

Name _____

Ch 1 Review

I. Evaluate the following limits algebraically.

1. $\lim_{x \rightarrow 0} \frac{\sin(5x)}{x} \cdot \frac{5}{5}$
 $\lim_{x \rightarrow 0} 5 \cdot \frac{\sin(5x)}{5x}$
 $= 5 \cdot 1 = \boxed{5}$

2. $\lim_{x \rightarrow 0} \frac{3 - \frac{1}{x+3}}{x}$
 $= \lim_{x \rightarrow 0} \frac{3 - (x+3)}{3(x+3)}$
 $= \lim_{x \rightarrow 0} \frac{\cancel{3} - x - \cancel{3}}{3(x+3)} \cdot \frac{1}{x}$
 $= \lim_{x \rightarrow 0} \frac{-1}{3(x+3)} = \boxed{-\frac{1}{9}}$

3. $\lim_{x \rightarrow -2} (3x^2 - 2x + 1)$
 $= 3(-2)^2 - 2(-2) + 1$
 $= 12 + 4 + 1$
 $= \boxed{17}$

4. $\lim_{x \rightarrow \infty} \frac{4x^2 + 6x - 3}{2x^2 + 1}$
 $= \lim_{x \rightarrow \infty} \frac{4 + \frac{6}{x} - \frac{3}{x^2}}{2 + \frac{1}{x^2}}$
 $= \lim_{x \rightarrow \infty} \frac{4}{2} = \boxed{2}$

5. $\lim_{x \rightarrow \infty} \left(3 + \frac{\sin x}{x} \right)$
 $= 3 + 0$
 $= \boxed{3}$

6. $\lim_{x \rightarrow 9} \frac{x^2 + 6x - 27}{x + 9}$
 $= \lim_{x \rightarrow 9} \frac{(x+9)(x-3)}{x+9}$
 $= \lim_{x \rightarrow 9} (x-3)$
 $= \boxed{-12}$

Note: HA $y=2$!

Note: HA $y=3$

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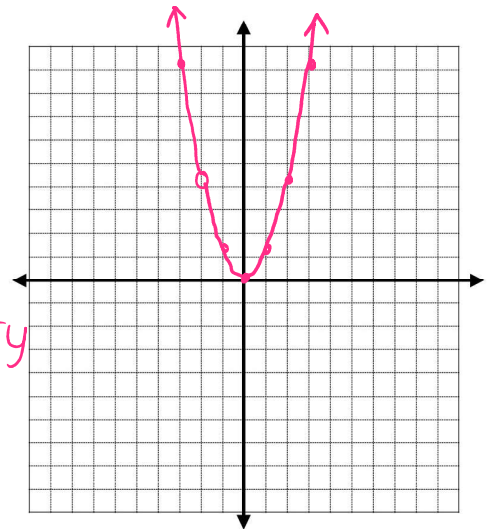
II. Given the function $f(x) = \frac{x^3 + 2x^2}{x+2} = \frac{x^2(x+2)}{\cancel{x+2}}$

1. Graph $f(x)$.

$= x^2, x \neq -2$

Find all points of discontinuity, identify the type of discontinuity and the condition for continuity that is not met.

$x = -2$, Removable hole discontinuity
 $f(-2)$ und
 $\lim_{x \rightarrow -2} f(x) = 4$



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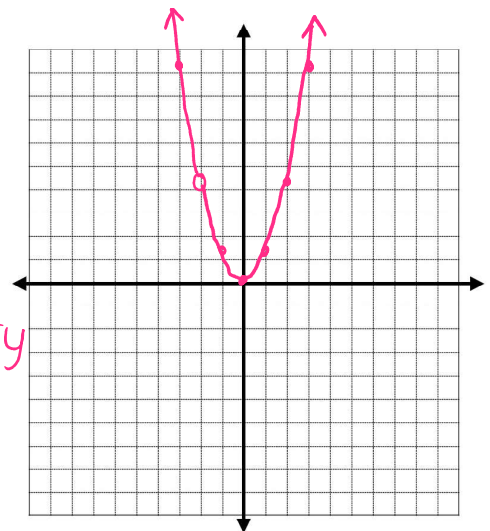
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2. Graph $h(x) = \frac{x^2 - 3x - 4}{x^2 + 4x + 3}$ and identify all asymptotes

