1. $\lim_{h\to 0}\dfrac{\cos\left(\dfrac{\pi}{3}+h\right)-\dfrac{1}{2}}{h}=$ $f'(\frac{\pi}{3})$ where $f(x)=\cos x$

$f'(x)=-\sin x$
$f'(\frac{\pi}{3})=-\sqrt{3}/2$

a. -1 b. $\boxed{-\dfrac{\sqrt{3}}{2}}$ c. $-\dfrac{1}{2}$ d. $\dfrac{1}{2}$ e. $\dfrac{\sqrt{3}}{2}$

2. $\lim_{x\to\frac{\pi}{4}}\dfrac{\tan(5x)-1}{x-\dfrac{\pi}{4}}$ $=f'(\pi/4)$ where $f(x)=\tan(5x)$

$f'(x)=\sec^2(5x)\cdot 5$
$f'(\pi/4)=5\sec^2(\frac{5\pi}{4})=5(-\sqrt{2})^2$
$=10$

a. 0 b. 2 c. $\dfrac{5}{2}$ d. $\boxed{10}$ e. nonexistent

3. Find the derivative of $f(\theta)=\cos^2(\theta)-\sin^2(\theta)$. $=\cos(2\theta)$

$f'(\theta)=-\sin(2\theta)\cdot 2$

a. $\cos(2\theta)$ b. $\boxed{-2\sin(2\theta)}$ c. $-\sin(2\theta)$ d. $-2\sin(2\theta)\cos(2\theta)$ e. $-2\cos(2\theta)$

Use the following table to answer questions #4-7.

x	$F(x)$	$F'(x)$	$F''(x)$	$G(x)$	$G'(x)$	$G''(x)$
3	5	4	-3	2	7	-2
5	8	6	10	-6	-4	11

4. If $H(x)=(F(x))^2$, then $H'(3)=$ $2(F(3))\cdot F'(3)=2(5)\cdot 4=40$

a. 0 b. 10 c. 25 d. $\boxed{40}$ e. 100

5. If $H(x)=\dfrac{F(x)}{G(x)}$, then $H'(3)=$ $\dfrac{F'(3)G(3)-G'(3)F(3)}{(G(3))^2}=\dfrac{4(2)-7(5)}{2^2}=\dfrac{8-35}{4}=\dfrac{-27}{4}$

a. $\boxed{-\dfrac{27}{4}}$ b. $-\dfrac{3}{2}$ c. 0 d. $\dfrac{4}{7}$ e. $\dfrac{43}{4}$

6. If $H(x)=G(F(x))$, then $H'(3)=$ $G'(F(3))\cdot F'(3)=G'(5)\cdot 4=-4\cdot 4=-16$

a. $\boxed{-16}$ b. -6 c. -4 d. 28 e. 43

7. If $H(x)=G(F(x))$, then $H''(3)=$

a. -33 b. 0 c. 6 d. 56 e. $\boxed{188}$

$H'(3)=G'(F(3))\cdot F'(3)$ Product Rule for 2nd deriv!

$H''(3)=G''(F(3))(F'(3))^2+G'(F(3))\cdot F''(3)$
$=G''(5)\cdot(4)^2+G'(5)\cdot(-3)=11\cdot 16+(-4)(-3)=188$

$f(x) = \begin{cases} -(x-3), & x < 3 \\ x-3, & x \geq 3 \end{cases}$ $\therefore f'(1) = -1$

8. Let $f(x) = |x-3|$, then $f'(1) =$

$\boxed{\text{a. } -1}$ b. 0 c. 1 d. 2 e. nonexistent

9. In the *xy*-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of *k*? $y = -x + k \Rightarrow m = -1$ $y' = 2x + 3 = -1$ $2x = -4$ $x = -2$

$\boxed{\text{a. } -3}$ b. -2 c. -1 d. 0 e. 1

$-2 + -1 = k$
$k = -3$
$y(-2) = 4 - 6 + 1 = -1$ $(-2, -1)$

10. Find the derivative of $f(x) = (\tan x)(\csc x)(1 - \sin^2 x)$

a. $f'(x) = \sin x$ $\boxed{\text{b. } f'(x) = -\sin x}$ c. $f'(x) = -\cos x$ d. $f'(x) = \cos x$

$f(x) = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \cos^2 x = \cos x$ $f'(x) = -\sin x$

For #11-14, let $s(t) = t^3 - 6t^2 + 9t$ be the position function, in feet, for a projectile's path through the air for $t \geq 0$, for t seconds.

