CHAPTER 2
Differentiation

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Section 2.1 The Derivative and the Tangent Line Problem

1. (a) At (x_1, y_1), slope $= 0$.

At (x_2, y_2), slope $\approx \frac{5}{2}$.

(b) At (x_1, y_1), slope $\approx -\frac{5}{2}$.

At (x_2, y_2), slope ≈ 2.

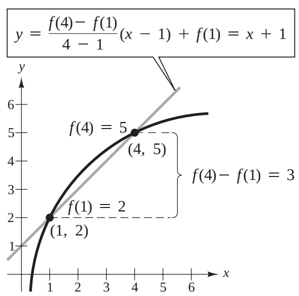
2. (a) At (x_1, y_1), slope $\approx \frac{2}{3}$.

At (x_2, y_2), slope $\approx -\frac{2}{5}$.

(b) At (x_1, y_1), slope $\approx \frac{4}{3}$.

At (x_2, y_2), slope $\approx \frac{5}{4}$.

3. (a), (b)



(c) $$y = \frac{f(4) - f(1)}{4 - 1}(x - 1) + f(1)$$
$$= \frac{3}{3}(x - 1) + 2$$
$$= 1(x - 1) + 2$$
$$= x + 1$$

4. (a) $$\frac{f(4) - f(1)}{4 - 1} = \frac{5 - 2}{3} = 1$$
$$\frac{f(4) - f(3)}{4 - 3} \approx \frac{5 - 4.75}{1} = 0.25$$
Thus, $\dfrac{f(4) - f(1)}{4 - 1} > \dfrac{f(4) - f(3)}{4 - 3}$.

(b) The slope of the tangent line at $(1, 2)$ equals $f'(1)$. This slope is steeper than the slope of the line through $(1, 2)$ and $(4, 5)$. Thus,
$$\frac{f(4) - f(1)}{4 - 1} < f'(1).$$

5. $f(x) = 3 - 2x$ is a line. Slope $= -2$

6. $g(x) = \frac{3}{2}x + 1$ is a line. Slope $= \frac{3}{2}$

7. Slope at $(1, -3)$ $$= \lim_{\Delta x \to 0} \frac{g(1 + \Delta x) - g(1)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(1 + \Delta x)^2 - 4 - (-3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{1 + 2(\Delta x) + (\Delta x)^2 - 1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} [2 + (\Delta x)] = 2$$

8. Slope at $(2, 1)$ $$= \lim_{\Delta x \to 0} \frac{g(2 + \Delta x) - g(2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{5 - (2 + \Delta x)^2 - 1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{5 - 4 - 4(\Delta x) - (\Delta x)^2 - 1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (-4 - \Delta x) = -4$$

9. Slope at $(0, 0)$ $$= \lim_{\Delta t \to 0} \frac{f(0 + \Delta t) - f(0)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{3(\Delta t) - (\Delta t)^2 - 0}{\Delta t}$$
$$= \lim_{\Delta t \to 0} (3 - \Delta t) = 3$$

10. Slope at $(-2, 7)$ $$= \lim_{\Delta t \to 0} \frac{h(-2 + \Delta t) - h(-2)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{(-2 + \Delta t)^2 + 3 - 7}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{4 - 4(\Delta t) + (\Delta t)^2 - 4}{\Delta t}$$
$$= \lim_{\Delta t \to 0} (-4 + \Delta t) = -4$$

11. $f(x) = 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3 - 3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 0 = 0$$

12. $g(x) = -5$

$$g'(x) = \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-5 - (-5)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{0}{\Delta x} = 0$$

13. $f(x) = -5x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-5(x + \Delta x) - (-5x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} -5 = -5$$

14. $f(x) = 3x + 2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{[3(x + \Delta x) + 2] - [3x + 2]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 3 = 3$$

15. $h(s) = 3 + \frac{2}{3}s$

$$h'(s) = \lim_{\Delta s \to 0} \frac{h(s + \Delta s) - h(s)}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{3 + \frac{2}{3}(s + \Delta s) - \left(3 + \frac{2}{3}s\right)}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{\frac{2}{3}\Delta s}{\Delta s} = \frac{2}{3}$$

16. $f(x) = 9 - \frac{1}{2}x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{[9 - (1/2)(x + \Delta x)] - [9 - (1/2)x]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(-\frac{1}{2}\right) = -\frac{1}{2}$$

17. $f(x) = 2x^2 + x - 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{[2(x + \Delta x)^2 + (x + \Delta x) - 1] - [2x^2 + x - 1]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(2x^2 + 4x\,\Delta x + 2(\Delta x)^2 + x + \Delta x - 1) - (2x^2 + x - 1)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{4x\,\Delta x + 2(\Delta x)^2 + \Delta x}{\Delta x} = \lim_{\Delta x \to 0} (4x + 2\,\Delta x + 1) = 4x + 1$$

18. $f(x) = 1 - x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{[1 - (x + \Delta x)^2] - [1 - x^2]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{1 - x^2 - 2x\,\Delta x - (\Delta x)^2 - 1 + x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-2x\,\Delta x - (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} (-2x - \Delta x) = -2x$$

19. $f(x) = x^3 - 12x$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{[(x + \Delta x)^3 - 12(x + \Delta x)] - [x^3 - 12x]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12x - 12\,\Delta x - x^3 + 12x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12\,\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} (3x^2 + 3x\,\Delta x + (\Delta x)^2 - 12) = 3x^2 - 12
\end{aligned}$$

20. $f(x) = x^3 + x^2$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{[(x + \Delta x)^3 + (x + \Delta x)^2] - [x^3 + x^2]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + x^2 + 2x\,\Delta x + (\Delta x)^2 - x^3 - x^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2x\,\Delta x + (\Delta x)^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} (3x^2 + 3x\,\Delta x + (\Delta x)^2 + 2x + (\Delta x)) = 3x^2 + 2x
\end{aligned}$$

21. $f(x) = \dfrac{1}{x - 1}$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x - 1} - \dfrac{1}{x - 1}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x - 1) - (x + \Delta x - 1)}{\Delta x(x + \Delta x - 1)(x - 1)} \\
&= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x(x + \Delta x - 1)(x - 1)} \\
&= \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x - 1)(x - 1)} \\
&= -\frac{1}{(x - 1)^2}
\end{aligned}$$

22. $f(x) = \dfrac{1}{x^2}$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\dfrac{1}{(x + \Delta x)^2} - \dfrac{1}{x^2}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^2 - (x + \Delta x)^2}{\Delta x(x + \Delta x)^2 x^2} \\
&= \lim_{\Delta x \to 0} \frac{-2x\,\Delta x - (\Delta x)^2}{\Delta x(x + \Delta x)^2 x^2} \\
&= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{(x + \Delta x)^2 x^2} \\
&= \frac{-2x}{x^4} \\
&= -\frac{2}{x^3}
\end{aligned}$$

23. $f(x) = \sqrt{x + 1}$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 1} - \sqrt{x + 1}}{\Delta x} \cdot \left(\frac{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}}{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}}\right) \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x + 1) - (x + 1)}{\Delta x\left[\sqrt{x + \Delta x + 1} + \sqrt{x + 1}\right]} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}} = \frac{1}{\sqrt{x + 1} + \sqrt{x + 1}} = \frac{1}{2\sqrt{x + 1}}
\end{aligned}$$

24. $f(x) = \dfrac{4}{\sqrt{x}}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{4}{\sqrt{x + \Delta x}} - \dfrac{4}{\sqrt{x}}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4\sqrt{x} - 4\sqrt{x + \Delta x}}{\Delta x \sqrt{x}\sqrt{x + \Delta x}} \cdot \left(\frac{\sqrt{x} + \sqrt{x + \Delta x}}{\sqrt{x} + \sqrt{x + \Delta x}}\right)$$

$$= \lim_{\Delta x \to 0} \frac{4x - 4(x + \Delta x)}{\Delta x \sqrt{x}\sqrt{x + \Delta x}\left(\sqrt{x} + \sqrt{x + \Delta x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{-4}{\sqrt{x}\sqrt{x + \Delta x}\left(\sqrt{x} + \sqrt{x + \Delta x}\right)}$$

$$= \frac{-4}{\sqrt{x}\sqrt{x}\left(\sqrt{x} + \sqrt{x}\right)} = \frac{-2}{x\sqrt{x}}$$

25. (a) $f(x) = x^2 + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[(x + \Delta x)^2 + 1] - [x^2 + 1]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\,\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

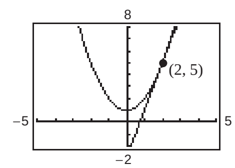
At $(2, 5)$, the slope of the tangent line is $m = 2(2) = 4$. The equation of the tangent line is

$$y - 5 = 4(x - 2)$$

$$y - 5 = 4x - 8$$

$$y = 4x - 3.$$

(b)



26. (a) $f(x) = x^2 + 2x + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[(x + \Delta x)^2 + 2(x + \Delta x) + 1] - [x^2 + 2x + 1]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\,\Delta x + (\Delta x)^2 + 2\,\Delta x}{\Delta x}$$

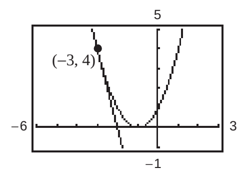
$$= \lim_{\Delta x \to 0} (2x + \Delta x + 2) = 2x + 2$$

At $(-3, 4)$, the slope of the tangent line is $m = 2(-3) + 2 = -4$. The equation of the tangent line is

$$y - 4 = -4(x + 3)$$

$$y = -4x - 8.$$

(b)



27. (a) $f(x) = x^3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$

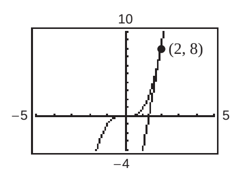
$$= \lim_{\Delta x \to 0} (3x^2 + 3x\,\Delta x + (\Delta x)^2) = 3x^2$$

At $(2, 8)$, the slope of the tangent is $m = 3(2)^2 = 12$. The equation of the tangent line is

$$y - 8 = 12(x - 2)$$

$$y = 12x - 16.$$

(b)

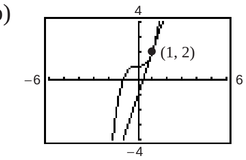


28. (a) $f(x) = x^3 + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{[(x + \Delta x)^3 + 1] - (x^3 + 1)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 + 1 - x^3 - 1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} [3x^2 + 3x(\Delta x) + (\Delta x)^2] = 3x^2$$

At (1, 2), the slope of the tangent line is $m = 3(1)^2 = 3$.
The equation of the tangent line is

$$y - 2 = 3(x - 1)$$
$$y = 3x - 1.$$

(b) 

29. (a) $f(x) = \sqrt{x}$

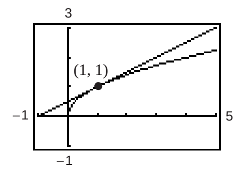
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

At (1, 1), the slope of the tangent line is

$$m = \frac{1}{2\sqrt{1}} = \frac{1}{2}.$$

The equation of the tangent line is

$$y - 1 = \frac{1}{2}(x - 1)$$
$$y = \frac{1}{2}x + \frac{1}{2}.$$

(b) 

30. (a) $f(x) = \sqrt{x - 1}$

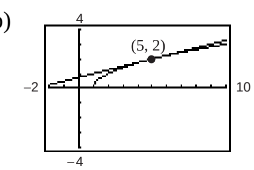
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x - 1} - \sqrt{x - 1}}{\Delta x} \cdot \left(\frac{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}}{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}}\right)$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x - 1) - (x - 1)}{\Delta x\left(\sqrt{x + \Delta x - 1} + \sqrt{x - 1}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}} = \frac{1}{2\sqrt{x - 1}}$$

At (5, 2), the slope of the tangent line is

$$m = \frac{1}{2\sqrt{5 - 1}} = \frac{1}{4}$$

The equation of the tangent line is

$$y - 2 = \frac{1}{4}(x - 5)$$
$$y = \frac{1}{4}x + \frac{3}{4}$$

(b) 

31. (a) $f(x) = x + \frac{4}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) + \frac{4}{x + \Delta x} - \left(x + \frac{4}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x(x + \Delta x)(x + \Delta x) + 4x - x^2(x + \Delta x) - 4(x + \Delta x)}{x(\Delta x)(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 2x^2(\Delta x) + x(\Delta x)^2 - x^3 - x^2(\Delta x) - 4(\Delta x)}{x(\Delta x)(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2(\Delta x) + x(\Delta x)^2 - 4(\Delta x)}{x(\Delta x)(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + x(\Delta x) - 4}{x(x + \Delta x)}$$

$$= \frac{x^2 - 4}{x^2} = 1 - \frac{4}{x^2}$$

At $(4, 5)$, the slope of the tangent line is

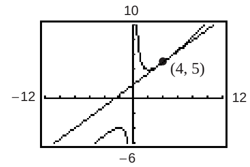
$$m = 1 - \frac{4}{16} = \frac{3}{4}.$$

The equation of the tangent line is

$$y - 5 = \frac{3}{4}(x - 4)$$

$$y = \frac{3}{4}x + 2.$$

(b)



32. (a) $f(x) = \frac{1}{x + 1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{x + \Delta x + 1} - \frac{1}{x + 1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + 1) - (x + \Delta x + 1)}{\Delta x(x + \Delta x + 1)(x + 1)}$$

$$= \lim_{\Delta x \to 0} -\frac{1}{(x + \Delta x + 1)(x + 1)}$$

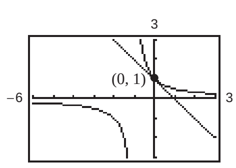
$$= -\frac{1}{(x + 1)^2}$$

At $(0, 1)$, the slope of the tangent line is

$$m = \frac{-1}{(0 + 1)^2} = -1.$$

The equation of the tangent line is $y = -x + 1$.

