1. $\int \frac{3x^2}{\sqrt{x^3+1}}dx =$ $u = x^3+1$, $\frac{du}{dx} = 3x^2$, $dx = \frac{du}{3x^2}$, $\int \frac{\cancel{3x^2}}{\sqrt{u}} \cdot \frac{du}{\cancel{3x^2}} = \int u^{-1/2}du = 2u^{1/2} + C$
$= 2\sqrt{x^3+1} + C$

$\boxed{\text{a. } 2\sqrt{x^3+1}+C}$ b. $\frac{3}{2}\sqrt{x^3+1}+C$ c. $\sqrt{x^3+1}+C$
d. $\ln\sqrt{x^3+1}+C$ e. $\ln(x^3+1)+C$

2. $\int (x^2+1)^2 dx =$ $\int(x^4+2x^2+1)\,dx = \frac{x^5}{5} + \frac{2x^3}{3} + x + C$

a. $\frac{(x^2+1)^3}{3}+C$ b. $\frac{(x^2+1)^3}{6x}+C$ c. $\left(\frac{x^3}{3}+x\right)^2+C$
d. $\frac{2x(x^2+1)^3}{3}+C$ $\boxed{\text{e. } \frac{x^5}{5}+\frac{2x^3}{3}+x+C}$

3. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0,2]$? $\text{Avy} = \frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\,dx$
$u = x^3+1$, $\frac{du}{dx} = 3x^2$, $dx = \frac{du}{3x^2}$, $x=0$, $u=1$; $x=2$, $u=9$

$\boxed{\text{a. } \frac{26}{9}}$ b. $\frac{52}{9}$ c. $\frac{26}{3}$ d. $\frac{52}{3}$ e. 24

$\text{Avy} = \frac{1}{2}\cdot\frac{1}{3}\int_1^9 u^{1/2}du = \frac{1}{\cancel{6}3}\left[\cancel{\tfrac{2}{3}}u^{3/2}\right]_1^9$
$= \frac{1}{9}(9^{3/2} - 1^{3/2}) = \frac{1}{9}(27-1)$

4. $\int_1^2 (4x^3-6x)dx =$ $(x^4-3x^2)\Big|_1^2 = 2^4 - 3(2)^2 - 1^4 + 3(1)^2$
$= 16 - 12 - 1 + 3 = 6$

a. 2 b. 4 $\boxed{\text{c. } 6}$ d. 36 e. 42

5. $\int_1^2 \frac{1}{x^2}dx =$ $\int_1^2 x^{-2}dx = -\frac{1}{x}\Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

a. $-\frac{1}{2}$ b. $\frac{7}{24}$ $\boxed{\text{c. } \frac{1}{2}}$ d. 1 e. $2\ln 2$

6. $\int_1^e \left(\frac{x^2-1}{x}\right)dx =$ $\int_1^e \left(x - \frac{1}{x}\right)dx = \left[\frac{x^2}{2} - \ln x\right]_1^e = \frac{e^2}{2} - \cancel{\ln(e)}^{1} - \frac{1}{2} + \cancel{\ln(1)}^{0} = \frac{e^2}{2} - \frac{3}{2}$

a. $e - \frac{1}{e}$ b. $e^2 - e$ c. $\frac{e^2}{2} - e + \frac{1}{2}$ d. $e^2 - 2$ $\boxed{\text{e. } \frac{e^2}{2} - \frac{3}{2}}$

7. What are all values of k for which $\int_{-3}^{k} x^2 dx = 0$?

$\frac{x^3}{3}\Big|_{-3}^{k} = 0$, $\frac{k^3}{3} - \frac{(-3)^3}{3} = 0$

$\frac{k^3}{3} + 9 = 0$, $\frac{k^3}{3} = -9$

a. $\boxed{-3}$ b. 0 c. 3 d. -3 and 3 e. -3, 0, and 3

$k^3 = -27$ $k = -3$

8. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval [-4, -1], what is the value of $\int_{-4}^{-1} f'(x)dx$ $= f(x)\Big|_{-4}^{-1}$