$v(t) = s'(t) = 3t^2 - 12t + 9$
$a(t) = s''(t) = 6t - 12$

11. What is the acceleration of the projectile after 8 seconds? $a(8) = 36$

a. $a = 72\, ft/s$ $\boxed{\text{b. } a = 36\, ft/s^2}$ c. $a = -36\, ft/s$ d. $a = 60\, ft/s^2$

12. What is the velocity of the projectile when $t = 4$? $v(4) = 9$

$\boxed{\text{a. } v = 9\, ft/s}$ b. $v = 12\, ft/s$ c. $v = 49\, ft/s$ d. $v = -12\, ft/s$

13. What is the average velocity of the projectile on the interval [0,4]? $\frac{s(4) - s(0)}{4 - 0} = \frac{4}{4} = 1$

a. $v = 4\, ft/s$ b. $v = 25\, ft/s$ $\boxed{\text{c. } v = 1\, ft/s}$ d. $v = -12\, ft/s$

14. What is the projectile's speed when the acceleration is zero? $a = 0 = 6t - 12$ $t = 2$ sec

a. $s = 45\, ft/s$ b. $s = 2\, ft/s$ c. $s = -3\, ft/s$ $\boxed{\text{d. } s = 3\, ft/s}$

$|v(2)| = |-3|$

15. Which of the following statements is/are FALSE?

a. Differentiability implies Continuity true
b. Continuity implies Differentiability false
c. If $f(x)$ is not differentiable at $x = c$, then $f(x)$ is not continuous at $x = c$. false
d. a. & c.
e. b. & c.

16. Let f be the function defined below, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

step ① $\Rightarrow$ implies continuity - step 2

$$f(x) = \begin{cases} cx + d, & x \le 2 \\ x^2 - cx, & x > 2 \end{cases}$$

Step ② $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$2c + d = 4 - 2c$

$2(2) + d = 4 - 2(2)$

$f'(x) = \begin{cases} c, & x \le 2 \\ 2x - c, & x > 2 \end{cases} \Rightarrow c = 4 - c$

$2c = 4$

$\boxed{c = 2}$

a. -4 $4 + d = 0$ $\boxed{d = -4}$ b. -2 c. 0 d. 2 e. 4

③ $\boxed{c + d = -2}$

17. If $f(x) = 2x^2 + 4$, which of the following will calculate the derivative of $f(x)$?

(a) $\dfrac{[2(x + \Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$

(b) $\lim\limits_{\Delta x \to 0} \dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$

(c) $\lim\limits_{\Delta x \to 0} \dfrac{[2(x + \Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$

(d) $\dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$

(e) None of these

18. Differentiate: $y = \dfrac{1 + \cos x}{1 - \cos x}$.

$y' = \dfrac{-\sin x(1 - \cos x) - \sin x(1 + \cos x)}{(1 - \cos x)^2} = \dfrac{-\sin x + \cancel{\sin x \cos x} - \sin x - \cancel{\sin x \cos x}}{(1 - \cos x)^2}$

$= \dfrac{-2\sin x}{(1 - \cos x)^2}$

(a) -1 (b) $-2 \csc x$ (c) $2 \csc x$

(d) $\dfrac{-2 \sin x}{(1 - \cos x)^2}$ (e) None of these

19. Find dy/dx for $y = (x^3)\sqrt{x + 1}$.

$y' = \dfrac{3x^2 (\sqrt{x+1})2}{2\sqrt{x+1}} + \dfrac{x^3}{2\sqrt{x+1}} = \dfrac{6x^2(x+1) + x^3}{2\sqrt{x+1}} = \dfrac{7x^3 + 6x^2}{2\sqrt{x+1}}$

(a) $\dfrac{3x^2}{2\sqrt{x + 1}}$ (b) $\dfrac{x^2(7x + 6)}{2\sqrt{x + 1}}$ (c) $3x^2\sqrt{x + 1}$

(d) $\dfrac{7x^3 + x^2}{2\sqrt{x + 1}}$ (e) None of these

20. The position equation for the movement of a particle is given by $s = (t^2 - 1)^3$ when s is measured in feet and t is measured in seconds. Find the acceleration at two seconds.