(b)



33. From Exercise 27 we know that $f'(x) = 3x^2$. Since the slope of the given line is 3, we have

$$3x^2 = 3$$

$$x = \pm 1.$$

Therefore, at the points $(1, 1)$ and $(-1, -1)$ the tangent lines are parallel to $3x - y + 1 = 0$. These lines have equations

$$y - 1 = 3(x - 1) \quad \text{and} \quad y + 1 = 3(x + 1)$$

$$y = 3x - 2 \qquad\qquad y = 3x + 2.$$

34. Using the limit definition of derivative, $f'(x) = 3x^2$. Since the slope of the given line is 3, we have

$$3x^2 = 3$$
$$x^2 = 1 \Rightarrow x = \pm 1.$$

Therefore, at the points $(1, 3)$ and $(-1, 1)$ the tangent lines are parallel to $3x - y - 4 = 0$. These lines have equations

$$y - 3 = 3(x - 1) \quad \text{and} \quad y - 1 = 3(x + 1)$$
$$y = 3x \qquad\qquad y = 3x + 4.$$

35. Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2x\sqrt{x}}.$$

Since the slope of the given line is $-\frac{1}{2}$, we have

$$-\frac{1}{2x\sqrt{x}} = -\frac{1}{2}$$
$$x = 1.$$

Therefore, at the point $(1, 1)$ the tangent line is parallel to $x + 2y - 6 = 0$. The equation of this line is

$$y - 1 = -\frac{1}{2}(x - 1)$$
$$y - 1 = -\frac{1}{2}x + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2}.$$

36. Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2(x - 1)^{3/2}}.$$

Since the slope of the given line is $-\frac{1}{2}$, we have

$$\frac{-1}{2(x - 1)^{3/2}} = -\frac{1}{2}$$
$$1 = (x - 1)^{3/2}$$
$$1 = x - 1 \implies x = 2.$$

At the point $(2, 1)$, the tangent line is parallel to $x + 2y + 7 = 0$. The equation of the tangent line is

$$y - 1 = -\frac{1}{2}(x - 2)$$
$$y = -\frac{1}{2}x + 2.$$

37. $f(x) = x \implies f'(x) = 1$ Matches (b).

38. $f(x) = x^2 \implies f'(x) = 2x$ Matches (d).

39. $f(x) = \sqrt{x} \implies f'(x)$ Matches (a).
(decreasing slope as $x \to \infty$)

40. f' does not exist at $x = 0$. Matches (c).

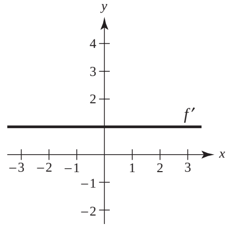
41. $g(5) = 2$ because the tangent line passes through $(5, 2)$.

$$g'(5) = \frac{2 - 0}{5 - 9} = \frac{2}{-4} = -\frac{1}{2}$$

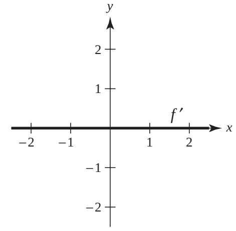
42. $h(-1) = 4$ because the tangent line passes through $(-1, 4)$.

$$h'(-1) = \frac{6 - 4}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

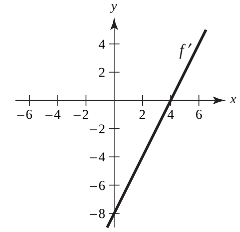
43. The slope of the graph of f is $1 \implies f'(x) = 1$.



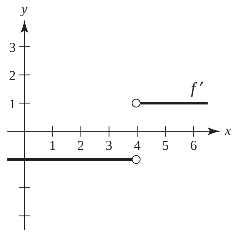
44. The slope of the graph of f is $0 \implies f'(x) = 0$.



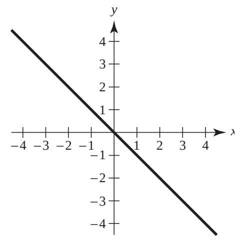
45. The slope of the graph of f is negative for $x < 4$, positive for $x > 4$, and 0 at $x = 4$.



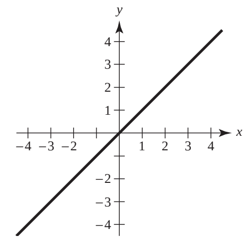
46. The slope of the graph of f is -1 for $x < 4$, 1 for $x > 4$, and undefined at $x = 4$.



47. Answers will vary.
Sample answer: $y = -x$



48. Answers will vary.
Sample answer: $y = x$



49. $f(x) = 5 - 3x$ and $c = 1$

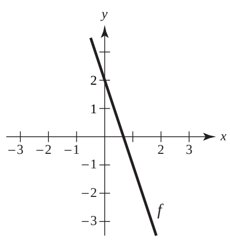
50. $f(x) = x^3$ and $c = -2$

51. $f(x) = -x^2$ and $c = 6$

52. $f(x) = 2\sqrt{x}$ and $c = 9$

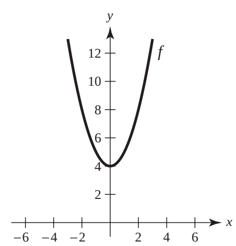
53. $f(0) = 2$ and $f'(x) = -3$, $-\infty < x < \infty$

$f(x) = -3x + 2$



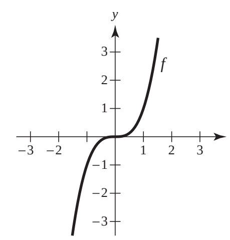
54. $f(0) = 4$, $f'(0) = 0$; $f'(x) < 0$ for $x < 0$, $f'(x) > 0$ for $x > 0$

$f(x) = x^2 + 4$



55. $f(0) = 0$; $f'(0) = 0$; $f'(x) > 0$ if $x \neq 0$

$f(x) = x^3$



56. (a) If $f'(c) = 3$ and f is odd, then $f'(-c) = f'(c) = 3$.

(b) If $f'(c) = 3$ and f is even, then $f'(-c) = -f'(c) = -3$.

57. Let (x_0, y_0) be a point of tangency on the graph of f. By the limit definition for the derivative, $f'(x) = 4 - 2x$. The slope of the line through (2, 5) and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{5 - y_0}{2 - x_0} = 4 - 2x_0$$

$$5 - y_0 = (2 - x_0)(4 - 2x_0)$$

$$5 - (4x_0 - x_0^2) = 8 - 8x_0 + 2x_0^2$$

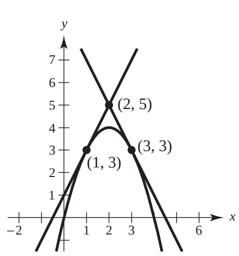
$$0 = x_0^2 - 4x_0 + 3$$

$$0 = (x_0 - 1)(x_0 - 3) \implies x_0 = 1, 3$$

Therefore, the points of tangency are (1, 3) and (3, 3), and the corresponding slopes are 2 and -2. The equations of the tangent lines are:

$$y - 5 = 2(x - 2) \qquad y - 5 = -2(x - 2)$$

$$y = 2x + 1 \qquad y = -2x + 9$$



58. Let (x_0, y_0) be a point of tangency on the graph of f. By the limit definition for the derivative, $f'(x) = 2x$. The slope of the line through $(1, -3)$ and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{-3 - y_0}{1 - x_0} = 2x_0$$

$$-3 - y_0 = (1 - x_0)2x_0$$

$$-3 - x_0^2 = 2x_0 - 2x_0^2$$

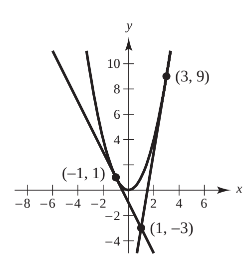
$$x_0^2 - 2x_0 - 3 = 0$$

$$(x_0 - 3)(x_0 + 1) = 0 \implies x_0 = 3, -1$$

Therefore, the points of tangency are (3, 9) and $(-1, 1)$, and the corresponding slopes are 6 and -2. The equations of the tangent lines are:

$$y + 3 = 6(x - 1) \qquad y + 3 = -2(x - 1)$$

$$y = 6x - 9 \qquad y = -2x - 1$$



59. (a) $g'(0) = -3$

(b) $g'(3) = 0$

(c) Because $g'(1) = -\frac{8}{3}$, g is decreasing (falling) at $x = 1$.

(d) Because $g'(-4) = \frac{7}{3}$, g is increasing (rising) at $x = -4$.

(e) Because $g'(4)$ and $g'(6)$ are both positive, $g(6)$ is greater than $g(4)$, and $g(6) - g(4) > 0$.

(f) No, it is not possible. All you can say is that g is decreasing (falling) at $x = 2$.

60. (a) $f(x) = x^2$

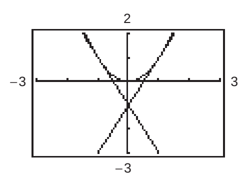
$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x \end{aligned}$$

At $x = -1$, $f'(-1) = -2$ and the tangent line is

$y - 1 = -2(x + 1)$ or $y = -2x - 1$.

At $x = 0$, $f'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $f'(1) = 2$ and the tangent line is $y = 2x - 1$.



For this function, the slopes of the tangent lines are always distinct for different values of x.

(b)
$$\begin{aligned} g'(x) &= \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x(3x^2 + 3x(\Delta x) + (\Delta x)^2)}{\Delta x} \\ &= \lim_{\Delta x \to 0} (3x^2 + 3x(\Delta x) + (\Delta x)^2) = 3x^2 \end{aligned}$$

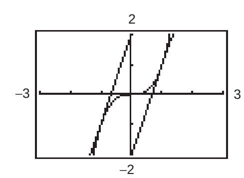
At $x = -1$, $g'(-1) = 3$ and the tangent line is

$y + 1 = 3(x + 1)$ or $y = 3x + 2$.

At $x = 0$, $g'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $g'(1) = 3$ and the tangent line is

$y - 1 = 3(x - 1)$ or $y = 3x - 2$.

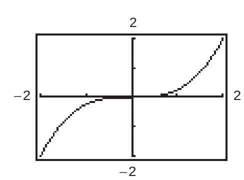


For this function, the slopes of the tangent lines are sometimes the same.

61. $f(x) = \frac{1}{4}x^3$

By the limit definition of the derivative we have $f'(x) = \frac{3}{4}x^2$.

