AP Calculus AB **Chapter 1 Test Review #1**

1. Consider the function $f(x)=\dfrac{3x^2+6x}{x^2-x} = \dfrac{3(x+2)}{x-1}$

a. Identify the domain, range, asymptotes, intercepts, and/or holes. Sketch the function.

b. Describe any points of discontinuity in terms of what part of the continuity definition fails.

c. If possible, create a piecewise function that "repairs" any discontinuities in $f(x)$.

A.

D: $(-\infty,0)\cup(0,1)\cup(1,\infty)$

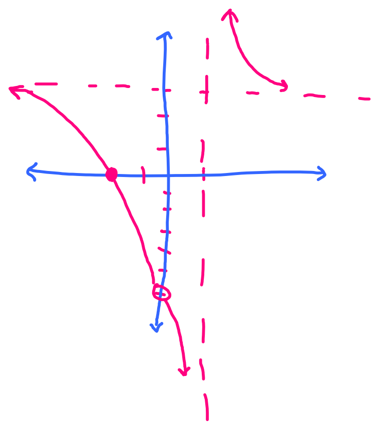
R: $(-\infty,-6)\cup(-6,3)\cup(3,\infty)$

V.A. $x=1$

H.A. $y=3$

Hole $(0,-6)$

x-int: $(-2,0)$ $(0,-6)$



B. $x=0$ Hole (removable)

$\lim_{x\to0} f(x)=-6$ but $f(0)$ is undefined

$x=1$ infinite (non-removable)

$\lim_{x\to1^-} f(x)=-\infty$ $\lim_{x\to1^+} f(x)=\infty$

C. $g(x)=\begin{cases}\dfrac{3x^2+6x}{x^2-x}, & x\neq0\\ -6, & x=0\end{cases}$

$\dfrac{(x+5)(x-3)}{x-3}=x+5$

2. What value should be assigned to k so that the function $f(x)=\begin{cases}\dfrac{x^2+2x-15}{x-3}, & x\neq3\\ k, & x=3\end{cases}$ is continuous.

$\lim_{x\to3} f(x)=x+5=8$ $f(3)=k$

$k=8 \rightarrow$ since $\lim_{x\to3} f(x)=f(3)$ guarantees the function is continuous, $k=8$.

3. Identify if the equation $e^{-x} \doteq x$ has any real solutions. No calculator allowed.

$f(x)=e^{-x}-x$

$f(1)=\frac{1}{e}-1<0$

$f(0)=e^0=1>0$

Since $f(x)$ is the difference of continuous function, $f(x)$ is continuous. Since $f(x)$ is continuous and $f(1)<0<f(0)$, by the IVT there exists a value c such that $f(c)=0$.

$\therefore$ $e^{-c}-c=0$ and $e^{-c}=c$

4. Given the function $f(x)=\begin{cases}2x, & 0<x<1\\ 1, & x=1\\ -x+3, & 1<x<2\end{cases}$ determine if each the following are true or false.

a. $f(1)$ does not exist

False, $f(1)=1$

b. $f(x)$ is a continuous function

False, $\lim_{x\to1} f(x)\neq f(1)$

$\lim_{x\to1^-} f(x)=2$ $\lim_{x\to1^+} f(x)=2$

c. $\lim_{x\to0^+} f(x)$ exists

True, $\lim_{x\to0^+} f(x)=2(0)=0$

5. For each of the following identify points of discontinuity, if any, and the type of discontinuity. Justify your answer using the definition of continuity.

a. $f(x)=\begin{cases}-2x, & x\le 2\\ x^2-4x+1, & x>2\end{cases}$

$f(2)=4$

$\left.\begin{array}{l}\lim\limits_{x\to 2^-} f(x)=-4\\ \lim\limits_{x\to 2^+} f(x)=3\end{array}\right\}$

$\boxed{\text{non-removable, jump disc. at } x=2 \to \lim\limits_{x\to 2} f(x) \text{ DNE}}$

b. $f(x)=\dfrac{|x-3|}{x-3}$

$\left.\begin{array}{l}\lim\limits_{x\to 3^-} f(x)=-1\\ \lim\limits_{x\to 3^+} f(x)=1\end{array}\right\}$

$\boxed{\text{non-removable, jump disc. at } x=3 \to \lim\limits_{x\to 3} f(x) \text{ DNE}}$

c. $y=\cot x$

V.A. $x=\pi n,\ n\in\mathbb{Z}$

$\boxed{\text{Infinite (non-removable) disc. at } x=\pi n,\ n\in\mathbb{Z} \quad \lim\limits_{x\to\pi n} f(x) \text{ DNE}}$

d. $f(x)=\dfrac{x+1}{x^2-4x+3}$

$(x-3)(x-1)$

V.A. $x=1,3$

$\boxed{\text{Infinite (non-removable) disc. at } x=1,3 \quad \lim\limits_{x\to 1} f(x) \text{ DNE} \ \&\ \lim\limits_{x\to 3} f(x) \text{ DNE}}$

