1. Evaluate $\int_1^{5e} \frac{1}{x}dx = \ln|x|\Big]_1^{5e} = \ln 5e - \ln 1 = \ln 5 + \ln e - 0 = \ln 5 + 1$

a) $\frac{1}{5e} - 1$ b) 0 c) $5e$

d) $1 + \ln 5$ e) None of These

2. Evaluate $\int \frac{x+2}{x+1}dx = \int \frac{u-1+2}{u} du = \int \frac{u+1}{u} du = \int 1 + \frac{1}{u} du = u + \ln|u| + C = x + 1 + \ln|x+1| + C$

$u = x+1 \to u-1 = x$

$\frac{du}{dx} = 1 \to du = dx$

a) $\frac{x^2+4x}{x^2+2x} + C$ b) $2x + C$ c) $x + C$

d) $(x+1) + \ln|x+1| + C$ e) None of These

3. Evaluate $\int \tan(3x)dx = \frac{1}{3}\int \tan(u)du = -\frac{1}{3}\ln|\cos(3x)| + C = \frac{1}{3}\ln|\sec(3x)| + C$

$u = 3x$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

a) $\frac{1}{3}\ln|\sec(3x)| + C$ b) $3\sec^2(3x) + C$ c) $\frac{1}{3}\sec^2(3x) + C$

d) $\ln|\cos(3x)| + C$ e) None of These

4. Evaluate $\int \frac{e^{1/(x+1)}}{(x+1)^2}dx = \int \frac{e^u}{(x+1)^2} \cdot -(x+1)^2 du = -\int e^u du = -e^{\frac{1}{x+1}} + C$

$(x+1)^{-1}$

$= \frac{-1}{(x+1)^2}$

$= -(x+1)^2$

a) $\frac{e^{1/(x+1)}}{2(x+1)} + C$ b) $\frac{e^{-x/(x+1)}}{(x+1)^2}$ c) $-e^{1/(x+1)} + C$

d) $\frac{e^{x/(x+1)}}{(x+1)^2}$ e) None of These

$u = x^2+9$

$\frac{du}{dx} = 2x \to dx = \frac{du}{2x}$

$u = x \to \frac{du}{dx} = 1$

$a = 3$

5. Evaluate $\int \frac{x+3}{x^2+9}dx = \int \frac{x}{x^2+9}dx + \int \frac{3}{x^2+9}dx = \int \frac{x}{u} \cdot \frac{du}{2x} + 3\int \frac{1}{u^2+a^2}du = \frac{1}{2}\int \frac{1}{u}du + 3\int \frac{1}{u^2+a^2}du$

a) $\ln|x-3| + C$ b) $\frac{1}{3}\arctan\frac{x}{3} + C$ c) $\frac{1}{2}\ln(x^2+9) + \arctan\frac{x}{3} + C$

d) $\ln(x^2+9) + \frac{1}{3}\arctan\frac{x}{3} + C$ e) None of These

$= \frac{1}{2}\ln|u| + 3 \cdot \frac{1}{a}\arctan(\frac{u}{a})$

$= \frac{1}{2}\ln|x^2+9| + \arctan(\frac{x}{3})$

6. Evaluate $\int \frac{\sin^2 x - \cos^2 x}{\sin x}dx$ (<u>HINT</u>: $\cos^2 x = 1 - \sin^2 x$)

a) $-2\cos x + \ln|\csc x + \cot x| + C$ b) $-\ln|\csc x + \cot x| + C$ c) $-\sec x + C$

d) $\cos x + \ln|\csc x + \cot x| + C$ e) None of These

$\int \frac{\sin^2 x - 1 + \sin^2 x}{\sin x}dx = \int \frac{2\sin^2 x - 1}{\sin x}dx = \int 2\sin x - \csc x \, dx$

$= -2\cos x + \ln|\csc x + \cot x| + C$

7. Evaluate $\int \frac{dx}{\sqrt{8+2x-x^2}}$ $= \int \frac{dx}{\sqrt{9-(x-1)^2}}$ $\quad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{x-1}{3}\right)+C$ $\quad 9-(x-1)^2$

$u = x-1$

$a = 3$

a) $\ln\sqrt{8+2x-x^2}+C$ b) $\arcsin\left(\frac{x-1}{3}\right)+C$ c) $\sqrt{8+2x-x^2}+C$

d) $\frac{1}{3}\arcsin\left(\frac{x-1}{3}\right)+C$ e) None of These

8. Evaluate $\boxed{\int_{\pi/2}^{\pi} e^{\cos(x)}\sin(x)\,dx = 1-\frac{1}{e}}$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$\int_{0}^{-1} e^u \cdot \sin x \cdot \frac{du}{-\sin x} = \int_{-1}^{0} e^u\,du$

$= e^0 - e^{-1}$

$= 1 - \frac{1}{e}$

a) $1-e$ b) 1 c) $e-1$

d) e^2-2 e) None of These

"a^x"

9. Evaluate $\int (\ln(3))^x\,dx = \frac{1}{\ln(\ln 3)}(\ln 3)^x + C$

a) $\frac{(\ln(3))^x}{\ln(3)}+C$ b) $(\ln(3))^x+C$ c) 3^x+C

d) $\frac{(\ln(3))^x}{\ln(\ln(3))}+C$ e) None of These

10. The function f is defined by $f(x)=\begin{cases} 2 & \text{for } x<0 \\ x2^{x^2+1} & \text{for } x\ge 0 \end{cases}$. What is the value of $\int_{-2}^{2} f(x)\,dx$?