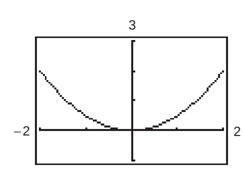
x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$f(x)$	-2	$-\frac{27}{32}$	$-\frac{1}{4}$	$-\frac{1}{32}$	0	$\frac{1}{32}$	$\frac{1}{4}$	$\frac{27}{32}$	2
$f'(x)$	3	$\frac{27}{16}$	$\frac{3}{4}$	$\frac{3}{16}$	0	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{27}{16}$	3



62. $f(x) = \frac{1}{2}x^2$

By the limit definition of the derivative we have $f'(x) = x$.

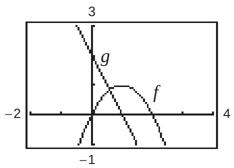
x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$f(x)$	2	1.125	0.5	0.125	0	0.125	0.5	1.125	2
$f'(x)$	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2



63. $g(x) = \dfrac{f(x + 0.01) - f(x)}{0.01}$

$= [2(x + 0.01) - (x + 0.01)^2 - 2x + x^2]100$

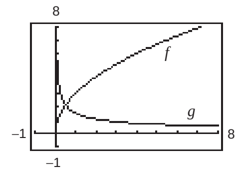
$= 2 - 2x - 0.01$



The graph of $g(x)$ is approximately the graph of $f'(x) = 2 - 2x$.

64. $g(x) = \dfrac{f(x + 0.01) - f(x)}{0.01}$

$= \left(3\sqrt{x + 0.01} - 3\sqrt{x}\right)100$



The graph of $g(x)$ is approximately the graph of $f'(x) = \dfrac{3}{2\sqrt{x}}$.

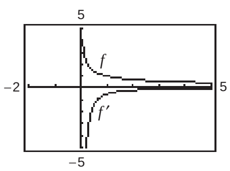
65. $f(2) = 2(4 - 2) = 4,\ f(2.1) = 2.1(4 - 2.1) = 3.99$

$f'(2) \approx \dfrac{3.99 - 4}{2.1 - 2} = -0.1 \quad [\text{Exact: } f'(2) = 0]$

66. $f(2) = \dfrac{1}{4}(2^3) = 2,\ f(2.1) = 2.31525$

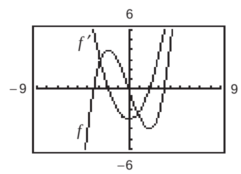
$f'(2) \approx \dfrac{2.31525 - 2}{2.1 - 2} = 3.1525 \quad [\text{Exact: } f'(2) = 3]$

67. $f(x) = \dfrac{1}{\sqrt{x}}$ and $f'(x) = \dfrac{-1}{2x^{3/2}}$.



As $x \to \infty$, f is nearly horizontal and thus $f' \approx 0$.

68. $f(x) = \dfrac{x^3}{4} - 3x$ and $f'(x) = \dfrac{3}{4}x^2 - 3$



69. $f(x) = 4 - (x - 3)^2$

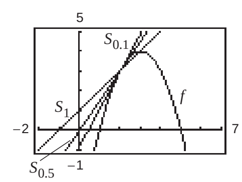
$S_{\Delta x}(x) = \dfrac{f(2 + \Delta x) - f(2)}{\Delta x}(x - 2) + f(2)$

$= \dfrac{4 - (2 + \Delta x - 3)^2 - 3}{\Delta x}(x - 2) + 3 = \dfrac{1 - (\Delta x - 1)^2}{\Delta x}(x - 2) + 3 = (-\Delta x + 2)(x - 2) + 3$

(a) $\Delta x = 1$: $S_{\Delta x} = (x - 2) + 3 = x + 1$

$\Delta x = 0.5$: $S_{\Delta x} = \left(\dfrac{3}{2}\right)(x - 2) + 3 = \dfrac{3}{2}x$

$\Delta x = 0.1$: $S_{\Delta x} = \left(\dfrac{19}{10}\right)(x - 2) + 3 = \dfrac{19}{10}x - \dfrac{4}{5}$



(b) As $\Delta x \to 0$, the line approaches the tangent line to f at $(2, 3)$.

70. $f(x) = x + \dfrac{1}{x}$

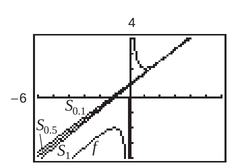
$S_{\Delta x}(x) = \dfrac{f(2 + \Delta x) - f(2)}{\Delta x}(x - 2) + f(2) = \dfrac{(2 + \Delta x) + \dfrac{1}{2 + \Delta x} - \dfrac{5}{2}}{\Delta x}(x - 2) + \dfrac{5}{2}$

$= \dfrac{2(2 + \Delta x)^2 + 2 - 5(2 + \Delta x)}{2(2 + \Delta x)\,\Delta x}(x - 2) + \dfrac{5}{2} = \dfrac{(2\Delta x + 3)}{2(2 + \Delta x)}(x - 2) + \dfrac{5}{2}$

(a) $\Delta x = 1$: $S_{\Delta x} = \dfrac{5}{6}(x - 2) + \dfrac{5}{2} = \dfrac{5}{6}x + \dfrac{5}{6}$

$\Delta x = 0.5$: $S_{\Delta x} = \dfrac{4}{5}(x - 2) + \dfrac{5}{2} = \dfrac{4}{5}x + \dfrac{9}{10}$

$\Delta x = 0.1$: $S_{\Delta x} = \dfrac{16}{21}(x - 2) + \dfrac{5}{2} = \dfrac{16}{21}x + \dfrac{41}{42}$



(b) As $\Delta x \to 0$, the line approaches the tangent line to f at $\left(2, \frac{5}{2}\right)$.

71. $f(x) = x^2 - 1, c = 2$

$$f'(2) = \lim_{x\to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2} \frac{(x^2 - 1) - 3}{x - 2} = \lim_{x\to 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x\to 2}(x + 2) = 4$$

72. $g(x) = x(x - 1) = x^2 - x, c = 1$

$$g'(1) = \lim_{x\to 1} \frac{g(x) - g(1)}{x - 1} = \lim_{x\to 1} \frac{x^2 - x - 0}{x - 1} = \lim_{x\to 1} \frac{x(x - 1)}{x - 1} = \lim_{x\to 1} x = 1$$

73. $f(x) = x^3 + 2x^2 + 1, c = -2$

$$f'(-2) = \lim_{x\to -2} \frac{f(x) - f(-2)}{x + 2} = \lim_{x\to -2} \frac{(x^3 + 2x^2 + 1) - 1}{x + 2} = \lim_{x\to -2} \frac{x^2(x + 2)}{x + 2} = \lim_{x\to -2} x^2 = 4$$

74. $f(x) = x^3 + 2x, c = 1$

$$f'(1) = \lim_{x\to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1} \frac{x^3 + 2x - 3}{x - 1} = \lim_{x\to 1} \frac{(x - 1)(x^2 + x + 3)}{x - 1} = \lim_{x\to 1}(x^2 + x + 3) = 5$$

75. $g(x) = \sqrt{|x|}, c = 0$

$g'(0) = \lim_{x\to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x\to 0} \frac{\sqrt{|x|}}{x}$. Does not exist.

As $x \to 0^-, \frac{\sqrt{|x|}}{x} = \frac{-1}{\sqrt{|x|}} \to -\infty$.

As $x \to 0^+, \frac{\sqrt{|x|}}{x} = \frac{1}{\sqrt{x}} \to \infty$.

76. $f(x) = \frac{1}{x}, c = 3$

$$f'(3) = \lim_{x\to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x\to 3} \frac{(1/x) - (1/3)}{x - 3} = \lim_{x\to 3} \frac{3 - x}{3x} \cdot \frac{1}{x - 3} = \lim_{x\to 3}\left(-\frac{1}{3x}\right) = -\frac{1}{9}$$

77. $f(x) = (x - 6)^{2/3}, c = 6$

$$f'(6) = \lim_{x\to 6} \frac{f(x) - f(6)}{x - 6} = \lim_{x\to 6} \frac{(x - 6)^{2/3} - 0}{x - 6} = \lim_{x\to 6} \frac{1}{(x - 6)^{1/3}}$$

Does not exist.

78. $g(x) = (x + 3)^{1/3}, c = -3$

$$g'(-3) = \lim_{x\to -3} \frac{g(x) - g(-3)}{x - (-3)} = \lim_{x\to -3} \frac{(x + 3)^{1/3} - 0}{x + 3} = \lim_{x\to -3} \frac{1}{(x + 3)^{2/3}}$$

Does not exist.

79. $h(x) = |x + 5|, c = -5$

$$h'(-5) = \lim_{x\to -5} \frac{h(x) - h(-5)}{x - (-5)} = \lim_{x\to -5} \frac{|x + 5| - 0}{x + 5} = \lim_{x\to -5} \frac{|x + 5|}{x + 5}$$

Does not exist.

80. $f(x) = |x - 4|, c = 4$

$$f'(4) = \lim_{x\to 4} \frac{f(x) - f(4)}{x - 4} = \lim_{x\to 4} \frac{|x - 4| - 0}{x - 4} = \lim_{x\to 4} \frac{|x - 4|}{x - 4}$$

Does not exist.

81. $f(x)$ is differentiable everywhere except at $x = -1$. (Discontinuity)

82. $f(x)$ is differentiable everywhere except at $x = \pm 3$. (Sharp turns in the graph)

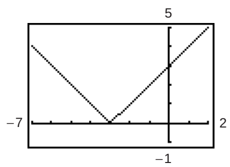
83. $f(x)$ is differentiable everywhere except at $x = 3$. (Sharp turn in the graph)

84. $f(x)$ is differentiable everywhere except at $x = \pm 2$. (Discontinuities)

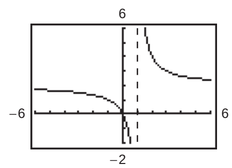
85. $f(x)$ is differentiable on the interval $(1, \infty)$. (At $x = 1$ the tangent line is vertical.)

86. $f(x)$ is differentiable everywhere except at $x = 0$. (Discontinuity)

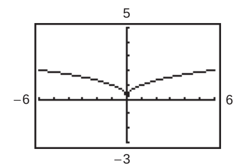
87. $f(x) = |x + 3|$ is differentiable for all $x \neq -3$. There is a sharp corner at $x = -3$.



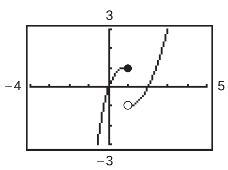
88. $f(x) = \dfrac{2x}{x - 1}$ is differentiable for all $x \neq 1$. f is not defined at $x = 1$. (Vertical asymptote)



89. $f(x) = x^{2/5}$ is differentiable for all $x \neq 0$. There is a sharp corner at $x = 0$.



90. f is differentiable for all $x \neq 1$. f is not continuous at $x = 1$.



91. $f(x) = |x - 1|$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{|x - 1| - 0}{x - 1} = -1.$$

The derivative from the right is

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{|x - 1| - 0}{x - 1} = 1.$$

The one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

92. $f(x) = \sqrt{1 - x^2}$

The derivative from the left does not exist because

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{\sqrt{1 - x^2} - 0}{x - 1} = \lim_{x \to 1^-} \frac{\sqrt{1 - x^2}}{x - 1} \cdot \frac{\sqrt{1 - x^2}}{\sqrt{1 - x^2}} = \lim_{x \to 1^-} -\frac{1 + x}{\sqrt{1 - x^2}} = -\infty. \quad \text{(Vertical tangent)}$$

The limit from the right does not exist since f is undefined for $x > 1$. Therefore, f is not differentiable at $x = 1$.

