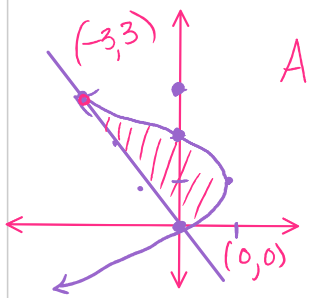
Set-up an integral that will find the area of the region bounded by the functions below. Use the integration capabilities of the graphing calculator to find the area.

$f' = 2-2y$

1. $f(y) = y(2-y)$, $g(y) = -y$

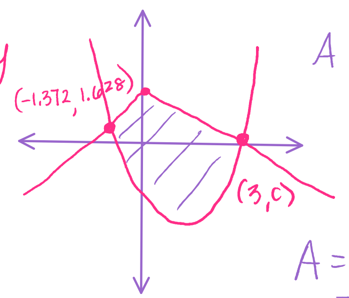


$$A = \int_0^3 \left[2y - y^2 - (-y)\right] dy$$

$$A = \int_0^3 (3y - y^2)\, dy$$

$$A = \boxed{\frac{9}{2}}$$

2. $y = x^2 - 2x - 3$, $y = 3 - |x|$



$$A = \int_{-1.372}^{0} (3 + x - (x^2 - 2x - 3))\, dx + \int_0^3 (3 - x - (x^2 - 2x - 3))\, dx$$

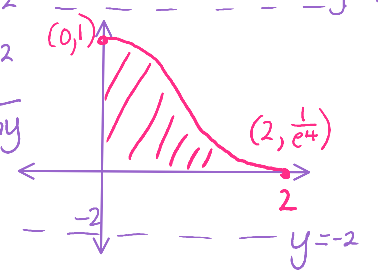
$$A = \int_{-1.372}^{0} (6 + 3x - x^2)\, dx + \int_0^3 (6 + x - x^2)\, dx$$

$$\boxed{A = 18.048}$$

Use a graphing calculator wherever applicable. Show the complete set-up of each integral.

3. Find the volume of the following solids formed by revolving the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 2$ about the appropriate axis of revolution. Use the integration capabilities of the graphing calculator to find the volume.

$y = e^{-x^2}$
$\ln y = -x^2$
$x = \sqrt{-\ln y}$



disk a. x-axis $V = \pi \int_0^2 (e^{-x^2})^2\, dx = 1.969$

$r = e^{-x^2}$

washer b. $y = -2$ $V = \pi \int_0^2 \left[(e^{-x^2} + 2)^2 - 2^2\right] dx = 13.053$

$R = e^{-x^2} + 2$

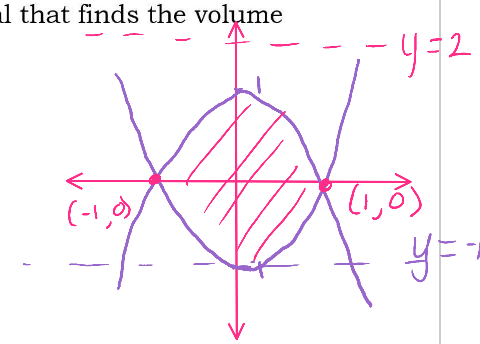
$r = 2$

washer c. $y = 3$ $V = \pi \int_0^2 \left[3^2 - (3 - e^{-x^2})^2\right] dx = 14.658$

$R = 3$

$r = 3 - e^{-x^2}$

4. Consider the region R bounded by the functions $y = x^2 - 1$ and $y = 1 - x^2$. Sketch the region & indicate point(s) of intersection. Set-up, but do not solve, an integral that finds the volume generated when R is revolved about:



a. $y = 2$ washer

$R = 2 - (x^2-1) = 3-x^2$

$r = 2-(1-x^2) = 1+x^2$

$$V = \pi \int_{-1}^{1} (3-x^2)^2 - (1+x^2)^2 \, dx$$

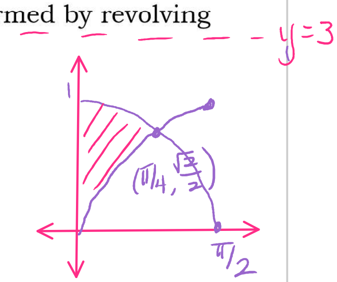
b. $y = -1$ washer

$R = 1-x^2-(-1) = 2-x^2$

$r = x^2-1-(-1) = x^2$

$$V = \pi \int_{-1}^{1} (2-x^2)^2 - (x^2)^2 \, dx$$

5. Consider the region bounded by $f(x) = \sin x$, $g(x) = \cos x$ and the y-axis in the first quadrant. Set up, but do not evaluate, integral expressions that find the volume of the solid formed by revolving the region around the given axis.



a. x-axis washer

$R = \cos x$

$r = \sin x$

$$V = \pi \int_{0}^{\pi/4} (\cos^2 x - \sin^2 x) \, dx$$

b. $y = 3$ washer

$R = 3 - \sin x$

$r = 3 - \cos x$

$$V = \pi \int_{0}^{\pi/4} \left[(3-\sin x)^2 - (3-\cos x)^2\right] dx$$

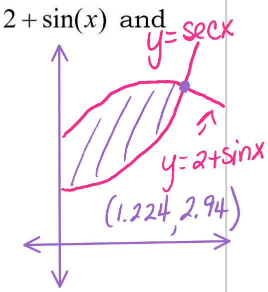
c. y-axis disk

$r_1 = \sin^{-1} y$

$r_2 = \cos^{-1} y$

$$V = \pi \int_{0}^{\sqrt{2}/2} (\sin^{-1} y)^2 \, dy + \pi \int_{\sqrt{2}/2}^{1} (\cos^{-1} y)^2 \, dy$$

6. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of $y = 2 + \sin(x)$ and $y = \sec(x)$. Use your calculator whenever applicable.



a. Find the area of R.

$$A = \int_{0}^{1.224} (2 + \sin x - \sec x) \, dx = 1.366$$

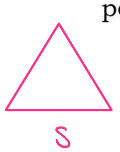
b. Find the volume of the solid generated when R is revolved about the x-axis.

washer $R = 2 + \sin x$

$r = \sec x$

$$V = \pi \int_{0}^{1.224} \left[(2+\sin x)^2 - \sec^2 x\right] dx = 16.404$$

c. Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are equilateral triangles.



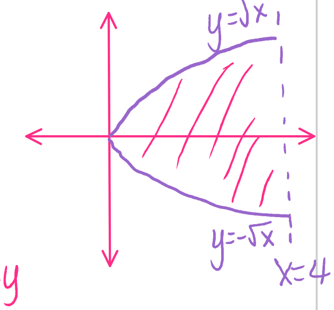
$A = \frac{\sqrt{3}}{4} s^2$

$A = \frac{\sqrt{3}}{4} (2 + \sin x - \sec x)^2$

$$V = \int_{0}^{1.224} \frac{\sqrt{3}}{4} (2 + \sin x - \sec x)^2 \, dx = 7.054$$

7. The base of a solid is a region that lies between $y=-\sqrt{x}$, $y=\sqrt{x}$, and $x=4$. Find the volume of the solid created with the following conditions for the cross sections.

$x=y^2$



a. The cross sections are semicircles that run parallel to the y-axis. dx

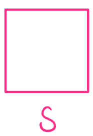
$d=\sqrt{x}-(-\sqrt{x})=2\sqrt{x}$

$r=\sqrt{x}$

$A=\frac{\pi}{2}r^2=\frac{\pi}{2}x$

$V=\int_0^4\left(\frac{\pi}{2}x\right)dx=4\pi$

b. The cross sections are squares that run perpendicular to the y-axis. dy

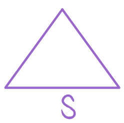


$S=4-y^2$

$A=S^2=(4-y^2)^2$

$V=\int_{-2}^{2}(4-y^2)^2\,dy=2\int_0^2(4-y^2)^2\,dy=\frac{512}{15}$

c. The cross sections are equilateral triangles that run perpendicular to the x-axis. dx