$\int_{-2}^{0} 2\,dx + \int_{0}^{2} x\cdot 2^{x^2+1}\,dx =$

$u = x^2+1$

$\frac{du}{dx} = 2x \rightarrow dx = \frac{du}{2x}$

a) $2+\frac{30}{\ln 2}$ b) $2+\frac{15}{\ln 2}$ c) $4+\frac{30}{\ln 2}$

d) $4+\frac{15}{\ln 2}$ e) None of These

$2x\Big]_{-2}^{0} + \frac{1}{2}\int_{1}^{5} 2^u\,du =$

$2x\Big]_{-2}^{0} + \frac{1}{2\ln 2}\cdot 2^u\Big]_{1}^{5} =$

$2(0)-2(-2)+\frac{1}{\ln 4}(2^5-2) =$

$4+\frac{1}{2\ln 2}(30) = 4+\frac{15}{\ln 2}$

11. Which of the following are equivalent to $\int \frac{\csc^2 x}{\cot^3 x}\,dx$?

✓ I. $\frac{1}{2\cos^2 x}+c$ ✓ II. $\frac{1}{2\cot^2 x}+c$ ✓ III. $\frac{\tan^2 x}{2}+c$

$\cot x$

$= -\csc^2 x$

$= \frac{du}{-\csc^2 x}$

a) I only b) II only c) III only

d) II and III only e) All of the above

$\int -\frac{1}{u^3}\,du = \frac{1}{2u^2}+C = \frac{1}{2\cot^2(x)}+C = \frac{\tan^2 x}{2}+C = \frac{\sin^2 x}{2\cos^2 x}+C = \frac{1-\cos^2 x}{2\cos^2 x}+C = \frac{1}{2\cos^2 x} - \frac{\cos^2 x}{2\cos^2 x} + C$

12. GC Calculator Active: A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \dfrac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

$\int_0^3 \frac{t+3}{\sqrt{t^3+1}}\,dt = v(3) - v(0)$

$6.710 = v(3) - 5$

$v(3) = 11.710$

a) 0.713 b) 1.134 c) 6.134

d) 6.710 e) 11.710

13. GC Calculator Active: The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \leq t \leq 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing? $r(t) < 0$ when $1.572 < t < 3.514$

a) $\int_{1.572}^{3.514} r(t)\,dt$ b) $\int_0^8 r(t)\,dt$ c) $\int_0^{2.667} r(t)\,dt$

$\int_{1.572}^{3.514} r(t)\,dt$

d) $\int_{1.572}^{3.514} r'(t)\,dt$ e) $\int_0^{2.667} r'(t)\,dt$

14. Using the substitution $u = 2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to $\int_1^5 \sqrt{u} \cdot \frac{du}{2}$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

a) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ b) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ c) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$

d) $\int_0^2 \sqrt{u}\,du$ e) $\int_1^5 \sqrt{u}\,du$

15. If $G(x)$ is the antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

$\int_2^4 f(t)\,dt = G(x)\Big]_2^4$

$\int_2^4 f(t)\,dt = G(4) - G(2)$

a) $f'(4)$ b) $-7 + f'(4)$ c) $\int_2^4 f(t)\,dt$

$\int_2^4 f(t)\,dt = G(4) + 7$

d) $\int_2^4 (-7 + f(t))\,dt$ e) $-7 + \int_2^4 f(t)\,dt$

$G(4) = -7 + \int_2^4 f(t)\,dt$

16. Explain how $\int \frac{2}{5-10x}\,dx$ is equivalent to both $-\frac{1}{5}\ln|5-10x| + c$ and $-\frac{1}{5}\ln|1-2x| + c$.

The constants of integration differ.

$$-\frac{1}{5}\ln|5-10x| + c = -\frac{1}{5}\ln|5(1-2x)| + c = -\frac{1}{5}\ln 5 - \frac{1}{5}\ln|1-2x| + c = -\frac{1}{5}\ln|1-2x| \underbrace{-\frac{1}{5}\ln 5 + c}$$

17. The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by function S, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 < t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

a) How much sand will the tide remove from the beach during the 6-hour period? Indicate units of measure.

b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

d) For $0 < t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(a) $\int_0^6 R(t)\,dt = 31.815$ or 31.816 yd^3

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer with units} \end{cases}$

(b) $Y(t) = 2500 + \int_0^t (S(x) - R(x))\,dx$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or -1.909 yd^3/hr

1 : answer

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.

