Ch 5 Review

1. $f(x) = x\sqrt{\ln x}$

$f'(x) = \sqrt{\ln x} + \frac{1}{2\sqrt{\ln x}} \cdot \frac{x}{x}$

$= \boxed{\frac{2(\ln x + 1}{2\sqrt{\ln x}} = \frac{\ln x^2 + 1}{2\sqrt{\ln x}}}$

2. $f(x) = \ln\left(x(x^2-3)^{2/3}\right)$

$f'(x) = \frac{1}{x(x^2-3)^{2/3}} \cdot \left((x^2-3)^{2/3} + \frac{2x \cdot 2x}{3(x^2-3)^{1/3}}\right)$

$= \frac{1}{x(x^2-3)^{2/3}} \quad \frac{3(x^2-3) + 4x^2}{3(x^2-3)^{1/3}} = \boxed{\frac{7x^2-9}{3x(x^2-3)}}$

3. $f(x) = \sqrt{e^{2x} + e^{-2x}}$

$f'(x) = \frac{1}{2\sqrt{e^{2x}+e^{-2x}}} \cdot (2e^{2x} - 2e^{-2x}) = \boxed{\frac{e^{2x} - e^{-2x}}{\sqrt{e^{2x}+e^{-2x}}}}$

4. $\frac{d}{dx}\left(y \ln(xy) + y^2 = 4\right)$

$\ln(xy)\frac{dy}{dx} + \frac{y}{xy}\left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$

$\ln(xy)\frac{dy}{dx} + \frac{y}{x} + \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$(\ln(xy) + 2y + 1)\frac{dy}{dx} = \frac{-y}{x}$

$\boxed{\frac{dy}{dx} = \frac{-y}{x(\ln(xy) + 2y + 1)}}$

5. $y = x^{2x+1}$

$\ln y = \ln x^{2x+1}$

$\frac{d}{dx}\left(\ln y = (2x+1)\ln x\right)$

$\frac{1}{y}\frac{dy}{dx} = 2\ln x + \frac{2x+1}{x}$

$\frac{dy}{dx} = y\left(2\ln x + \frac{2x+1}{x}\right)$

$\boxed{\frac{dy}{dx} = x^{2x+1}\left(\ln x^2 + \frac{2x+1}{x}\right)}$

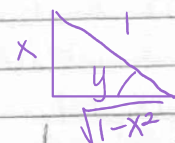
rewriting using log

6. $f(x) = 5x \log_4 x^2$

$f'(x) = 5\log_4 x^2 + 5x \cdot \dfrac{1}{x^2 \ln(4)} \cdot 2x = 5\log_4 x^2 + \dfrac{10}{\ln 4}$

$= \boxed{\dfrac{5\ln x^2 + 10}{\ln 4}}$

$y = \arcsin x$
$x = \sin y$



composition of functions

*7. $f(x) = \tan(\arcsin x)$

$f'(x) = \sec^2(\arcsin x) \cdot \dfrac{1}{\sqrt{1-x^2}} = \dfrac{\left(\frac{1}{\sqrt{1-x^2}}\right)^2}{\sqrt{1-x^2}} = \boxed{\dfrac{1}{(1-x^2)^{3/2}}}$

alternate method on last page

comp of functions

8. $f(x) = \frac{1}{2} \arctan e^{4x}$

$f'(x) = \dfrac{4e^{4x}}{2(e^{8x}+1)} = \boxed{\dfrac{2e^{4x}}{e^{8x}+1}}$

using prop of logs

9. $f(x) = \ln\left(\dfrac{5x\sqrt{\cos x}}{7x^3}\right) = \ln 5x + \frac{1}{2}\ln(\cos x) - \ln 7 - 3\ln x$

$f'(x) = \dfrac{1}{x} + \dfrac{1}{2\cos x} \cdot -\sin x - \dfrac{3}{x}$

$f'(x) = \boxed{-\dfrac{2}{x} - \dfrac{\tan x}{2}}$

simp before deriv

10. $g(x) = 5^{3x} \ln e^{\sqrt{x}} = 5^{3x} \cdot \sqrt{x}$

$g'(x) = 3 \cdot 5^{3x} \ln(5)\sqrt{x} + 5^{3x} \cdot \dfrac{1}{2\sqrt{x}}$

$g'(x) = 5^{3x}\left(\dfrac{6\ln(5)x + 1}{2\sqrt{x}}\right) = \boxed{5^{3x}\left(\dfrac{\ln(5^6)x + 1}{2\sqrt{x}}\right)}$

simplifying trig functions

11. $h(x) = \arctan(\sec x)$

$h'(x) = \dfrac{1}{\sec^2 x + 1} \cdot \sec x \tan x = \dfrac{1}{\frac{1}{\cos^2 x} + 1} \cdot \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x}$