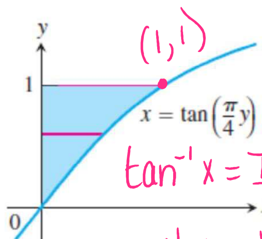
$A=\frac{\sqrt{3}}{4}s^2$

$A=\frac{\sqrt{3}}{4}(2\sqrt{x})^2=\sqrt{3}x$

$V=\int_0^4(\sqrt{3}x)\,dx=8\sqrt{3}$

8. Set up, but do not solve, an integral that finds the volume of the solid generated by revolving the function around the given axes.

a) y-axis and $y=1$



$\tan^{-1}x=\frac{\pi}{4}y$

$y=\frac{4}{\pi}\tan^{-1}x$

y axis: dy, disk

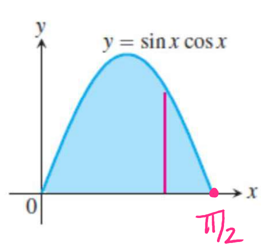
$V=\pi\int_0^1\tan^2\left(\frac{\pi}{4}y\right)dy$

$y=1$ dx, disk

$r=1-\frac{4}{\pi}\tan^{-1}(x)$

$V=\pi\int_0^1\left(1-\frac{4}{\pi}\tan^{-1}x\right)^2dx$

b) x-axis & $y=-1$



$y=-1$

x axis: dx, disk

$V=\pi\int_0^{\pi/2}(\sin^2x\cos^2x)\,dx$

$y=-1$: dx, washer

$R=\sin x\cos x+1$

$r=1$

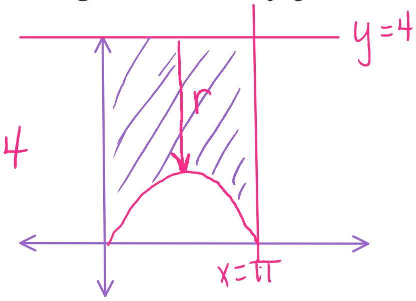
$V=\pi\int_0^{\pi/2}\left[(\sin x\cos x+1)^2-1^2\right]dx$

Multiple Choice: You may use your calculator but be sure to show your work!

9. The integral $\pi\int_0^{\pi}(4-\sin x)^2\,dx$ represents the volume generated when the region bounded by $y=4$, $y=\sin(x)$, $x=0$, and $x=\pi$ is revolved about:

$r = 4 - \sin x$

disk about $y=4$



a. the x-axis ~~using~~ washers

$\boxed{\text{b. the line } y=4 \text{ using disks}}$

c. the line $y=4$ ~~using~~ washers

d. the line x ~~using~~ washers

10. The integral $\int^{\pi}$ represents the volume generated when the region bounded by $y=4$, $y=\sin(x)$, $x=0$, and $x=\pi$ is revolved about:

around x axis – washer

$R=4$, $r=\sin x$

$V=\pi\int_0^{\pi}(4^2-\sin^2 x)\,dx$

$\boxed{\text{a. the } x\text{-axis using washers}}$

b. the line $x=\pi$ using washers

c. the line $x=\pi$ using disks

d. none of the above

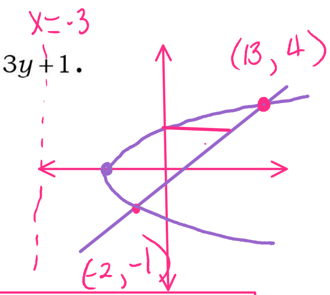
For problems 11 and 12, region *R* is bounded by $f(y)=y^2-3$ **and** $g(y)=3y+1$.

