Part A. Directions: Answer these questions *without* using your calculator.

1. $\lim_{x\to 2}\frac{x^2-4}{x^2+4}$ is $\frac{2^2-4}{2^2+4}=\frac{0}{8}=0$

(A) 1 (B) 0 (C) $-\frac{1}{2}$ (D) −1 (E) ∞

2. $\lim_{x\to\infty}\frac{4-x^2}{x^2-1}$ is   H.A. $y=-1$

(A) 1 (B) 0 (C) −4 (D) −1 (E) ∞

3. $\lim_{x\to 3}\frac{x-3}{x^2-2x-3}$ is $\lim_{x\to 3}\frac{(x-3)}{(x-3)(x+1)}=\lim_{x\to 3}\frac{1}{x+1}=\frac{1}{4}$

(A) 0 (B) 1 (C) $\frac{1}{4}$ (D) ∞ (E) none of these

4. $\lim_{x\to 0}\frac{x}{x}$ is $\lim_{x\to 0} 1=1$

(A) 1 (B) 0 (C) ∞ (D) −1 (E) nonexistent

5. $\lim_{x\to 2}\frac{x^3-8}{x^2-4}$ is $\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}=\lim_{x\to 2}\frac{x^2+2x+4}{x+2}=\frac{12}{4}=3$

(A) 4 (B) 0 (C) 1 (D) 3 (E) ∞

6. $\lim_{x\to\infty}\frac{4-x^2}{4x^2-x-2}$ is   H.A. $y=-\frac{1}{4}$

(A) −2 (B) $-\frac{1}{4}$ (C) 1 (D) 2 (E) nonexistent

7. $\lim_{x\to -\infty}\frac{5x^3+27}{20x^2+10x+9}$ is   Slant Asymptote

(A) −∞ (B) −1 (C) 0 (D) 3 (E) ∞

8. $\lim_{x\to\infty}\frac{3x^2+27}{x^3-27}$ is   H.A. $y=0$

(A) 3 (B) ∞ (C) 1 (D) −1 (E) 0

9. $\lim_{x\to\infty}\frac{2^{-x}}{2^x}$ is $\lim_{x\to\infty}\frac{1}{2^x\cdot 2^x}=\lim_{x\to\infty}\frac{1}{2^{2x}}=0$ ($2^{2x}\to\infty$)

(A) −1 (B) 1 (C) 0 (D) ∞ (E) none of these

10. $\lim_{x\to -\infty}\frac{2^{-x}}{2^x}$ is $\lim_{x\to -\infty}\frac{1}{2^{2x}}=\infty$ ($2^{2x}\to 0$)

(A) −1 (B) 1 (C) 0 (D) ∞ (E) none of these

11. $\lim_{x\to 0}\frac{5\sin 5x}{5x}=5(1)$

(A) $=0$ (B) $=\frac{1}{5}$ (C) $=1$ (D) $=5$ (E) does not exist

12. $\lim_{x\to 0}\frac{\sin 2x}{3x}=\lim_{x\to 0}\frac{2}{3}\cdot\frac{\sin(2x)}{2x}=\frac{2}{3}(1)=\frac{2}{3}$

(A) $=0$ (B) $=\frac{2}{3}$ (C) $=1$ (D) $=\frac{3}{2}$ (E) does not exist

13. The graph of $y = \arctan x$ has

(A) vertical asymptotes at $x = 0$ and $x = \pi$
(B) horizontal asymptotes at $y = \pm\frac{\pi}{2}$
(C) horizontal asymptotes at $y = 0$ and $y = \pi$
(D) vertical asymptotes at $x = \pm\frac{\pi}{2}$
(E) none of these

$\tan(x)$ V.A. $x = \frac{\pi}{2} + \pi n, n \in$ $\tan^{-1}(x)$ H.A. $y = \pm\frac{\pi}{2}$

14. The graph of $y = \frac{x^2-9}{3x-9}$ has

$y = \frac{(x-3)(x+3)}{3(x-3)} = \frac{x+3}{3}$ Hole $x=3$

(A) a vertical asymptote at $x = 3$ (B) a horizontal asymptote at $y = \frac{1}{3}$
(C) a removable discontinuity at $x = 3$ (D) an infinite discontinuity at $x = 3$
(E) none of these

15. $\lim_{x\to 0} \frac{\sin x}{x^2+3x}$ is

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{x+3} = 1\left(\frac{1}{3}\right) = \frac{1}{3}$

(A) 1 (B) $\frac{1}{3}$ (C) 3 (D) ∞ (E) $\frac{1}{4}$

16. $\lim_{x\to 0} \sin\frac{1}{x}$ is

(A) ∞ (B) 1 (C) nonexistent (D) -1 (E) none of these

$\lim_{x\to 0} \frac{1}{x} = \infty$ $\lim_{x\to\infty} \sin(x)$ osciliates

17. Which statement is true about the curve $y = \frac{2x^2+4}{2+7x-4x^2}$? $= \frac{2(x^2+2)}{-(4x+1)(x-2)}$

(A) The line $x = -\frac{1}{4}$ is a vertical asymptote.
(B) The line $x = 1$ is a vertical asymptote.
(C) The line $y = -\frac{1}{4}$ is a horizontal asymptote.
(D) The graph has no vertical or horizontal asymptote.
(E) The line $y = 2$ is a horizontal asymptote.