93. $f(x) = \begin{cases} (x - 1)^3, & x \leq 1 \\ (x - 1)^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{(x - 1)^3 - 0}{x - 1}$$
$$= \lim_{x \to 1^-} (x - 1)^2 = 0.$$

The derivative from the right is

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{(x - 1)^2 - 0}{x - 1}$$
$$= \lim_{x \to 1^+} (x - 1) = 0.$$

These one-sided limits are equal. Therefore, f is differentiable at $x = 1$. $(f'(1) = 0)$

94. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{x - 1}{x - 1} = \lim_{x\to 1^-} 1 = 1.$$

The derivative from the right is

$$\lim_{x\to 1^+} = \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x\to 1^+} (x + 1) = 2.$$

These one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

95. Note that f is continuous at $x = 2$. $f(x) = \begin{cases} x^2 + 1, & x \le 2 \\ 4x - 3, & x > 2 \end{cases}$

The derivative from the left is $\lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^-} \frac{(x^2 + 1) - 5}{x - 2} = \lim_{x\to 2^-} (x + 2) = 4.$

The derivative from the right is $\lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^+} \frac{(4x - 3) - 5}{x - 2} = \lim_{x\to 2^+} 4 = 4.$

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $\left(f'(2) = 4\right)$

96. Note that f is continuous at $x = 2$. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x < 2 \\ \sqrt{2x}, & x \ge 2 \end{cases}$

The derivative from the left is

$$\lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^-} \frac{\left(\frac{1}{2}x + 1\right) - 2}{x - 2} = \lim_{x\to 2^-} \frac{\frac{1}{2}(x - 2)}{x - 2} = \frac{1}{2}.$$

The derivative from the right is

$$\lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^+} \frac{\sqrt{2x} - 2}{x - 2} \cdot \frac{\sqrt{2x} + 2}{\sqrt{2x} + 2}$$

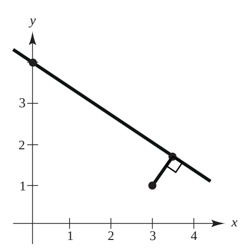
$$= \lim_{x\to 2^+} \frac{2x - 4}{(x - 2)\left(\sqrt{2x} + 2\right)} = \lim_{x\to 2^+} \frac{2(x - 2)}{(x - 2)\left(\sqrt{2x} + 2\right)} = \lim_{x\to 2^+} \frac{2}{\sqrt{2x} + 2} = \frac{1}{2}.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $\left(f'(2) = \frac{1}{2}\right)$

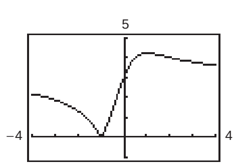
97. (a) The distance from $(3, 1)$ to the line $mx - y + 4 = 0$ is

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

$$= \frac{|m(3) - 1(1) + 4|}{\sqrt{m^2 + 1}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}.$$

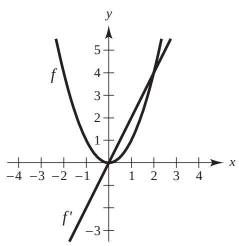


(b)

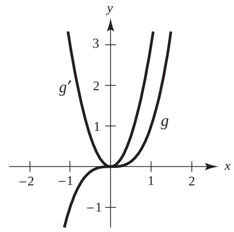


The function d is not differentiable at $m = -1$. This corresponds to the line $y = -x + 4$, which passes through the point $(3, 1)$.

98. (a) $f(x) = x^2$ and $f'(x) = 2x$



(b) $g(x) = x^3$ and $g'(x) = 3x^2$



(c) The derivative is a polynomial of degree 1 less than the original function. If $h(x) = x^n$, then $h'(x) = nx^{n-1}$.

(d) If $f(x) = x^4$, then

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^4 - x^4}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^4 + 4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4 - x^4}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x(4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3)}{\Delta x} \\
&= \lim_{\Delta x \to 0} (4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3) = 4x^3.
\end{aligned}$$

Hence, if $f(x) = x^4$, then $f'(x) = 4x^3$ which is consistent with the conjecture. However, this is not a proof since you must verify the conjecture for all integer values of n, $n \geq 2$.

99. False. The slope is $\displaystyle\lim_{\Delta x \to 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x}$.

100. False. $y = |x - 2|$ is continuous at $x = 2$, but is not differentiable at $x = 2$. (Sharp turn in the graph)

101. False. If the derivative from the left of a point does not equal the derivative from the right of a point, then the derivative does not exist at that point. For example, if $f(x) = |x|$, then the derivative from the left at $x = 0$ is -1 and the derivative from the right at $x = 0$ is 1. At $x = 0$, the derivative does not exist.

102. True—see Theorem 2.1.

103. $f(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Using the Squeeze Theorem, we have $-|x| \leq x \sin(1/x) \leq |x|$, $x \neq 0$. Thus, $\lim_{x \to 0} x \sin(1/x) = 0 = f(0)$ and f is continuous at $x = 0$. Using the alternative form of the derivative, we have

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x \sin(1/x) - 0}{x - 0} = \lim_{x \to 0}\left(\sin \frac{1}{x}\right).$$

Since this limit does not exist ($\sin(1/x)$ oscillates between -1 and 1), the function is not differentiable at $x = 0$.

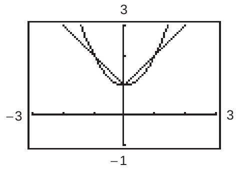
$$g(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Using the Squeeze Theorem again, we have $-x^2 \leq x^2 \sin(1/x) \leq x^2$, $x \neq 0$. Thus, $\lim_{x \to 0} x^2 \sin(1/x) = 0 = g(0)$ and g is continuous at $x = 0$. Using the alternative form of the derivative again, we have

$$\lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \to 0} \frac{x^2 \sin(1/x) - 0}{x - 0} = \lim_{x \to 0} x \sin \frac{1}{x} = 0.$$

Therefore, g is differentiable at $x = 0$, $g'(0) = 0$.

104.



As you zoom in, the graph of $y_1 = x^2 + 1$ appears to be locally the graph of a horizontal line, whereas the graph of $y_2 = |x| + 1$ always has a sharp corner at $(0, 1)$. y_2 is not differentiable at $(0, 1)$.

Section 2.2 Basic Differentiation Rules and Rates of Change

1. (a) $y = x^{1/2}$, $y' = \frac{1}{2}x^{-1/2}$, $y'(1) = \frac{1}{2}$

(b) $y = x^3$, $y' = 3x^2$, $y'(1) = 3$

2. (a) $y = x^{-1/2}$, $y' = -\frac{1}{2}x^{-3/2}$, $y'(1) = -\frac{1}{2}$

(b) $y = x^{-1}$, $y' = -x^{-2}$, $y'(1) = -1$

3. $y = 8$, $y' = 0$

4. $f(x) = -2$, $f'(x) = 0$

5. $y = x^6$, $y' = 6x^5$

6. $y = x^8$, $y' = 8x^7$

7. $y = \dfrac{1}{x^7} = x^{-7}$, $y' = -7x^{-8} = \dfrac{-7}{x^8}$

8. $y = \dfrac{1}{x^8} = x^{-8}$, $y' = -8x^{-9} = \dfrac{-8}{x^9}$

9. $y = \sqrt[5]{x} = x^{1/5}$, $y' = \dfrac{1}{5}x^{-4/5} = \dfrac{1}{5x^{4/5}}$

10. $y = \sqrt[4]{x} = x^{1/4}$, $y' = \dfrac{1}{4}x^{-3/4} = \dfrac{1}{4x^{3/4}}$

11. $f(x) = x + 1$, $f'(x) = 1$

12. $g(x) = 3x - 1$, $g'(x) = 3$

13. $f(t) = -2t^2 + 3t - 6$, $f'(t) = -4t + 3$

14. $y = t^2 + 2t - 3$, $y' = 2t + 2$

15. $g(x) = x^2 + 4x^3$, $g'(x) = 2x + 12x^2$

16. $y = 8 - x^3$, $y' = -3x^2$

17. $s(t) = t^3 - 2t + 4$, $s'(t) = 3t^2 - 2$

18. $f(x) = 2x^3 - x^2 + 3x$, $f'(x) = 6x^2 - 2x + 3$

19. $y = \dfrac{\pi}{2}\sin\theta - \cos\theta$, $y' = \dfrac{\pi}{2}\cos\theta + \sin\theta$

20. $g(t) = \pi\cos t$, $g'(t) = -\pi\sin t$

21. $y = x^2 - \dfrac{1}{2}\cos x$, $y' = 2x + \dfrac{1}{2}\sin x$

22. $y = 5 + \sin x$, $y' = \cos x$

23. $y = \dfrac{1}{x} - 3\sin x$, $y' = -\dfrac{1}{x^2} - 3\cos x$

24. $y = \dfrac{5}{(2x)^3} + 2\cos x = \dfrac{5}{8}x^{-3} + 2\cos x$

$y' = \dfrac{5}{8}(-3)x^{-4} - 2\sin x = \dfrac{-15}{8x^4} - 2\sin x$

	Function	*Rewrite*	*Differentiate*	*Simplify*
25.	$y = \dfrac{5}{2x^2}$	$y = \dfrac{5}{2}x^{-2}$	$y' = -5x^{-3}$	$y' = \dfrac{-5}{x^3}$
26.	$y = \dfrac{2}{3x^2}$	$y = \dfrac{2}{3}x^{-2}$	$y' = -\dfrac{4}{3}x^{-3}$	$y' = -\dfrac{4}{3x^3}$
27.	$y = \dfrac{3}{(2x)^3}$	$y = \dfrac{3}{8}x^{-3}$	$y' = \dfrac{-9}{8}x^{-4}$	$y' = \dfrac{-9}{8x^4}$

	Function	Rewrite	Differentiate	Simplify
28.	$y = \dfrac{\pi}{(3x)^2}$	$y = \dfrac{\pi}{9}x^{-2}$	$y' = -\dfrac{2\pi}{9}x^{-3}$	$y' = -\dfrac{2\pi}{9x^3}$
29.	$y = \dfrac{\sqrt{x}}{x}$	$y = x^{-1/2}$	$y' = -\dfrac{1}{2}x^{-3/2}$	$y' = -\dfrac{1}{2x^{3/2}}$
30.	$y = \dfrac{4}{x^{-3}}$	$y = 4x^3$	$y' = 12x^2$	$y' = 12x^2$

31. $f(x) = \dfrac{3}{x^2} = 3x^{-2}$, $(1, 3)$

$f'(x) = -6x^{-3} = \dfrac{-6}{x^3}$

$f'(1) = -6$

32. $f(t) = 3 - \dfrac{3}{5t}$, $\left(\dfrac{3}{5}, 2\right)$

$f'(t) = \dfrac{3}{5t^2}$

$f'\left(\dfrac{3}{5}\right) = \dfrac{5}{3}$

33. $f(x) = -\dfrac{1}{2} + \dfrac{7}{5}x^3$, $\left(0, -\dfrac{1}{2}\right)$

$f'(x) = \dfrac{21}{5}x^2$

$f'(0) = 0$

34. $y = 3x^3 - 6$, $(2, 18)$

$y' = 9x^2$

$y'(2) = 36$

35. $y = (2x + 1)^2$, $(0, 1)$

$= 4x^2 + 4x + 1$

$y' = 8x + 4$

$y'(0) = 4$

36. $f(x) = 3(5 - x)^2$, $(5, 0)$

$= 3x^2 - 30x + 75$

$f'(x) = 6x - 30$

$f'(5) = 0$

37. $f(\theta) = 4 \sin \theta - \theta$, $(0, 0)$

$f'(\theta) = 4 \cos \theta - 1$

$f'(0) = 4(1) - 1 = 3$

38. $g(t) = 2 + 3 \cos t$, $(\pi, -1)$

$g'(t) = -3 \sin t$

$g'(\pi) = 0$

39. $f(x) = x^2 + 5 - 3x^{-2}$

$f'(x) = 2x + 6x^{-3} = 2x + \dfrac{6}{x^3}$

40. $f(x) = x^2 - 3x - 3x^{-2}$

$f'(x) = 2x - 3 + 6x^{-3}$

$= 2x - 3 + \dfrac{6}{x^3}$

41. $g(t) = t^2 - \dfrac{4}{t^3} = t^2 - 4t^{-3}$

$g'(t) = 2t + 12t^{-4} = 2t + \dfrac{12}{t^4}$

42. $f(x) = x + x^{-2}$

$f'(x) = 1 - 2x^{-3}$

$= 1 - \dfrac{2}{x^3}$

43. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2} = x - 3 + 4x^{-2}$

$f'(x) = 1 - \dfrac{8}{x^3} = \dfrac{x^3 - 8}{x^3}$

44. $h(x) = \dfrac{2x^2 - 3x + 1}{x} = 2x - 3 + x^{-1}$

$h'(x) = 2 - \dfrac{1}{x^2} = \dfrac{2x^2 - 1}{x^2}$

45. $y = x(x^2 + 1) = x^3 + x$

$y' = 3x^2 + 1$

46. $y = 3x(6x - 5x^2) = 18x^2 - 15x^3$

$y' = 36x - 45x^2$

47. $f(x) = \sqrt{x} - 6\sqrt[3]{x} = x^{1/2} - 6x^{1/3}$

$f'(x) = \dfrac{1}{2}x^{-1/2} - 2x^{-2/3} = \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^{2/3}}$