$= \boxed{\dfrac{\sin x}{1 + \cos^2 x}}$

Ch 5 Rev cont

ccos 12. $f(x) = \arccos(7x^4)$

$f'(x) = \dfrac{-1}{\sqrt{1-(7x^4)^2}} \cdot 28x^3 = \boxed{\dfrac{-28x^3}{\sqrt{1-49x^8}}}$

13. $f(x) = \operatorname{arccsc}^2(e^x)$

$f'(x) = 2\operatorname{arccsc}(e^x) \cdot \dfrac{-1}{e^x\sqrt{e^{2x}-1}} \cdot e^x$

$= \boxed{\dfrac{-2\operatorname{arcsc}(e^x)}{\sqrt{e^{2x}-1}}}$

d of e^x as of arg of inverse

14. $g(x) = 5\sin(2x)\arcsin(2^{4x})$

$g'(x) = 10\cos(2x)\arcsin(2^{4x}) + \dfrac{5\sin(2x)}{\sqrt{1-(2^{4x})^2}} \cdot 2^{4x}\ln(2) \cdot 4$

$= \boxed{10\cos(2x)\arcsin(2^{4x}) + \dfrac{20\ln(2)(2^{4x})\sin(2x)}{\sqrt{1-2^{8x}}}}$

$\boxed{d.}$ 15. $f(x) = \ln(3x+2)^k$, $f'(2) = 3$

ew app

$= k\ln(3x+2)$

$f'(x) = \dfrac{k}{3x+2} \cdot 3 \Rightarrow f'(2) = \dfrac{3k}{8} = 3$

$\boxed{k=8}$

$\boxed{a.}$ 16. $f(x) = e^x \ln x$

simplifying exp functions

$f'(x) = e^x \ln x + \dfrac{e^x}{x}$

$f'(e) = e^e \ln e + \dfrac{e^e}{e} = \boxed{e^e + e^{e-1}}$

$\boxed{d}$ 17. $y = \ln(3x+5)$

riv

$\dfrac{dy}{dx} = \dfrac{3}{3x+5}$ → $\dfrac{d^2y}{dx^2} = \dfrac{-3}{(3x+5)^2} \cdot 3 = \boxed{\dfrac{-9}{(3x+5)^2}}$

$\boxed{a}$ 18. $f(x) = x^2 \ln x$

$f'(x) = 2x\ln x + \dfrac{x^2}{x}$

$f'(x) = 2x\ln x + x = 2$

GC ⇒ $x \approx 1.305$

GC use

e] 19. $f(x) = x^{-1/3}$

inverse first

$(x)^{-3} = (y^{-1/3})^{-3}$

$\frac{1}{x^3} = y$

$\boxed{y' = \frac{-3}{x^4}}$

e] 20. $y = \arcsin\left(\frac{3x}{4}\right)$

simplifying

$y' = \frac{1}{\sqrt{1-\left(\frac{3x}{4}\right)^2}} \cdot \frac{3}{4} = \frac{3}{\sqrt{16-9x^2}}$

d] 21. $y = x^{1-x}$

$\ln y = \ln x^{1-x}$

$\frac{d}{dx}(\ln y = ((1-x)\ln x)$

$\frac{1}{y}\frac{dy}{dx} = -\ln x + \frac{(1-x)}{x}$

$\frac{dy}{dx} = y\left(-\ln x + \frac{(1-x)}{x}\right)$

$\frac{dy}{dx} = x^{1-x}\left(-\ln x + \frac{1-x}{x}\right)$

of ... to ... $\frac{d}{dx}$ ... nd ... deriv

d] 22. $y = \frac{x^3}{3^x}$

$y' = \frac{3x^2 3^x - 3^x \ln(3) x^3}{3^{2x}} = \frac{3^x x^2 (3 - \ln(3)x)}{3^{2x}}$

$= \boxed{\frac{x^2(3-\ln(3)x)}{3^x}}$

d] 23. $f(x) = 5^{\frac{x}{2}}$ tan line at $x=2$ $\quad f(2) = 5^{2/2} = 5 \quad (2,5)$