e. $y=\dfrac{1}{x^2+1}$

$x^2+1\neq 0$

$\boxed{\text{continuous on } (-\infty,\infty)}$

f. $f(x)=\sqrt{2x+3}$

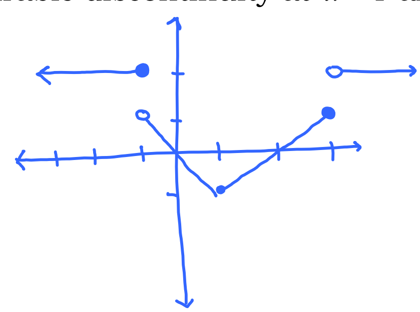
$2x+3\ge 0$

$x\ge -\frac{3}{2}$

$\boxed{\text{continuous on } [-\tfrac{3}{2},\infty)}$

6. Sketch and give the equation of one function that has both a repairable discontinuity at $x=1$ and non-repairable discontinuities at $x=3$ and $x=-1$.

$f(x)=\begin{cases}2, & x\le -1\\ -x, & -1<x<1\\ x-2, & 1<x\le 3\\ 2, & x>3\end{cases}$



7. Sketch a graph of a *function* that satisfies the stated conditions. State the domain and range.

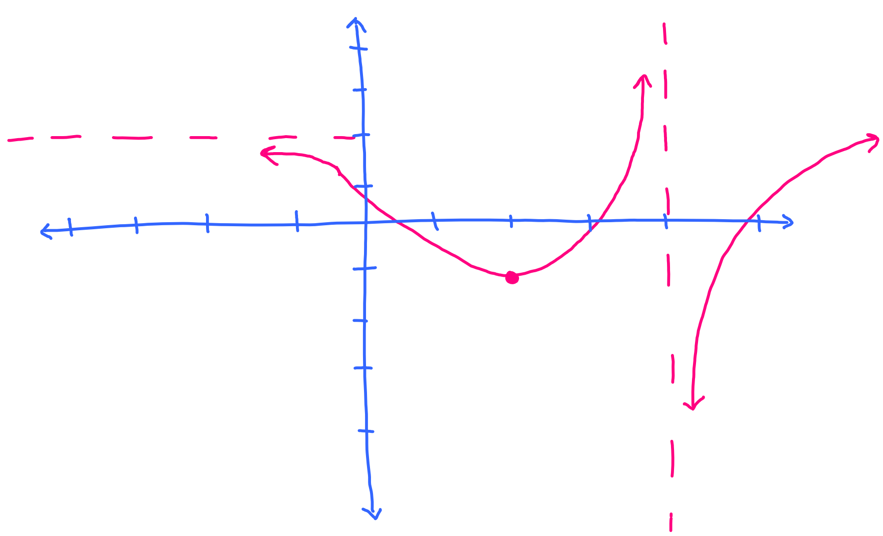
I. $\lim\limits_{x\to 2} f(x)=-1$

II. $\lim\limits_{x\to 4^+} f(x)=-\infty$

III. $\lim\limits_{x\to 4^-} f(x)=\infty$

IV. $\lim\limits_{x\to \infty} f(x)=\infty$

V. $\lim\limits_{x\to -\infty} f(x)=2$

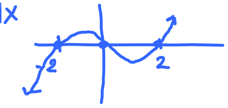


D: $(-\infty,4)\cup(4,\infty)$

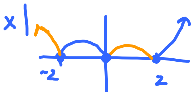
R: $(-\infty,\infty)$

$x(\cancel{x}-2)(x+2)$

x^3-4x



$|x^3-4x|$



8. Given the function $f(x)=\begin{cases} |x^3-4x|, & x<1 \\ x^2-2x-2, & x\ge 1 \end{cases}$, find the right-hand, left-hand and normal limits of f at $x=1$. Discuss the continuity of the function.

$\lim\limits_{x\to 1^-} f(x) = -(1^3-4(1)) = 3$

$\lim\limits_{x\to 1^+} f(x) = 1^2-2(1)-2 = -3$

$\lim\limits_{x\to 1} f(x) = DNE$

$f(1) = -3$

Since the limit does not exist at $x=1$, the function is not continuous at $x=1$. There is a non-removable, jump discontinuity.

9. Evaluate the following limits:

a. $\lim\limits_{x\to -\infty} \dfrac{3+\cos x}{x} \boxed{=0}$

$-1 \le \cos x \le 1$

$2 \le 3+\cos x \le 4$

$\frac{2}{x} \le \frac{3+\cos x}{x} \le \frac{4}{x}$

$\lim\limits_{x\to -\infty} \frac{2}{x} = 0$ $\quad \lim\limits_{x\to -\infty} \frac{4}{x} = 0$

b. $\lim\limits_{x\to -\infty} \dfrac{e^x}{4+5e^{3x}} \boxed{=0}$

$e^x \to 0$, $4+5e^{3x} \to 4$

c. $\lim\limits_{x\to \infty} \dfrac{x-\cos x}{x} = \lim\limits_{x\to\infty} 1 - \dfrac{\cos x}{x}$