48. $f(x) = \sqrt[3]{x} + \sqrt[5]{x} = x^{1/3} + x^{1/5}$

$f'(x) = \dfrac{1}{3}x^{-2/3} + \dfrac{1}{5}x^{-4/5} = \dfrac{1}{3x^{2/3}} + \dfrac{1}{5x^{4/5}}$

49. $h(s) = s^{4/5} - s^{2/3}$

$h'(s) = \dfrac{4}{5}s^{-1/5} - \dfrac{2}{3}s^{-1/3} = \dfrac{4}{5s^{1/5}} - \dfrac{2}{3s^{1/3}}$

50. $f(t) = t^{2/3} - t^{1/3} + 4$

$f'(t) = \dfrac{2}{3}t^{-1/3} - \dfrac{1}{3}t^{-2/3} = \dfrac{2}{3t^{1/3}} - \dfrac{1}{3t^{2/3}}$

51. $f(x) = 6\sqrt{x} + 5\cos x = 6x^{1/2} + 5\cos x$

$f'(x) = 3x^{-1/2} - 5\sin x = \dfrac{3}{\sqrt{x}} - 5\sin x$

52. $f(x) = \dfrac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$f'(x) = \dfrac{-2}{3}x^{-4/3} - 3\sin x = \dfrac{-2}{3x^{4/3}} - 3\sin x$

53. (a) $y = x^4 - 3x^2 + 2$

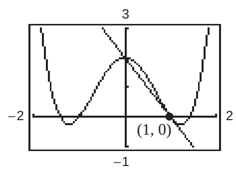
$y' = 4x^3 - 6x$

At $(1, 0)$: $y' = 4(1)^3 - 6(1) = -2$

Tangent line: $y - 0 = -2(x - 1)$

$2x + y - 2 = 0$

(b)



54. (a) $y = x^3 + x$

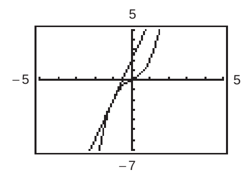
$y' = 3x^2 + 1$

At $(-1, -2)$: $y' = 3(-1)^2 + 1 = 4$

Tangent line: $y + 2 = 4(x + 1)$

$4x - y + 2 = 0$

(b)



55. (a) $f(x) = \dfrac{2}{\sqrt[4]{x^3}} = 2x^{-3/4}$

$f'(x) = \dfrac{-3}{2}x^{-7/4} = \dfrac{-3}{2x^{7/4}}$

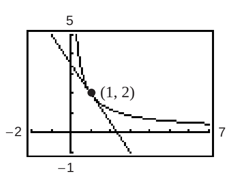
At $(1, 2)$: $f'(1) = \dfrac{-3}{2}$

Tangent line: $y - 2 = -\dfrac{3}{2}(x - 1)$

$y = -\dfrac{3}{2}x + \dfrac{7}{2}$

$3x + 2y - 7 = 0$

(b)



56. (a) $y = (x^2 + 2x)(x + 1)$

$= x^3 + 3x^2 + 2x$

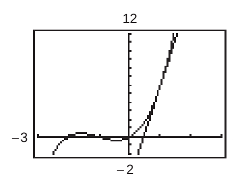
$y' = 3x^2 + 6x + 2$

At $(1, 6)$: $y' = 3(1)^2 + 6(1) + 2 = 11$

Tangent line: $y - 6 = 11(x - 1)$

$0 = 11x - y - 5$

(b)



57. $y = x^4 - 8x^2 + 2$

$y' = 4x^3 - 16x$

$= 4x(x^2 - 4)$

$= 4x(x - 2)(x + 2)$

$y' = 0 \implies x = 0, \pm 2$

Horizontal tangents: $(0, 2)$, $(2, -14)$, $(-2, -14)$

58. $y = x^3 + x$

$y' = 3x^2 + 1 > 0$ for all x.

Therefore, there are no horizontal tangents.

59. $y = \dfrac{1}{x^2} = x^{-2}$

$y' = -2x^{-3} = \dfrac{-2}{x^3}$ cannot equal zero.

Therefore, there are no horizontal tangents.

60. $y = x^2 + 1$

$y' = 2x = 0 \implies x = 0$

At $x = 0$, $y = 1$.

Horizontal tangent: $(0, 1)$

61. $y = x + \sin x,\ 0 \le x < 2\pi$

$y' = 1 + \cos x = 0$

$\cos x = -1 \implies x = \pi$

At $x = \pi$: $y = \pi$

Horizontal tangent: (π, π)

62. $y = \sqrt{3}x + 2\cos x,\ 0 \le x < 2\pi$

$y' = \sqrt{3} - 2\sin x = 0$

$\sin x = \dfrac{\sqrt{3}}{2} \implies x = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

At $x = \dfrac{\pi}{3}$: $y = \dfrac{\sqrt{3}\pi + 3}{3}$

At $x = \dfrac{2\pi}{3}$: $y = \dfrac{2\sqrt{3}\pi - 3}{3}$

Horizontal tangents: $\left(\dfrac{\pi}{3}, \dfrac{\sqrt{3}\pi + 3}{3}\right), \left(\dfrac{2\pi}{3}, \dfrac{2\sqrt{3}\pi - 3}{3}\right)$

63. $x^2 - kx = 4x - 9$ Equate functions.

$2x - k = 4$ Equate derivatives.

Hence, $k = 2x - 4$ and

$x^2 - (2x - 4)x = 4x - 9 \implies -x^2 = -9 \implies x = \pm 3.$

For $x = 3$, $k = 2$ and for $x = -3$, $k = -10$.

64. $k - x^2 = -4x + 7$ Equate functions.

$-2x = -4$ Equate derivatives.

Hence, $x = 2$ and $k - 4 = -8 + 7 \implies k = 3.$

65. $\dfrac{k}{x} = -\dfrac{3}{4}x + 3$ Equate functions.

$-\dfrac{k}{x^2} = -\dfrac{3}{4}$ Equate derivatives.

Hence, $k = \frac{3}{4}x^2$ and

$\dfrac{\frac{3}{4}x^2}{x} = \dfrac{-3}{4}x + 3 \implies \dfrac{3}{4}x = -\dfrac{3}{4}x + 3$

$\implies \dfrac{3}{2}x = 3 \implies x = 2 \implies k = 3.$

66. $k\sqrt{x} = x + 4$ Equate functions.

$\dfrac{k}{2\sqrt{x}} = 1$ Equate derivatives.

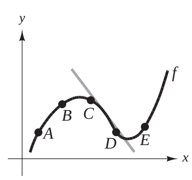
Hence, $k = 2\sqrt{x}$ and

$\left(2\sqrt{x}\right)\sqrt{x} = x + 4 \implies 2x = x + 4 \implies x = 4 \implies k = 4.$

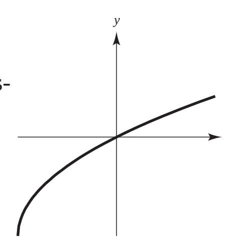
67. (a) The slope appears to be steepest between A and B.

(b) The average rate of change between A and B is **greater** than the instantaneous rate of change at B.

(c)



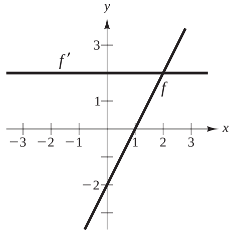
68. The graph of a function f such that $f' > 0$ for all x and the rate of change the function is decreasing (i.e., $f'' < 0$) would, in general, look like the graph at the right.



69. $g(x) = f(x) + 6 \implies g'(x) = f'(x)$

70. $g(x) = -5f(x) \implies g'(x) = -5f'(x)$

71.

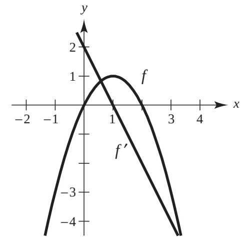


If f is linear then its derivative is a constant function.

$$f(x) = ax + b$$

$$f'(x) = a$$

72.



If f is quadratic, then its derivative is a linear function.

$$f(x) = ax^2 + bx + c$$

$$f'(x) = 2ax + b$$

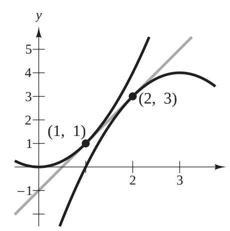
73. Let (x_1, y_1) and (x_2, y_2) be the points of tangency on $y = x^2$ and $y = -x^2 + 6x - 5$, respectively. The derivatives of these functions are:

$$y' = 2x \implies m = 2x_1 \quad \text{and} \quad y' = -2x + 6 \implies m = -2x_2 + 6$$

$$m = 2x_1 = -2x_2 + 6$$

$$x_1 = -x_2 + 3$$

Since $y_1 = x_1^2$ and $y_2 = -x_2^2 + 6x_2 - 5$:



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-x_2^2 + 6x_2 - 5) - (x_1^2)}{x_2 - x_1} = -2x_2 + 6$$

$$\frac{(-x_2^2 + 6x_2 - 5) - (-x_2 + 3)^2}{x_2 - (-x_2 + 3)} = -2x_2 + 6$$

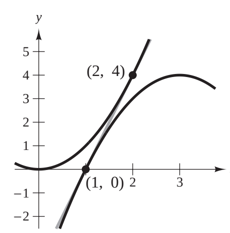
$$(-x_2^2 + 6x_2 - 5) - (x_2^2 - 6x_2 + 9) = (-2x_2 + 6)(2x_2 - 3)$$

$$-2x_2^2 + 12x_2 - 14 = -4x_2^2 + 18x_2 - 18$$

$$2x_2^2 - 6x_2 + 4 = 0$$

$$2(x_2 - 2)(x_2 - 1) = 0$$

$$x_2 = 1 \text{ or } 2$$



$x_2 = 1 \implies y_2 = 0,\ x_1 = 2$ and $y_1 = 4$

Thus, the tangent line through $(1, 0)$ and $(2, 4)$ is

$$y - 0 = \left(\frac{4 - 0}{2 - 1}\right)(x - 1) \implies y = 4x - 4.$$

$x_2 = 2 \implies y_2 = 3,\ x_1 = 1$ and $y_1 = 1$

Thus, the tangent line through $(2, 3)$ and $(1, 1)$ is

$$y - 1 = \left(\frac{3 - 1}{2 - 1}\right)(x - 1) \implies y = 2x - 1.$$

74. m_1 is the slope of the line tangent to $y = x$. m_2 is the slope of the line tangent to $y = 1/x$. Since

$$y = x \implies y' = 1 \implies m_1 = 1 \text{ and } y = \frac{1}{x} \implies y' = \frac{-1}{x^2} \implies m_2 = \frac{-1}{x^2}.$$

The points of intersection of $y = x$ and $y = 1/x$ are

$$x = \frac{1}{x} \implies x^2 = 1 \implies x = \pm 1.$$

At $x = \pm 1$, $m_2 = -1$. Since $m_2 = -1/m_1$, these tangent lines are perpendicular at the points of intersection.

75. $f(x) = 3x + \sin x + 2$

$f'(x) = 3 + \cos x$

Since $|\cos x| \leq 1$, $f'(x) \neq 0$ for all x and f does not have a horizontal tangent line.