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

$= f(-1) - f(-4)$

$= -1.5 - .75$

$= -2.25$

a. 4.5 b. 2.25 c. 0 d. $\boxed{-2.25}$ e. -4.5

9. $\int \frac{1}{x^2} dx =$ $-\frac{1}{x} + C$

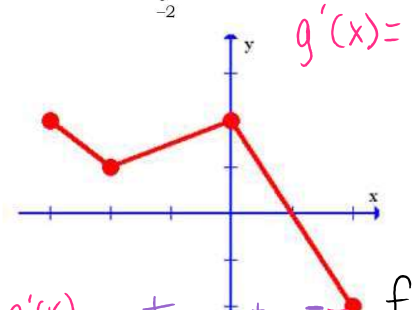
a. $\ln x^2 + c$ b. $-\ln x^2 + c$ c. $x^{-1} + c$ d. $\boxed{-x^{-1} + c}$ e. $-2x^{-3} + c$

10. The graph of a piecewise linear function f is given below. If $g(x) = \int_{-2}^{x} f(t)dt$, which of the following values is greatest?

$g'(x) = f(x)$

a. $g(-3)$ b. $g(-2)$ c. $g(0)$

d. $\boxed{g(1)}$ e. $g(2)$



max occurs at $x=1$

g'(x) + + −

g(x) -3 ↗ ↘ 2

11. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

a. $f'(4)$ b. $-7 + f'(4)$ c. $\int_2^4 f(t)dt$ d. $\int_2^4 (-7 + f(t))dt$ e. $\boxed{-7 + \int_2^4 f(t)dt}$

$\int_2^4 f(x)dx = G(x)\Big|_2^4 = G(4) - G(2)$

$G(4) = \int_2^4 f(x)dx + G(2) = \int_2^4 f(x)dx - 7$

$$= \int_1^3 2\,dx + \int_3^5 (x-1)\,dx = 2x\Big|_1^3 + \left(\frac{x^2}{2}-x\right)\Big|_3^5$$

$$= 6-2+\frac{25}{2}-5-\frac{9}{2}+3$$

$$= \frac{16}{2}+2$$

$$= 10$$

12. The function f is defined by $f(x)=\begin{cases}2 & \text{for } x<3 \\ x-1 & \text{for } x\geq 3\end{cases}$. What is the value of $\int_1^5 f(x)\,dx$?

a. 2 b. 6 c. 8 $\boxed{\text{d. } 10}$ e. 12

$\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ $dx=2\sqrt{x}\,du$

$x=1,\ u=1$ $x=4$ $u=2$

$2\int_1^2 e^u\,du$

13. Using the substitution $u=\sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ is equal to which of the following?

a. $2\int_1^{16} e^u\,du$ b. $2\int_1^4 e^u\,du$ $\boxed{\text{c. } 2\int_1^2 e^u\,du}$ d. $\frac{1}{2}\int_1^2 e^u\,du$ e. $\int_1^4 e^u\,du$

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

14. A tank contains 50 liters of oil at time $t=4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t=15$ hours?

a. 64.9 b. 68.2 $\boxed{\text{c. } 114.9}$ d. 116.6 e. 118.2

$l = \underbrace{50}_{\text{initial}} + \underbrace{\int_4^{15} R(t)\,dt}_{\text{amt added}} = 50+3(6.2)+5(5.9)+3(5.6)=114.9$

★ 15. Let g be a function with first derivative given by $g'(x)=\int_0^x e^{-t^2}dt$. Which of the following must be true on the interval $0<x<2$?

$g''(x)=e^{-x^2}=\frac{1}{e^{x^2}}>0$

Always concave up!

$g'(x)=\int_0^x e^{-t^2}dt$ represents area between curve & x axis. Since $y=e^{-x^2}>0$ for all x, $g'(x)>0$, incr!

$\boxed{\text{a. } g \text{ is increasing, and the graph of } g \text{ is concave up}}$
~~b.~~ g is increasing, and the graph of g is concave down
c. g is decreasing, and the graph of g is concave up
~~d.~~ g is decreasing, and the graph of g is concave down
~~e.~~ g is decreasing, and the graph of g has a point of inflection on $0<x<2$

16. A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle is given by $a(t)$. Which of the following gives the average velocity of the particle from time $t=0$ to time $t=8$?

avg value of velocity!

a. $\cancel{\frac{a(8)-a(0)}{8}}$ $\boxed{\text{b. } \frac{1}{8}\int_0^8 v(t)\,dt}$ c. $\cancel{\frac{1}{8}\int_0^8 |v(t)|\,dt}$ d. $\frac{1}{2}\int_0^8 v(t)\,dt$ e. $\cancel{\frac{v(0)+v(8)}{2}}$