$\cos x \to [-1,1]$, $x \to \infty$

$= 1 - 0$

$\boxed{=1}$

d. $\lim\limits_{x\to \infty} \dfrac{3x^2-x+1}{x+3} \boxed{=\infty}$

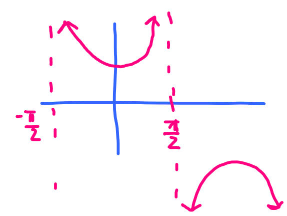
Slant Asymptote

$\text{"}\dfrac{3x^2}{x}\text{"} = \text{"}3x\text{"}$

e. $\lim\limits_{x\to \infty} \cos\left(\dfrac{1}{x}\right) \boxed{=1}$

$\lim\limits_{x\to\infty} \frac{1}{x} = 0$

f. $\lim\limits_{x\to \frac{\pi}{2}^+} \sec x \boxed{=-\infty}$



g. $\lim\limits_{x\to \infty} \dfrac{3x^2-x^3+\sqrt{x}}{5x^2+3x^3} \boxed{=\dfrac{-1}{3}}$

$\text{"}\dfrac{-x^3}{3x^3}\text{"}$

h. $\lim\limits_{\theta\to 0} \dfrac{2\theta}{\tan(5\theta)} =$

$\lim\limits_{\theta\to 0} \dfrac{2\theta\cos(5\theta)}{\sin(5\theta)} =$

$\lim\limits_{\theta\to 0} \dfrac{2\cos(5\theta)}{5} \cdot \dfrac{5\theta}{\sin(5\theta)} =$

$\dfrac{2(\cos(0))}{5} \cdot 1 = \boxed{\dfrac{2}{5}}$

i. $\lim\limits_{x\to 4} \dfrac{|x-4|}{x-4} \boxed{=DNE}$

$\lim\limits_{x\to 4^-} \dfrac{|x-4|}{x-4} = \lim\limits_{x\to 4^-} \dfrac{-(x-4)}{x-4} = -1$

$\lim\limits_{x\to 4^+} \dfrac{|x-4|}{x-4} = \lim\limits_{x\to 4^+} \dfrac{x-4}{x-4} = 1$

H.A. $y=0$

j. $\lim\limits_{x\to5}\frac{\sqrt{x+11}-4}{x-5}\cdot\frac{\sqrt{x+11}+4}{\sqrt{x+11}+4}=$

$\lim\limits_{x\to5}\frac{x+11-16}{(x-5)(\sqrt{x+11}+4)}=$

$\lim\limits_{x\to5}\frac{1}{\sqrt{x+11}+4}=\frac{1}{\sqrt{16}+4}=\boxed{\frac{1}{8}}$

k. $\lim\limits_{x\to\infty}\frac{4x+2}{x^3}=\boxed{0}$

l. $\lim\limits_{x\to0}\frac{2(x+2)\,\frac{1}{2+x}-\frac{1}{2}\,2(x+2)}{x\cdot 2(x+2)}=$

$\lim\limits_{x\to0}\frac{2-x-2}{2x(x+2)}=$

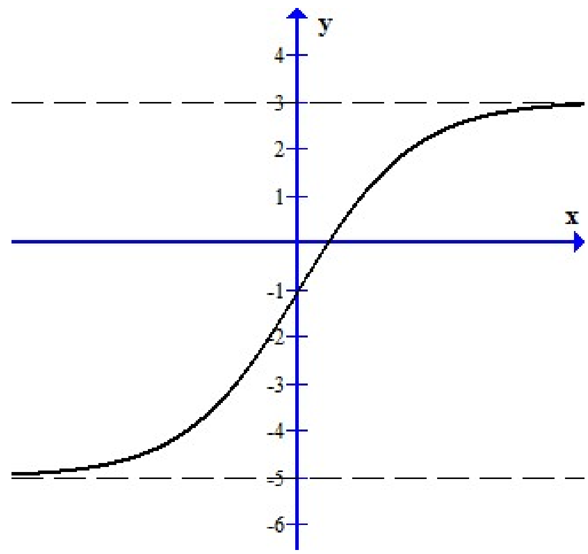
$\lim\limits_{x\to0}\frac{-1}{2(x+2)}=\boxed{-\frac{1}{4}}$

10. The graph of the function $y=\frac{ae^x+b}{e^x+1}$ is shown below. The graph has horizontal asymptotes at $y=3$ and $y=-5$. What is the value of a and b?

$\underbrace{\lim\limits_{x\to\infty}\frac{ae^x+b}{e^x+1}=3}$

"$\frac{ae^x}{e^x}$" $=3$

$a=3$



$\lim\limits_{x\to-\infty}\frac{a\cancel{e^x}^{\,0}+b}{\cancel{e^x}_{\,0}+1}=-5$

$b=-5$

$\boxed{\begin{array}{l}a=3\\ b=-5\end{array}}$