76. $f(x) = x^5 + 3x^3 + 5x$

$f'(x) = 5x^4 + 9x^2 + 5$

Since $5x^4 + 9x^2 \geq 0$, $f'(x) \geq 5$. Thus, f does not have a tangent line with a slope of 3.

77. $f(x) = \sqrt{x}, \quad (-4, 0)$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = \frac{0 - y}{-4 - x}$$

$$4 + x = 2\sqrt{x}y$$

$$4 + x = 2\sqrt{x}\sqrt{x}$$

$$4 + x = 2x$$

$$x = 4, y = 2$$

The point $(4, 2)$ is on the graph of f.

Tangent line: $y - 2 = \dfrac{0 - 2}{-4 - 4}(x - 4)$

$$4y - 8 = x - 4$$

$$0 = x - 4y + 4$$

78. $f(x) = \dfrac{2}{x}, \quad (5, 0)$

$$f'(x) = -\frac{2}{x^2}$$

$$-\frac{2}{x^2} = \frac{0 - y}{5 - x}$$

$$-10 + 2x = -x^2y$$

$$-10 + 2x = -x^2\left(\frac{2}{x}\right)$$

$$-10 + 2x = -2x$$

$$4x = 10$$

$$x = \frac{5}{2}, y = \frac{4}{5}$$

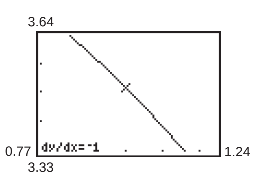
The point $\left(\frac{5}{2}, \frac{4}{5}\right)$ is on the graph of f. The slope of the tangent line is $f'\left(\frac{5}{2}\right) = -\frac{8}{25}$.

Tangent line: $y - \dfrac{4}{5} = -\dfrac{8}{25}\left(x - \dfrac{5}{2}\right)$

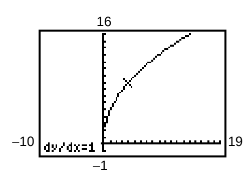
$$25y - 20 = -8x + 20$$

$$8x + 25y - 40 = 0$$

79. $f'(1) = -1$



80. $f'(4) = 1$

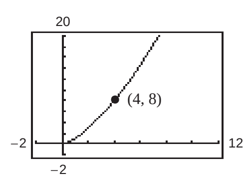


81. (a) One possible secant is between $(3.9, 7.7019)$ and $(4, 8)$:

$$y - 8 = \frac{8 - 7.7019}{4 - 3.9}(x - 4)$$

$$y - 8 = 2.981(x - 4)$$

$$y = S(x) = 2.981x - 3.924$$



(b) $f'(x) = \dfrac{3}{2}x^{1/2} \implies f'(4) = \dfrac{3}{2}(2) = 3$

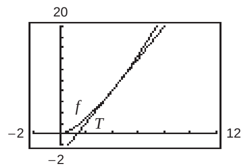
$$T(x) = 3(x - 4) + 8 = 3x - 4$$

$S(x)$ is an approximation of the tangent line $T(x)$.

—CONTINUED—

81. —CONTINUED—

(c) As you move further away from $(4, 8)$, the accuracy of the approximation T gets worse.



(d)

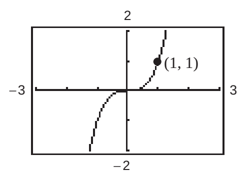
Δx	-3	-2	-1	-0.5	-0.1	0	0.1	0.5	1	2	3
$f(4+\Delta x)$	1	2.828	5.196	6.548	7.702	8	8.302	9.546	11.180	14.697	18.520
$T(4+\Delta x)$	-1	2	5	6.5	7.7	8	8.3	9.5	11	14	17

82. (a) Nearby point: $(1.0073138, 1.0221024)$

Secant line: $y - 1 = \dfrac{1.0221024 - 1}{1.0073138 - 1}(x - 1)$

$$y = 3.022(x - 1) + 1$$

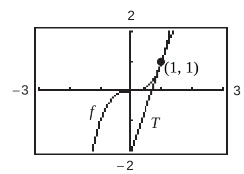
(Answers will vary.)



(b) $f'(x) = 3x^2$

$T(x) = 3(x - 1) + 1 = 3x - 2$

(c) The accuracy worsens as you move away from $(1, 1)$.



(d)

Δx	-3	-2	-1	-0.5	-0.1	0	0.1	0.5	1	2	3
$f(x)$	-8	-1	0	0.125	0.729	1	1.331	3.375	8	27	64
$T(x)$	-8	-5	-2	-0.5	0.7	1	1.3	2.5	4	7	10

The accuracy decreases more rapidly than in Exercise 81 because $y = x^3$ is less “linear” than $y = x^{3/2}$.

83. False. Let $f(x) = x^2$ and $g(x) = x^2 + 4$. Then $f'(x) = g'(x) = 2x$, but $f(x) \neq g(x)$.

84. True. If $f(x) = g(x) + c$, then $f'(x) = g'(x) + 0 = g'(x)$.

85. False. If $y = \pi^2$, then $dy/dx = 0$. (π^2 is a constant.)

86. True. If $y = x/\pi = (1/\pi) \cdot x$, then $dy/dx = (1/\pi)(1) = 1/\pi$.

87. True. If $g(x) = 3f(x)$, then $g'(x) = 3f'(x)$.

88. False. If $f(x) = \dfrac{1}{x^n} = x^{-n}$, then $f'(x) = -nx^{-n-1} = \dfrac{-n}{x^{n+1}}$.

89. $f(t) = 2t + 7, [1, 2]$

$f'(t) = 2$

Instantaneous rate of change is the constant 2.
Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{[2(2) + 7] - [2(1) + 7]}{1} = 2$$

(These are the same because f is a line of slope 2.)

90. $f(t) = t^2 - 3, \ [2, 2.1]$

$f'(t) = 2t$

Instantaneous rate of change:

$$(2, 1) \implies f'(2) = 2(2) = 4$$

$$(2.1, 1.41) \implies f'(2.1) = 4.2$$

Average rate of change:

$$\frac{f(2.1) - f(2)}{2.1 - 2} = \frac{1.41 - 1}{0.1} = 4.1$$

91. $f(x) = -\dfrac{1}{x}, \ [1, 2]$

$f'(x) = \dfrac{1}{x^2}$

Instantaneous rate of change:

$(1, -1) \implies f'(1) = 1$

$\left(2, -\dfrac{1}{2}\right) \implies f'(2) = \dfrac{1}{4}$

Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{(-1/2) - (-1)}{2 - 1} = \frac{1}{2}$$

92. $f(x) = \sin x, \ \left[0, \dfrac{\pi}{6}\right]$

$f'(x) = \cos x$

Instantaneous rate of change:

$(0, 0) \implies f'(0) = 1$

$\left(\dfrac{\pi}{6}, \dfrac{1}{2}\right) \implies f'\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2} \approx 0.866$

Average rate of change:

$$\frac{f(\pi/6) - f(0)}{(\pi/6) - 0} = \frac{(1/2) - 0}{(\pi/6) - 0} = \frac{3}{\pi} \approx 0.955$$

93. (a) $s(t) = -16t^2 + 1362$

$v(t) = -32t$

(b) $\dfrac{s(2) - s(1)}{2 - 1} = 1298 - 1346 = -48$ ft/sec

(c) $v(t) = s'(t) = -32t$

When $t = 1$: $v(1) = -32$ ft/sec

When $t = 2$: $v(2) = -64$ ft/sec

(d) $-16t^2 + 1362 = 0$

$$t^2 = \frac{1362}{16} \implies t = \frac{\sqrt{1362}}{4} \approx 9.226 \text{ sec}$$

(e) $v\left(\dfrac{\sqrt{1362}}{4}\right) = -32\left(\dfrac{\sqrt{1362}}{4}\right)$

$= -8\sqrt{1362} \approx -295.242$ ft/sec

94.

$$\begin{aligned} s(t) &= -16t^2 - 22t + 220 \\ v(t) &= -32t - 22 \\ v(3) &= -118 \text{ ft/sec} \\ s(t) &= -16t^2 - 22t + 220 \\ &= 112 \ \text{(height after falling 108 ft)} \\ -16t^2 - 22t + 108 &= 0 \\ -2(t - 2)(8t + 27) &= 0 \\ t &= 2 \\ v(2) &= -32(2) - 22 \\ &= -86 \text{ ft/sec} \end{aligned}$$

95.

$$\begin{aligned} s(t) &= -4.9t^2 + v_0 t + s_0 \\ &= -4.9t^2 + 120t \\ v(t) &= -9.8t + 120 \\ v(5) &= -9.8(5) + 120 = 71 \text{ m/sec} \\ v(10) &= -9.8(10) + 120 = 22 \text{ m/sec} \end{aligned}$$

96. $s(t) = -4.9t^2 + v_0 t + s_0$

$= -4.9t^2 + s_0 = 0$ when $t = 6.8$.

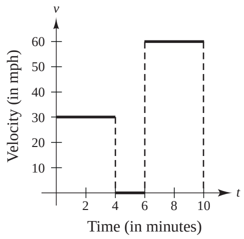
$s_0 = 4.9t^2 = 4.9(6.8)^2 \approx 226.6$ m

97. From (0, 0) to (4, 2), $s(t) = \frac{1}{2}t \implies v(t) = \frac{1}{2}$ mi/min.

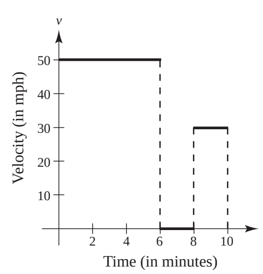
$v(t) = \frac{1}{2}(60) = 30$ mph for $0 < t < 4$

Similarly, $v(t) = 0$ for $4 < t < 6$. Finally, from (6, 2) to (10, 6),

$s(t) = t - 4 \implies v(t) = 1$ mi/in $= 60$ mph.



98.



(The velocity has been converted to miles per hour.)

99. $v = 40 \text{ mph} = \frac{2}{3} \text{ mi/min}$

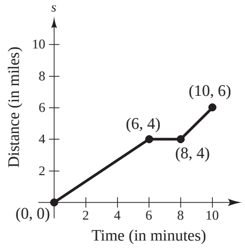
$\left(\frac{2}{3} \text{ mi/min}\right)(6 \text{ min}) = 4 \text{ mi}$

$v = 0 \text{ mph} = 0 \text{ mi/min}$

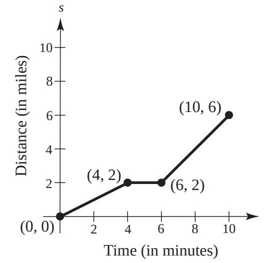
$(0 \text{ mi/min})(2 \text{ min}) = 0 \text{ mi}$

$v = 60 \text{ mph} = 1 \text{ mi/min}$

$(1 \text{ mi/min})(2 \text{ min}) = 2 \text{ mi}$



100. This graph corresponds with Exercise 97.



101. (a) Using a graphing utility,

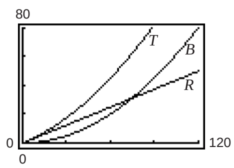
$R = 0.417v - 0.02.$

(b) Using a graphing utility,

$B = 0.00557v^2 + 0.0014v + 0.04.$

(c) $T = R + B = 0.00557v^2 + 0.418v + 0.02$

(d)



(e) $\frac{dT}{dv} = 0.01114v + 0.418$

For $v = 40$, $T'(40) \approx 0.86$.

For $v = 80$, $T'(80) \approx 1.31$.

For $v = 100$, $T'(100) \approx 1.53$.

(f) For increasing speeds, the total stopping distance increases.

102. $C = (\text{gallons of fuel used})(\text{cost per gallon})$

$= \left(\frac{15{,}000}{x}\right)(1.55) = \frac{23{,}250}{x}$

$\frac{dC}{dx} = -\frac{23{,}250}{x^2}$

x	10	15	20	25	30	35	40
C	2325	1550	1163	930	775	664	581
$\frac{dC}{dx}$	−233	−103	−58	−37	−26	−19	−15

The driver who gets 15 miles per gallon would benefit more. The rate of change at $x = 15$ is larger in absolute value than that at $x = 35$.