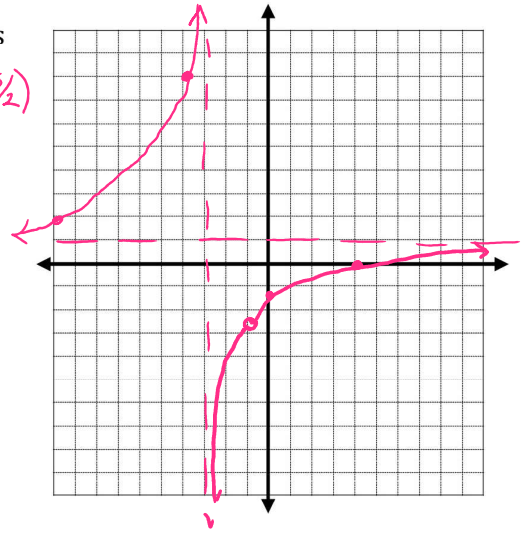
$$h(x) = \frac{(x-4)(x+1)}{(x+3)(x+1)} = \frac{x-4}{x+3}, \quad \text{hole } (-1, -\frac{5}{2}), \quad x = -1$$

$$\lim_{x \rightarrow \pm\infty} \frac{x-4}{x+3} = \lim_{x \rightarrow \pm\infty} \frac{1 - \frac{4}{x}}{1 + \frac{3}{x}} = 1$$

HA: $y=1$

VA: $x = -3$, Nonremovable infinite disc

$$\lim_{x \rightarrow -3^-} \left(\frac{x-4}{x+3} \right) = \infty, \quad \lim_{x \rightarrow -3^+} \left(\frac{x-4}{x+3} \right) = -\infty$$



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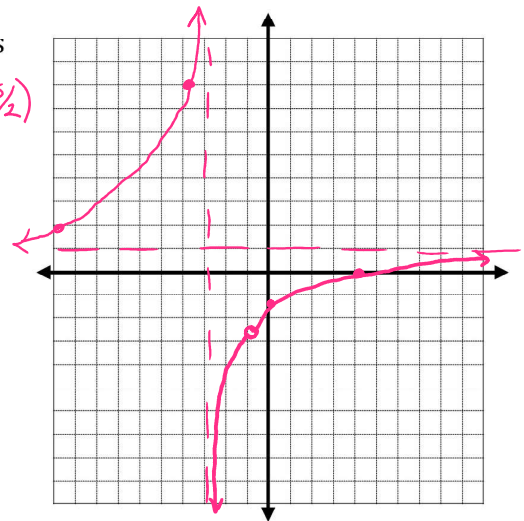
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III. Determine the value of c so that $f(x)$ is continuous on the real number line.

1. $f(x) = \begin{cases} x+3 & x \leq -1 \\ 2x-c & x > -1 \end{cases}$

$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x)$ Check: $f(-1) = 2$

$-1+3 = -2-c$ $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x)$

$2 = -2-c$ $2 = -2 - (-4)$

$4 = -c \Rightarrow c = -4$ $2 = 2 \checkmark$

$f(-1) = \lim_{x \rightarrow -1} f(x)$

2. $f(x) = \begin{cases} x+2c, & x \leq 1 \\ cx^2+x+1 & x > 1 \end{cases}$

$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$ Check: $f(1) = 1+2 = 3$

$1+2c = c^2+2$ $\lim_{x \rightarrow 1^-} f(x) = 3$

$0 = c^2 - 2c + 1$ $\lim_{x \rightarrow 1^+} f(x) = 1^2+1+1 = 3$ } $\lim_{x \rightarrow 1} f(x) = 3$

$0 = (c-1)^2$ $f(1) = \lim_{x \rightarrow 1} f(x) \checkmark$

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