a^x w/ base

$f'(x) = 5^{\frac{x}{2}}\ln(5) \cdot \frac{1}{2}$

$f'(2) = \frac{5\ln(5)}{2} \qquad y - 5 = \frac{5\ln(5)}{2}(x-2)$

$y = \frac{5}{2}\ln(5)x - 5\ln(5) + 5$

$\boxed{y = \frac{5}{2}\left[\ln(5)x - 2\ln(5) + 2\right]}$

Ch 5 Review

24. monotonic → one to one → inverse exists

recognizing inverse

$$(f^{-1})'(b) = \frac{1}{f'(a)} = \frac{9}{4}$$

25. $y = \ln(3-x^2)$ $x=1$ $y(1)=\ln(2)$

tan line

$(1, \ln(2))$

$$y' = \frac{1}{3-x^2} \cdot -2x$$

$$y'(1) = \frac{-2}{2} = -1 = m$$

$$y - \ln(2) = -(x-1)$$

$$\boxed{y = -x+1+\ln(2)}$$

26. a) $f(x) = \tan x$ $\left(c, \frac{\sqrt{3}}{3}\right)$

$(f^{-1})'(x)$ $\left(\frac{\sqrt{3}}{3}, c\right)$

$$\frac{1}{\sqrt{3}} = \tan c$$

$$c = \frac{\pi}{6}$$

$$f'(x) = \sec^2 x$$

$$f'(\pi/6) = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$$

$$(f^{-1})'\left(\frac{\sqrt{3}}{3}\right) = \boxed{\frac{3}{4}}$$

b) $x = \tan y$

$y = \arctan x$

$(f^{-1})(x) = \arctan x$

$$(f^{-1})'(x) = \frac{1}{x^2+1}$$

$$(f^{-1})'\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)^2+1} = \frac{1}{\frac{1}{3}+\frac{3}{3}}$$

$$= \frac{1}{\frac{4}{3}} = \boxed{\frac{3}{4}}$$

27. a) $y = \log_3 x$ $(27, 3)$

$$y' = \frac{1}{x\ln(3)}$$ $$m = \frac{1}{27\ln(3)}$$

$$\boxed{y - 3 = \frac{1}{27\ln(3)}(x-27)}$$

$y(2) = 5^0$

b. $y = 5^{x-2}$ $(2, 1)$ $\boxed{y - 1 = \ln(5)(x-2)}$

$y' = 5^{x-2}\ln(5)$

$y'(2) = 5^0 \ln(5) = \ln(5) = m$

c. $\frac{d}{dx}\left(1 + \ln(xy) = e^{x-y}\right)$ $(1,1)$

$\frac{1}{xy}\left(y + x\frac{dy}{dx}\right) = e^{x-y}\left(x - \frac{dy}{dx}\right)$

$1\left(1 + \frac{dy}{dx}\right) = e^0\left(1 - \frac{dy}{dx}\right)$

$1 + \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$2\frac{dy}{dx} = 0 \longrightarrow \boxed{y = 1}$

28. $f(x) = x^5 + 2x - 1$ $(f^{-1})'(x)$

$(C, 2)$ $(2, C)$

$2 = x^5 + 2x - 1$

on GC $\Rightarrow x = 1$ $\rightarrow (f^{-1})'(2) = \boxed{\frac{1}{7}}$

$f'(x) = 5x^4 + 2$

$f'(1) = 7$

29. $y = \arctan x$

a) $y' = \frac{1}{x^2+1} > 0$ for all x! Always +, ∴ always moving right

b) $y'' = \frac{-1}{(x^2+1)^2} \cdot 2x = \frac{-2x}{(x^2+1)^2}$ for $x \geq 0$, $y'' < 0$ ∴ $y' > 0$ decelerating

c) $\lim_{t\to\infty} \arctan t = \pi/2$

30. $f(x) = \cos x + 3x$

$f'(x) = -\sin x + 3$

nce $-1 \leq -\sin x \leq 1$

$f'(x) > 0$ for all x ∴ always increasing one-to-one

30. $f(x) = \cos x + 3x$

a) $f'(x) = -\sin x + 3$

Since $-1 \le \sin x \le 1$

$f'(x) > 0$ for all x, $\therefore$ increasing for all x & one-to-one.

b) $\boxed{f(0) = 1}$ $\boxed{f'(0) = 3}$

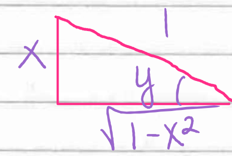
c) $\dfrac{f(x) = \cos x + 3x}{(0,1)}$ $\dfrac{f^{-1}(x)}{(1,0)}$

$\therefore f^{-1}(1) = 0$

$\boxed{f'(0) = 3}$ $\boxed{(f^{-1})'(1) = \frac{1}{3}}$

7. Alternate method:

$f(x) = \tan(\underbrace{\arcsin x}_{y})$



$y = \arcsin x$

$\sin y = x$

$f(x) = \dfrac{x}{\sqrt{1-x^2}}$

$f'(x) = \dfrac{\dfrac{(\sqrt{1-x^2})^2}{\sqrt{1-x^2}} - x\left(\dfrac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}\right)}{(\sqrt{1-x^2})^2}$

$f'(x) = \dfrac{1-x^2+x^2}{\sqrt{1-x^2}} \cdot \dfrac{1}{1-x^2}$

$\boxed{f'(x) = \dfrac{1}{(1-x^2)^{3/2}}}$