103. $V = s^3, \frac{dV}{ds} = 3s^2$

When $s = 4$ cm, $\frac{dV}{ds} = 48 \text{ cm}^2$ per cm change in s.

104. $A = s^2, \frac{dA}{ds} = 2s$

When $s = 4$ m,

$\frac{dA}{ds} = 8$ square meters per meter change in s.

105. $s(t) = -\frac{1}{2}at^2 + c$ and $s'(t) = -at$

Average velocity: $$\frac{s(t_0 + \Delta t) - s(t_0 - \Delta t)}{(t_0 + \Delta t) - (t_0 - \Delta t)} = \frac{[-(1/2)a(t_0 + \Delta t)^2 + c] - [-(1/2)a(t_0 - \Delta t)^2 + c)]}{2\Delta t}$$

$$= \frac{-(1/2)a(t_0^2 + 2t_0\Delta t + (\Delta t)^2) + (1/2)a(t_0^2 - 2t_0\Delta t + (\Delta t)^2)}{2\,\Delta t}$$

$$= \frac{-2at_0\,\Delta t}{2\,\Delta t}$$

$$= -at_0$$

$$= s'(t_0) \quad \text{instantaneous velocity at } t = t_0$$

106. $$C = \frac{1{,}008{,}000}{Q} + 6.3Q$$

$$\frac{dC}{dQ} = -\frac{1{,}008{,}000}{Q^2} + 6.3$$

$C(351) - C(350) \approx 5083.095 - 5085 \approx -\$1.91$

When $Q = 350$, $\frac{dC}{dQ} \approx -\$1.93$.

107. $N = f(p)$

(a) $f'(1.479)$ is the rate of change of gallons of gasoline sold when the price is \$1.479 per gallon.

(b) $f'(1.479)$ is usually negative. As prices go up, sales go down.

108. $\frac{dT}{dt} = K(T - T_a)$

109. $y = ax^2 + bx + c$

Since the parabola passes through (0, 1) and (1, 0), we have:

$(0, 1): 1 = a(0)^2 + b(0) + c \implies c = 1$

$(1, 0): 0 = a(1)^2 + b(1) + 1 \implies b = -a - 1$

Thus, $y = ax^2 + (-a - 1)x + 1$. From the tangent line $y = x - 1$, we know that the derivative is 1 at the point (1, 0).

$y' = 2ax + (-a - 1)$

$1 = 2a(1) + (-a - 1)$

$1 = a - 1$

$a = 2$

$b = -a - 1 = -3$

Therefore, $y = 2x^2 - 3x + 1$.

110. $y = \frac{1}{x},\ x > 0$

$y' = -\frac{1}{x^2}$

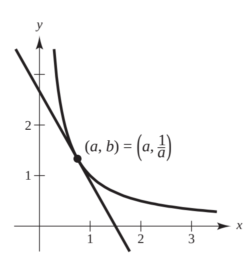
At (a, b), the equation of the tangent line is

$$y - \frac{1}{a} = -\frac{1}{a^2}(x - a) \quad \text{or} \quad y = -\frac{x}{a^2} + \frac{2}{a}.$$

The x-intercept is $(2a, 0)$.

The y-intercept is $\left(0, \frac{2}{a}\right)$.

The area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(2a)\left(\frac{2}{a}\right) = 2$.



111. $y = x^3 - 9x$

$y' = 3x^2 - 9$

Tangent lines through $(1, -9)$:

$$y + 9 = (3x^2 - 9)(x - 1)$$
$$(x^3 - 9x) + 9 = 3x^3 - 3x^2 - 9x + 9$$
$$0 = 2x^3 - 3x^2 = x^2(2x - 3)$$
$$x = 0 \text{ or } x = \tfrac{3}{2}$$

The points of tangency are $(0, 0)$ and $\left(\frac{3}{2}, -\frac{81}{8}\right)$. At $(0, 0)$, the slope is $y'(0) = -9$. At $\left(\frac{3}{2}, -\frac{81}{8}\right)$, the slope is $y'\left(\frac{3}{2}\right) = -\frac{9}{4}$.

Tangent lines:

$$y - 0 = -9(x - 0) \quad \text{and} \quad y + \tfrac{81}{8} = -\tfrac{9}{4}\left(x - \tfrac{3}{2}\right)$$
$$y = -9x \qquad\qquad y = -\tfrac{9}{4}x - \tfrac{27}{4}$$
$$9x + y = 0 \qquad\qquad 9x + 4y + 27 = 0$$

112. $y = x^2$

$y' = 2x$

(a) Tangent lines through $(0, a)$:

$$y - a = 2x(x - 0)$$
$$x^2 - a = 2x^2$$
$$-a = x^2$$
$$\pm\sqrt{-a} = x$$

The points of tangency are $\left(\pm\sqrt{-a}, -a\right)$. At $\left(\sqrt{-a}, -a\right)$, the slope is $y'\left(\sqrt{-a}\right) = 2\sqrt{-a}$.
At $\left(-\sqrt{-a}, -a\right)$, the slope is $y'\left(-\sqrt{-a}\right) = -2\sqrt{-a}$.

Tangent lines: $y + a = 2\sqrt{-a}\left(x - \sqrt{-a}\right)$ and $y + a = -2\sqrt{-a}\left(x + \sqrt{-a}\right)$

$$y = 2\sqrt{-a}\,x + a \qquad\qquad y = -2\sqrt{-a}\,x + a$$

Restriction: a must be negative.

(b) Tangent lines through $(a, 0)$:

$$y - 0 = 2x(x - a)$$
$$x^2 = 2x^2 - 2ax$$
$$0 = x^2 - 2ax = x(x - 2a)$$

The points of tangency are $(0, 0)$ and $(2a, 4a^2)$. At $(0, 0)$, the slope is $y'(0) = 0$. At $(2a, 4a^2)$, the slope is $y'(2a) = 4a$.

Tangent lines: $y - 0 = 0(x - 0)$ and $y - 4a^2 = 4a(x - 2a)$

$$y = 0 \qquad\qquad y = 4ax - 4a^2$$

Restriction: None, a can be any real number.

113. $f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$

f must be continuous at $x = 2$ to be differentiable at $x = 2$.

$$\left.\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} ax^3 = 8a \\ \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} (x^2 + b) = 4 + b \end{aligned}\right\} \quad \begin{aligned} 8a &= 4 + b \\ 8a - 4 &= b \end{aligned}$$

$$f'(x) = \begin{cases} 3ax^2, & x < 2 \\ 2x, & x > 2 \end{cases}$$

For f to be differentiable at $x = 2$, the left derivative must equal the right derivative.

$$3a(2)^2 = 2(2)$$
$$12a = 4$$
$$a = \tfrac{1}{3}$$
$$b = 8a - 4 = -\tfrac{4}{3}$$

114. $f(x) = \begin{cases} \cos x, & x < 0 \\ ax + b, & x \ge 0 \end{cases}$

$$f(0) = b = \cos(0) = 1 \Longrightarrow b = 1$$

$$f'(x) = \begin{cases} -\sin x, & x < 0 \\ a, & x > 0 \end{cases}$$

Hence, $a = 0$.

Answer: $a = 0, b = 1$

115. $f_1(x) = |\sin x|$ is differentiable for all $x \ne n\pi$, n an integer.

$f_2(x) = \sin|x|$ is differentiable for all $x \ne 0$.

You can verify this by graphing f_1 and f_2 and observing the locations of the sharp turns.

116. Let $f(x) = \cos x$.

$$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{\cos x(\cos \Delta x - 1)}{\Delta x} - \lim_{\Delta x\to 0} \sin x\left(\frac{\sin \Delta x}{\Delta x}\right) \\ &= 0 - \sin x(1) = -\sin x \end{aligned}$$

Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

1. $g(x) = (x^2 + 1)(x^2 - 2x)$

$$\begin{aligned} g'(x) &= (x^2 + 1)(2x - 2) + (x^2 - 2x)(2x) \\ &= 2x^3 - 2x^2 + 2x - 2 + 2x^3 - 4x^2 \\ &= 4x^3 - 6x^2 + 2x - 2 \end{aligned}$$

2. $f(x) = (6x + 5)(x^3 - 2)$

$$\begin{aligned} f'(x) &= (6x + 5)(3x^2) + (x^3 - 2)(6) \\ &= 18x^3 + 15x^2 + 6x^3 - 12 \\ &= 24x^3 + 15x^2 - 12 \end{aligned}$$

3. $h(t) = \sqrt[3]{t}(t^2 + 4) = t^{1/3}(t^2 + 4)$

$$\begin{aligned} h'(t) &= t^{1/3}(2t) + (t^2 + 4)\frac{1}{3}t^{-2/3} \\ &= 2t^{4/3} + \frac{t^2 + 4}{3t^{2/3}} \\ &= \frac{7t^2 + 4}{3t^{2/3}} \end{aligned}$$

4. $g(s) = \sqrt{s}(4 - s^2) = s^{1/2}(4 - s^2)$

$$\begin{aligned} g'(s) &= s^{1/2}(-2s) + (4 - s^2)\frac{1}{2}s^{-1/2} \\ &= -2s^{3/2} + \frac{4 - s^2}{2s^{1/2}} \\ &= \frac{4 - 5s^2}{2s^{1/2}} \end{aligned}$$

5. $f(x) = x^3 \cos x$

$f'(x) = x^3(-\sin x) + \cos x(3x^2)$

$= 3x^2 \cos x - x^3 \sin x$

6. $g(x) = \sqrt{x} \sin x$

$g'(x) = \sqrt{x} \cos x + \sin x\left(\dfrac{1}{2\sqrt{x}}\right) = \sqrt{x} \cos x + \dfrac{1}{2\sqrt{x}} \sin x$

7. $f(x) = \dfrac{x}{x^2 + 1}$

$f'(x) = \dfrac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \dfrac{1 - x^2}{(x^2 + 1)^2}$

8. $g(t) = \dfrac{t^2 + 2}{2t - 7}$

$g'(t) = \dfrac{(2t - 7)(2t) - (t^2 + 2)(2)}{(2t - 7)^2} = \dfrac{2t^2 - 14t - 4}{(2t - 7)^2}$

9. $h(x) = \dfrac{\sqrt[3]{x}}{x^3 + 1} = \dfrac{x^{1/3}}{x^3 + 1}$

$h'(x) = \dfrac{(x^3 + 1)\frac{1}{3}x^{-2/3} - x^{1/3}(3x^2)}{(x^3 + 1)^2}$

$= \dfrac{(x^3 + 1) - x(9x^2)}{3x^{2/3}(x^3 + 1)^2}$

$= \dfrac{1 - 8x^3}{3x^{2/3}(x^3 + 1)^2}$

10. $h(s) = \dfrac{s}{\sqrt{s} - 1}$

$h'(s) = \dfrac{\left(\sqrt{s} - 1\right)(1) - s\left(\frac{1}{2}s^{-1/2}\right)}{\left(\sqrt{s} - 1\right)^2}$

$= \dfrac{\sqrt{s} - 1 - \frac{1}{2}\sqrt{s}}{\left(\sqrt{s} - 1\right)^2} = \dfrac{\sqrt{s} - 2}{2\left(\sqrt{s} - 1\right)^2}$

11. $g(x) = \dfrac{\sin x}{x^2}$

$g'(x) = \dfrac{x^2(\cos x) - \sin x(2x)}{(x^2)^2} = \dfrac{x \cos x - 2 \sin x}{x^3}$

12. $f(t) = \dfrac{\cos t}{t^3}$

$f'(t) = \dfrac{t^3(-\sin t) - \cos t(3t^2)}{(t^3)^2} = -\dfrac{t \sin t + 3 \cos t}{t^4}$