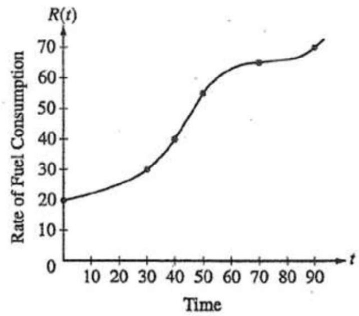
The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

$3: \begin{cases} 1: \text{sets } Y'(t) = 0 \\ 1: \text{critical } t\text{-value} \\ 1: \text{answer with justification} \end{cases}$

18.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values for $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

a. Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

b. The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

c. Approximate the value of $\int_0^{90} R(t)dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)dt$? Explain your reasoning.

d. For $0 \le b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)dt$ and $\frac{1}{b}\int_0^{b} R(t)dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a) $R'(45) \approx \frac{R(50) - R(40)}{50 - 40} = \frac{55 - 40}{10}$ $= 1.5$ gal/min^2	2 : { 1 : a difference quotient using numbers from table and interval that contains 45; 1 : 1.5 gal/min^2 }
(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.	2 : { 1 : $R''(45) = 0$; 1 : reason }
(c) $\int_0^{90} R(t)\,dt \approx (30)(20) + (10)(30) + (10)(40)$ $+(20)(55) + (20)(65) = 3700$ Yes, this approximation is less because the graph of R is increasing on the interval.	2 : { 1 : value of left Riemann sum; 1 : "less" with reason }
(d) $\int_0^{b} R(t)\,dt$ is the total amount of fuel in gallons consumed for the first b minutes. $\frac{1}{b}\int_0^{b} R(t)\,dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.	3 : { 2 : meanings { 1 : meaning of $\int_0^{b} R(t)\,dt$; 1 : meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$; < -1 > if no reference to time b }; 1 : units in both answers }

19. Complete the following questions from Sample Exam II in your AP Prep Book:
(NOTE: Be sure to follow directions in each section for GC.)

a. OE - #1, 3

b. MC - #4, 10, 15, 16, 33, 40

1a) $T(x)=70$

$x=8.575\approx 8.5$ 8:30am

b) $T(x)=77$

$x=10.507\approx 10.5$ 10:30am

c) $C(8.5)=.16\int_{8.5}^{18} T(x)-70\,dx=\$18.26$

d) $C(10.5)=.16\int_{10.5}^{18} T(x)-77\,dx=\$8.79$

$\$18.26-\$8.79=\$9.47$

3a) $\underbrace{(5,6)\quad h'(5)=.7}$

$y-6=.7(x-5)$

$L(x)=.7(x-5)+6$

$L(4)=.7(4-5)+6=5.3$

$h''(t)>0\rightarrow$ concave up $\rightarrow L(4)$ underapproximation

b) $V=10(20)h$

$\frac{dV}{dt}=200\frac{dh}{dt}$

$\left.\frac{dV}{dt}\right|_{t=2}=200(.5)$

$=100\ in^3/min$

c) $\int_0^{15} h'(t)dt=2(.4)+3(.5)+4(.7)+4(1)+2(1.1)=$

11.3in

From $t=0$ to $t=15$, the depth changed by 11.3 in.

d) $h''(t)>0\rightarrow h'(t)$ increasing $\rightarrow$ under approximation

B 4. $\int_0^1 e^{-2x}dx=-\frac{1}{2}e^{-2x}\Big]_0^1=-\frac{1}{2}e^{-2}+\frac{1}{2}e^{0}$

$=\frac{-1}{2e^2}+\frac{1}{2}$

$=\frac{1}{2}-\frac{1}{2e^2}$

B 10. $\int 2\tan(x)dx=$

$2\ln|\sec x|=$

$\ln(\sec^2 x)$

A 15. $\int 8-2t\,dt=-t^2+8t+C=s(t)$

$v(t)=0=8-2t$

$\underbrace{t=4}$

changes direction at origin

$s(4)=-4^2+8(4)+C=0$

$16+C=0$

$C=-16$

$s(t)=-t^2+8t-16$

B 16. $u^2+1=x$

$u=\sqrt{x-1}$

$\frac{du}{dx}=\frac{1}{2\sqrt{x-1}}$

$dx=2\sqrt{x-1}\,du$

$\int_2^5\frac{\sqrt{x-1}}{x}dx=\int^2\frac{u}{x}\cdot 2\sqrt{x-1}\,du$

$=\int_1^2\frac{2u^2}{u^2+1}du$

A 33. $\int_0^3 m(t)dt=10.667$

D 40. largest area enclosed by graphs