17. Let f be a function such that $\int_6^{12} f(2x)\,dx = 10$. Which of the following must be true?

a. $\int_{12}^{24} f(t)\,dt = 5$

b. $\boxed{\int_{12}^{24} f(t)\,dt = 20}$

c. $\int_{6}^{12} f(t)\,dt = 5$

d. $\int_{6}^{12} f(t)\,dt = 20$

e. $\int_{3}^{6} f(t)\,dt = 5$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$x = 6 \quad u = 12$

$x = 12 \quad u = 24$

$\frac{1}{2}\int_{12}^{24} f(u)\,du = \int_6^{12} f(x)\,dx = 10$

$\int_{12}^{24} f(u)\,du = 2(10) = 20$

18. If $F(x) = \int_2^x \sqrt{t^3 - 1}\,dt$, then $F'(2) =$

$F'(x) = \sqrt{x^3 - 1}, \quad F'(2) = \sqrt{7}$

a. 0 b. $\frac{1}{2\sqrt{7}}$ c. $\frac{6}{\sqrt{7}}$ d. $\boxed{\sqrt{7}}$ e. $12\sqrt{7}$

#20 Alternate Method:

$s(t) = \int 6\sin\left(\frac{t}{2}\right)dt$

$s(t) = -12\cos\left(\frac{t}{2}\right) + C$

$(0,1) \quad 1 = -12\cos(0) + C$

$13 = C$

$s(t) = -12\cos\left(\frac{t}{2}\right) + 13$

$s(\pi) = 13$

19. If $F(x) = \int_{3x}^{\sqrt{\pi}} \cos(t^2)\,dt$, then $F'(x) =$ $-\cos(9x^2) \cdot 3$

$F(x) = -\int_{\sqrt{\pi}}^{3x} \cos(t^2)\,dt$

a. $-1 - \cos(9x^2)$ b. $3\cos(9x^2)$ c. $\boxed{-3\cos(9x^2)}$ d. $3\sin(9x^2)$ e. $-3\sin(9x^2)$

20. A particle travels on the x-axis with velocity given by $v(t) = 6\sin\left(\frac{t}{2}\right)$. If the particle is at $x = 1$ when $t = 0$, then its position when $t = \pi$ is $x =$

* Remember $s'(t) = v(t)$

$\int_0^{\pi} v(t)\,dt = s(t)\Big|_0^{\pi} = s(\pi) - s(0)$

$12 = s(\pi) - 1 \Rightarrow s(\pi) = 13$

$u = \frac{t}{2} \quad du = \frac{1}{2}dt \quad dt = 2du$

a. -11 b. -5 c. -2 d. 7 e. $\boxed{13}$

$6\int_0^{\pi} \sin\left(\frac{t}{2}\right)dt = 12\int_0^{\pi/2} \sin u\,du = -12\cos u\Big|_0^{\pi/2} = -12\left(\cos\left(\frac{\pi}{2}\right) - \cos(0)\right) = 12$

21. The expression $\frac{1}{25}\left[\ln\left(1 + \frac{1}{25}\right) + \ln\left(1 + \frac{2}{25}\right) + \ldots + \ln\left(1 + \frac{25}{25}\right)\right]$ is a Riemann approximation for

$\Delta x = \frac{1}{25} = \frac{b-a}{n}$ Lower bound $f(x) = \ln x$

a. $\int_0^1 \ln x\,dx$ b. $\frac{1}{25}\int_0^1 \ln x\,dx$ c. $\boxed{\int_1^2 \ln x\,dx}$ d. $\int_0^1 \ln\left(1 + \frac{x}{25}\right)dx$ e. $\int_0^{25} \ln x\,dx$

22. $\lim_{n\to\infty} \frac{1}{n}\left[\left|\frac{3}{n}\right| + \left|\frac{6}{n}\right| + \left|\frac{9}{n}\right| + \ldots + \left|\frac{3n}{n}\right|\right] =$

$\Delta x = \frac{1}{n} = \frac{b-a}{n} \qquad f(x) = |3x|$

a. $\boxed{\int_0^1 |3x|\,dx}$ b. $\int_1^2 |3x|\,dx$ c. $\int_1^3 |x|\,dx$ d. $3\int_0^1 \left|\frac{1}{x}\right|dx$ e. $\int_0^1 |3 + x|\,dx$