18. $\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)}$ is

H.A. $y = -2$

(A) -4 (B) -2 (C) 1 (D) 2 (E) nonexistent

19. $\lim_{x\to 0} \frac{|x|}{x}$ is

(A) 0 (B) nonexistent (C) 1 (D) -1 (E) none of these

$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = -1$
$\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+} \frac{x}{x} = 1$
$\lim_{x\to 0} \frac{|x|}{x} = \text{DNE}$

20. $\lim_{x\to\infty} x\sin\frac{1}{x}$ is

(A) 0 (B) ∞ (C) nonexistent (D) -1 (E) 1

$\lim_{x\to\infty} \frac{\sin(\frac{1}{x})}{\frac{1}{x}} = 1$ $\lim_{x\to\infty} \frac{1}{x} = 0$

21. $\lim_{x\to\pi} \frac{\sin(\pi - x)}{\pi - x}$ is $=1$

(A) 1 (B) 0 (C) ∞ (D) nonexistent (E) none of these

22. Let $f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 4 & \text{if } x = 1. \end{cases}$ $= (x+1)$

Which of the following statements is (are) true?

I. $\lim_{x\to 1} f(x)$ exists ✓
II. $f(1)$ exists ✓
III. f is continuous at $x = 1$

$\lim_{x\to 1} f(x) = 2$
$f(1) = 4$

(A) I only (B) II only (C) I and II
(D) none of them (E) all of them

23. If $\begin{cases} f(x) = \dfrac{x^2 - x}{2x} \text{ for } x \neq 0, \\ f(0) = k, \end{cases}$

and if f is continuous at $x = 0$, then $k =$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

24. Suppose $\begin{cases} f(x) = \dfrac{3x(x-1)}{x^2 - 3x + 2} \text{ for } x \neq 1, 2, \\ f(1) = -3, \\ f(2) = 4. \end{cases}$

Then $f(x)$ is continuous

(A) except at $x = 1$ (B) except at $x = 2$ (C) except at $x = 1$ or 2
(D) except at $x = 0$, 1, or 2 (E) at each real number

25. The graph of $f(x) = \frac{4}{x^2 - 1}$ has

(A) one vertical asymptote, at $x = 1$
(B) the y-axis as vertical asymptote
(C) the x-axis as horizontal asymptote and $x = \pm 1$ as vertical asymptotes
(D) two vertical asymptotes, at $x = \pm 1$, but no horizontal asymptote
(E) no asymptote

26. The graph of $y = \frac{2x^2 + 2x + 3}{4x^2 - 4x}$ has

(A) a horizontal asymptote at $y = +\frac{1}{2}$ but no vertical asymptote
(B) no horizontal asymptote but two vertical asymptotes, at $x = 0$ and $x = 1$
(C) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = 0$ and $x = 1$
(D) a horizontal asymptote at $x = 2$ but no vertical asymptote
(E) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = \pm 1$

27. Let $f(x) = \begin{cases} \dfrac{x^2 + x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$.

Which of the following statements is (are) true?

I. $f(0)$ exists
II. $\lim_{x \to 0} f(x)$ exists
III. f is continuous at $x = 0$

(A) I only (B) II only (C) I and II only
(D) all of them (E) none of them

Part B. Directions: Some of the following questions require the use of a graphing calculator.

28. If $[x]$ is the greatest integer not greater than x, then $\lim_{x \to 1/2} [x]$ is

(A) $\frac{1}{2}$ (B) 1 (C) nonexistent (D) 0 (E) none of these

29. (With the same notation) $\lim_{x \to -2} [x]$ is

(A) -3 (B) -2 (C) -1 (D) 0 (E) none of these

30. $\lim_{x \to \infty} \sin x$

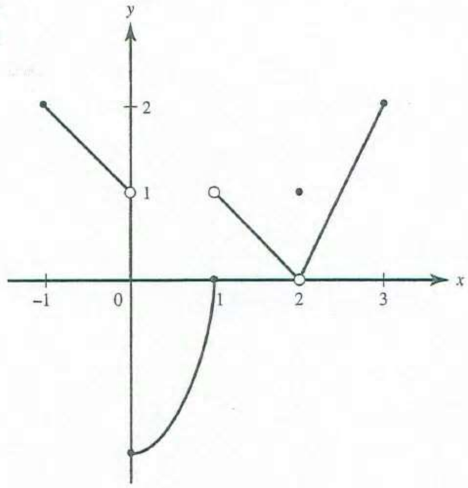
(A) is -1 (B) is infinity (C) oscillates between -1 and 1
(D) is zero (E) does not exist

31. The function $f(x) = \begin{cases} x^2/x & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

(A) is continuous everywhere
(B) is continuous except at $x = 0$
(C) has a removable discontinuity at $x = 0$
(D) has an infinite discontinuity at $x = 0$
(E) has $x = 0$ as a vertical asymptote

Questions 32–36 are based on the function f shown in the graph and defined below:

$$f(x) = \begin{cases} 1 - x & (-1 \leqslant x < 0) \\ 2x^2 - 2 & (0 \leqslant x \leqslant 1) \\ -x + 2 & (1 < x < 2) \\ 1 & (x = 2) \\ 2x - 4 & (2 < x \leqslant 3) \end{cases}$$



32. $\lim_{x \to 2} f(x)$

(A) equals 0 (B) equals 1 (C) equals 2
(D) does not exist (E) none of these

33. The function f is defined on $[-1,3]$

(A) if $x \neq 0$ (B) if $x \neq 1$ (C) if $x \neq 2$
(D) if $x \neq 3$ (E) at each x in $[-1,3]$

34. The function f has a removable discontinuity at

(A) $x = 0$ (B) $x = 1$ (C) $x = 2$ (D) $x = 3$ (E) none of these

35. On which of the following intervals is f continuous?

(A) $-1 \leqslant x \leqslant 0$ (B) $0 < x < 1$ (C) $1 \leqslant x \leqslant 2$
(D) $2 \leqslant x \leqslant 3$ (E) none of these

36. The function f has a jump discontinuity at

(A) $x = -1$ (B) $x = 1$ (C) $x = 2$
(D) $x = 3$ (E) none of these