$y^2-3=3y+1$

$y^2-3y-4=0$

$(y-4)(y+1)=0$

$y=4,-1$



11. Which of the following expressions gives the area of region *R*?

← Bounds incorrect →

a. $\int_{-2}^{13}(3y+1)-\cancel{(y^2-3)}\,dy$

b. $\int_{-2}^{13}\cancel{(y^2-3)}-(3y+1)\,dy$

$\boxed{\text{c. } \int_{-1}^{4}(3y+1)-(y^2-3)\,dy}$

d. $\int_{-1}^{4}(y^2-3)-(3y+1)\,dy$

e. $\int_{-1}^{4}(3y+1)+(y^2-3)\,dy$

12. Which of the following expressions gives the volume when R is rotated about the line $x=-3$?

washer $R = 3 + 3y+1 = 3y+4$ $r = 3 + y^2 - 3 = y^2$

a. $\pi\int_{-2}^{13}\left[\cancel{(3y+4)-(y^2)}\right]^2 dy$

b. $\pi\int_{-2}^{13}\left[\cancel{(6-y^2)-(2-3y)}\right]^2 dy$

c. $\pi\int_{-1}^{4}(2-3y)^2-(6-y^2)^2\,dy$

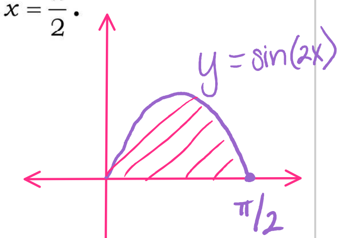
d. $\pi\int_{-1}^{4}(y^2-3)^2-(3y+1)^2\,dy$

$\boxed{\text{e. } \pi\int_{-1}^{4}\left[(3y+4)^2-(y^2)^2\right]dy}$

For problems 13 and 14, region Q is bounded by $y = \sin(2x)$, $y = 0$, $x = 0$ **and** $x = \frac{\pi}{2}$.

$A = \int_0^{\pi/2} \sin(2x)\,dx = -\frac{1}{2}\cos(2x)\Big|_0^{\pi/2} = -\frac{1}{2}(\cos\pi - \cos 0)$

$= -\frac{1}{2}(-2) = 1$



13. What is the area of region Q?

a. 0 b. $\frac{1}{2}$ c. $\frac{\pi}{2}$ $\boxed{\text{d. } 1}$ e. 2

14. Which of the following expressions gives the volume of a solid whose base in the xy-plane is the region Q and whose cross sections, perpendicular to the x-axis, are squares?

a. $\pi\int_0^{\frac{\pi}{2}}(1-\cos^2(2x))dx$ $\boxed{\text{b. } \int_0^{\frac{\pi}{2}}\sin^2(2x)dx}$ c. $\int_0^{\frac{\pi}{2}}(1-\cos(2x))dx$

d. $\int_0^{\frac{\pi}{2}}(1-\cos(2x^2))dx$ e. $\pi\int_0^{\frac{\pi}{2}}(\sin(2x)^2)dx$

s

$A = s^2 = \sin^2(2x)$

15. GC Allowed: The base of a solid is bounded by $y = \tan^2(x)$ and $y = 4 - x^2$ in the xy-plane. Each cross section perpendicular to the x-axis is a rectangle with one side in the xy-plane and whose height is 2. What is the volume of the solid?

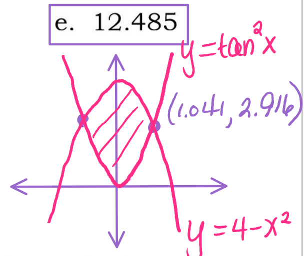
a. 3.121 b. 4.454 c. 6.211 d. 6.243 $\boxed{\text{e. } 12.485}$

2
s

$A = 2s$

$A = 2(4 - x^2 - \tan^2 x)$

$V = 2\int_0^{1.041} 2(4 - x^2 - \tan^2 x)\,dx$



16. GC Allowed: The region bounded by the graphs $y = \sqrt{x^3 - x^2}$, $y = 0$, and $x = 3$ is revolved around the x-axis to form a solid. Find the volume of the solid.

$\boxed{\text{a. } \frac{34\pi}{3}}$ b. $\frac{27\pi}{2}$ c. 13π d. $\frac{\pi}{12}(971 - 216\sqrt{3})$ e. None of these

dx, disk $r = \sqrt{x^3 - x^2}$

$V = \pi\int_1^3 \left(\sqrt{x^3 - x^2}\right)^2 dx = \frac{34\pi}{3}$