{1}{6}$

6) $\dfrac{1}{4}$ 7) -1   8) $-\dfrac{1}{3}$ 9) $-\infty$ 10) $-\infty$ 11) ∞ 12) $\dfrac{4}{5}$

13) ∞ 14) x = 2, Non-Remov, Jump, $\lim_{x\to 2}f(x)=DNE$

15) x = -1, Non-Remov, Infinite, $f(-1)=und$, $\lim_{x\to -1^-}f(x)=-\infty$, $\lim_{x\to -1^+}f(x)=\infty$

16) x = -1, Removable hole, $f(-1)=und$ 17) $c=-\dfrac{1}{2}$ 18) $x=0$

19) $x=\pm 2$ 20) None   21) $\dfrac{2}{3}$ 22) 0   23) 3

24) Vertical x = 4, Horizontal y = 2   25) None

26) False   27) True   28) False