(a) 342 units/sec^2 (b) 18 units/sec^2 (c) 288 units/sec^2

(d) 90 units/sec^2 (e) None of these

$a(2) = 6(3)(19) = 342$

$v(t) = 3(t^2 - 1)^2 \cdot 2t = 6t(t^2 - 1)^2$

$a(t) = 6(t^2 - 1)^2 + 6t \cdot 2(t^2 - 1) \cdot 2t$

$= 6(t^2 - 1)(t^2 - 1 + 4t^2) = 6(t^2 - 1)(5t^2 - 1)$

$f'(x) = 7(2x^2+5)^6 \cdot 4x$
$= 28x(2x^2+5)^6$

21. Find $f'(x)$ for $f(x) = (2x^2 + 5)^7$.

(a) $7(4x)^6$ (b) $(4x)^7$ (c) $28x(2x^2 + 5)^6$
(d) $7(2x^2 + 5)^6$ (e) None of these

$y' = \underline{2\cos 4x \sin 4x} \cdot 4 = 4\cos 8x$
$y'' = -4\sin 8x\,(8) = -32\sin 8x$

22. Find $\dfrac{d^2y}{dx^2}$ for $y = \cos^2 4x$.

(a) $-8\cos 4x$ (b) $32\sin 4x$ (c) $4\cos 8x$
(d) $-32\cos 8x$ (e) None of these

23. Find $\dfrac{dy}{dx}$ if $(y^2 - 3xy + x^2 = 7)\dfrac{d}{dx}$

$2y\dfrac{dy}{dx} - 3y - 3x\dfrac{dy}{dx} + 2x = 0$

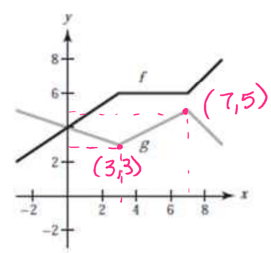
$(2y-3x)\dfrac{dy}{dx} = 3y - 2x$

$\dfrac{dy}{dx} = \dfrac{3y-2x}{2y-3x}$

(a) $\dfrac{2x + y}{3x - 2y}$ (b) $\dfrac{3y - 2x}{2y - 3x}$ (c) $\dfrac{2x}{3 - 2y}$
(d) $\dfrac{2x}{y}$ (e) None of these

24. Let $p(x) = f(x)g(x)$. Use the figure to find $p'(5)$.

(a) 7 (b) 3
(c) 0 (d) 24
(e) None of these



$m = \frac{2}{4} = \frac{1}{2}$

$p'(5) = f'(5)g(5) + f(5)g'(5)$
$= 0(4) + 6\left(\frac{1}{2}\right) = 3$

25. A point moves along the curve $\frac{d}{dt}(y = 2x^2 + 1)$ in such a way that the y value is decreasing at the rate of 2 units per second. At what rate is x changing when $x = \frac{3}{2}$?

$\dfrac{dy}{dt} = -2\dfrac{u}{sec}$

(a) increasing $\frac{1}{3}$ unit/sec (b) decreasing $\frac{1}{3}$ unit/sec (c) decreasing $\frac{7}{2}$ unit/sec
(d) increasing $\frac{7}{2}$ unit/sec (e) None of these

$\dfrac{dy}{dt} = 4x\dfrac{dx}{dt} \Rightarrow \dfrac{dx}{dt} = \dfrac{1}{4x}\dfrac{dy}{dt}$
$= \dfrac{1}{4\left(\frac{3}{2}\right)} \cdot -2 = -\dfrac{1}{3}\dfrac{u}{sec}$

26. Assume $f'(c) = -4$. Find $f'(-c)$ if f is an odd function.

(a) 4 (b) 0 (c) -3 (d) -4 (e) None of these

If f is odd, then $(f(-x) = -f(x))\frac{d}{dx}$

(d) $\dfrac{1}{2}\cot^2\dfrac{t}{2}$ (e) None of these

$-f'(-x) = -f'(x)$
$f'(-x) = f'(x)$

27. Find all points on the graph of $f(x) = -x^3 + 3x^2 - 2$ at which there is a horizontal tangent line.

$m = 0$

(a) $(0, -2), (2, 2)$ (b) $(0, -2)$ (c) $(1, 0), (0, -2)$
(d) $(2, 2)$ (e) None of these

$f'(x) = -3x^2 + 6x = 0$
$-3x(x-2) = 0$
$x = 0, 2$