13. $f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$

$f'(x) = (x^3 - 3x)(4x + 3) + (2x^2 + 3x + 5)(3x^2 - 3)$

$= 10x^4 + 12x^3 - 3x^2 - 18x - 15$

$f'(0) = -15$

14. $f(x) = (x^2 - 2x + 1)(x^3 - 1)$

$f'(x) = (x^2 - 2x + 1)(3x^2) + (x^3 - 1)(2x - 2)$

$= 3x^2(x - 1)^2 + 2(x - 1)^2(x^2 + x + 1)$

$= (x - 1)^2(5x^2 + 2x + 2)$

$f'(1) = 0$

15. $f(x) = \dfrac{x^2 - 4}{x - 3}$

$f'(x) = \dfrac{(x - 3)(2x) - (x^2 - 4)(1)}{(x - 3)^2}$

$= \dfrac{2x^2 - 6x - x^2 + 4}{(x - 3)^2}$

$= \dfrac{x^2 - 6x + 4}{(x - 3)^2}$

$f'(1) = \dfrac{1 - 6 + 4}{(1 - 3)^2} = -\dfrac{1}{4}$

16. $f(x) = \dfrac{x + 1}{x - 1}$

$f'(x) = \dfrac{(x - 1)(1) - (x + 1)(1)}{(x - 1)^2}$

$= \dfrac{x - 1 - x - 1}{(x - 1)^2}$

$= -\dfrac{2}{(x - 1)^2}$

$f'(2) = -\dfrac{2}{(2 - 1)^2} = -2$

17. $f(x) = x \cos x$

$f'(x) = (x)(-\sin x) + (\cos x)(1) = \cos x - x \sin x$

$f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2} - \dfrac{\pi}{4}\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\sqrt{2}}{8}(4 - \pi)$

18. $f(x) = \dfrac{\sin x}{x}$

$$f'(x) = \frac{(x)(\cos x) - (\sin x)(1)}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

$$f'\left(\frac{\pi}{6}\right) = \frac{(\pi/6)\left(\sqrt{3}/2\right) - (1/2)}{\pi^2/36}$$

$$= \frac{3\sqrt{3}\pi - 18}{\pi^2}$$

$$= \frac{3\left(\sqrt{3}\pi - 6\right)}{\pi^2}$$

	Function	Rewrite	Differentiate	Simplify
19.	$y = \dfrac{x^2 + 2x}{3}$	$y = \dfrac{1}{3}x^2 + \dfrac{2}{3}x$	$y' = \dfrac{2}{3}x + \dfrac{2}{3}$	$y' = \dfrac{2x + 2}{3}$
20.	$y = \dfrac{5x^2 - 3}{4}$	$y = \dfrac{5}{4}x^2 - \dfrac{3}{4}$	$y' = \dfrac{10}{4}x$	$y' = \dfrac{5x}{2}$
21.	$y = \dfrac{7}{3x^3}$	$y = \dfrac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\dfrac{7}{x^4}$
22.	$y = \dfrac{4}{5x^2}$	$y = \dfrac{4}{5}x^{-2}$	$y' = -\dfrac{8}{5}x^{-3}$	$y' = -\dfrac{8}{5x^3}$
23.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4\sqrt{x},\ \ x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}$
24.	$y = \dfrac{3x^2 - 5}{7}$	$y = \dfrac{3}{7}x^2 - \dfrac{5}{7}$	$y' = \dfrac{6x}{7}$	$y' = \dfrac{6}{7}x$

25. $f(x) = \dfrac{3 - 2x - x^2}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(-2 - 2x) - (3 - 2x - x^2)(2x)}{(x^2 - 1)^2}$$

$$= \frac{2x^2 - 4x + 2}{(x^2 - 1)^2} = \frac{2(x - 1)^2}{(x^2 - 1)^2}$$

$$= \frac{2}{(x + 1)^2}, \quad x \neq 1$$

26. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(3x^2 + 3) - (x^3 + 3x + 2)(2x)}{(x^2 - 1)^2}$$

$$= \frac{x^4 - 6x^2 - 4x - 3}{(x^2 - 1)^2}$$

27. $f(x) = x\left(1 - \dfrac{4}{x + 3}\right) = x - \dfrac{4x}{x + 3}$

$$f'(x) = 1 - \frac{(x + 3)4 - 4x(1)}{(x + 3)^2}$$

$$= \frac{(x^2 + 6x + 9) - 12}{(x + 3)^2}$$

$$= \frac{x^2 + 6x - 3}{(x + 3)^2}$$

28. $f(x) = x^4\left[1 - \dfrac{2}{x + 1}\right] = x^4\left[\dfrac{x - 1}{x + 1}\right]$

$$f'(x) = x^4\left[\frac{(x + 1) - (x - 1)}{(x + 1)^2}\right] + \left[\frac{x - 1}{x + 1}\right](4x^3)$$

$$= x^4\left[\frac{2}{(x + 1)^2}\right] + \left[\frac{x^2 - 1}{(x + 1)^2}\right](4x^3)$$

$$= 2x^3\left[\frac{2x^2 + x - 2}{(x + 1)^2}\right]$$

29. $f(x) = \dfrac{2x + 5}{\sqrt{x}} = 2x^{1/2} + 5x^{-1/2}$

$f'(x) = x^{-1/2} - \dfrac{5}{2}x^{-3/2} = x^{-3/2}\left[x - \dfrac{5}{2}\right] = \dfrac{2x - 5}{2x\sqrt{x}} = \dfrac{2x - 5}{2x^{3/2}}$

30. $f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right) = x^{1/3}(x^{1/2} + 3)$

$f'(x) = x^{1/3}\left(\dfrac{1}{2}x^{-1/2}\right) + (x^{1/2} + 3)\left(\dfrac{1}{3}x^{-2/3}\right)$

$= \dfrac{5}{6}x^{-1/6} + x^{-2/3}$

$= \dfrac{5}{6x^{1/6}} + \dfrac{1}{x^{2/3}}$

Alternate solution:

$f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right)$

$= x^{5/6} + 3x^{1/3}$

$f'(x) = \dfrac{5}{6}x^{-1/6} + x^{-2/3}$

$= \dfrac{5}{6x^{1/6}} + \dfrac{1}{x^{2/3}}$

31. $h(s) = (s^3 - 2)^2 = s^6 - 4s^3 + 4$

$h'(s) = 6s^5 - 12s^2 = 6s^2(s^3 - 2)$

32. $h(x) = (x^2 - 1)^2 = x^4 - 2x^2 + 1$

$h'(x) = 4x^3 - 4x = 4x(x^2 - 1)$

33. $f(x) = \dfrac{2 - (1/x)}{x - 3} = \dfrac{2x - 1}{x(x - 3)} = \dfrac{2x - 1}{x^2 - 3x}$

$f'(x) = \dfrac{(x^2 - 3x)2 - (2x - 1)(2x - 3)}{(x^2 - 3x)^2} = \dfrac{2x^2 - 6x - 4x^2 + 8x - 3}{(x^2 - 3x)^2}$

$= \dfrac{-2x^2 + 2x - 3}{(x^2 - 3x)^2} = -\dfrac{2x^2 - 2x + 3}{x^2(x - 3)^2}$

34. $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x + 1}\right) = 2x - \dfrac{x^2}{x + 1}$

$g'(x) = 2 - \dfrac{(x + 1)2x - x^2(1)}{(x + 1)^2} = \dfrac{2(x^2 + 2x + 1) - x^2 - 2x}{(x + 1)^2} = \dfrac{x^2 + 2x + 2}{(x + 1)^2}$

35. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$

$f'(x) = (9x^2 + 4)(x - 5)(x + 1) + (3x^3 + 4x)(1)(x + 1) + (3x^3 + 4x)(x - 5)(1)$

$= (9x^2 + 4)(x^2 - 4x - 5) + 3x^4 + 3x^3 + 4x^2 + 4x + 3x^4 - 15x^3 + 4x^2 - 20x$

$= 9x^4 - 36x^3 - 41x^2 - 16x - 20 + 6x^4 - 12x^3 + 8x^2 - 16x$

$= 15x^4 - 48x^3 - 33x^2 - 32x - 20$

36. $f(x) = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$

$f'(x) = (2x - 1)(x^2 + 1)(x^2 + x + 1) + (x^2 - x)(2x)(x^2 + x + 1) + (x^2 - x)(x^2 + 1)(2x + 1)$

$= (2x - 1)(x^4 + x^3 + 2x^2 + x + 1) + (x^2 - x)(2x^3 + 2x^2 + 2x) + (x^2 - x)(2x^3 + x^2 + 2x + 1)$

$= 2x^5 + x^4 + 3x^3 + x - 1 + 2x^5 - 2x^2 + 2x^5 - x^4 + x^3 - x^2 - x$

$= 6x^5 + 4x^3 - 3x^2 - 1$

37. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$

$$f'(x) = \frac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2}$$

$$= \frac{-4xc^2}{(x^2 - c^2)^2}$$

38. $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$

$$f'(x) = \frac{(c^2 + x^2)(-2x) - (c^2 - x^2)(2x)}{(c^2 + x^2)^2}$$

$$= \frac{-4xc^2}{(c^2 + x^2)^2}$$

39. $f(t) = t^2 \sin t$

$$f'(t) = t^2 \cos t + 2t \sin t$$

$$= t(t \cos t + 2 \sin t)$$

40. $f(\theta) = (\theta + 1) \cos \theta$

$$f'(\theta) = (\theta + 1)(-\sin \theta) + (\cos \theta)(1)$$

$$= \cos \theta - (\theta + 1) \sin \theta$$

41. $f(t) = \dfrac{\cos t}{t}$

$$f'(t) = \frac{-t \sin t - \cos t}{t^2} = -\frac{t \sin t + \cos t}{t^2}$$

42. $f(x) = \dfrac{\sin x}{x}$

$$f'(x) = \frac{x \cos x - \sin x}{x^2}$$

43. $f(x) = -x + \tan x$

$$f'(x) = -1 + \sec^2 x = \tan^2 x$$

44. $y = x + \cot x$

$$y' = 1 - \csc^2 x = -\cot^2 x$$

45. $g(t) = \sqrt[4]{t} + 8 \sec t = t^{1/4} + 8 \sec t$

$$g'(t) = \frac{1}{4}t^{-3/4} + 8 \sec t \tan t = \frac{1}{4t^{3/4}} + 8 \sec t \tan t$$

46. $h(s) = \dfrac{1}{s} - 10 \csc s$

$$h'(s) = -\frac{1}{s^2} + 10 \csc s \cot s$$

47. $y = \dfrac{3(1 - \sin x)}{2 \cos x} = \dfrac{3}{2}(\sec x - \tan x)$

$$y' = \frac{3}{2}(\sec x \tan x - \sec^2 x) = \frac{3}{2} \sec x(\tan x - \sec x)$$

$$= \frac{3}{2}(\sec x \tan x - \tan^2 x - 1)$$

48. $y = \dfrac{\sec x}{x}$

$$y' = \frac{x \sec x \tan x - \sec x}{x^2}$$

$$= \frac{\sec x(x \tan x - 1)}{x^2}$$

49. $y = -\csc x - \sin x$

$$y' = \csc x \cot x - \cos x$$

$$= \frac{\cos x}{\sin^2 x} - \cos x$$

$$= \cos x(\csc^2 x - 1)$$

$$= \cos x \cot^2 x$$

50. $y = x \sin x + \cos x$

$$y' = x \cos x + \sin x - \sin x = x \cos x$$

51. $f(x) = x^2 \tan x$

$$f'(x) = x^2 \sec^2 x + 2x \tan x$$

$$= x(x \sec^2 x + 2 \tan x)$$

52. $f(x) = \sin x \cos x$

$$f'(x) = \sin x(-\sin x) + \cos x(\cos x)$$

$$= \cos 2x$$

53. $y = 2x \sin x + x^2 \cos x$

$$y' = 2x \cos x + 2 \sin x + x^2(-\sin x) + 2x \cos x$$

$$= 4x \cos x + 2 \sin x - x^2 \sin x$$

54. $h(\theta) = 5\theta \sec \theta + \theta \tan \theta$

$$h'(\theta) = 5\theta \sec \theta \tan \theta + 5 \sec \theta + \theta \sec^2 \theta + \tan \theta$$

55. $g(x) = \left(\frac{x+1}{x+2}\right)(2x-5)$

$g'(x) = \frac{2x^2+8x-1}{(x+2)^2}$ (Form of answer may vary.)

56. $f(x) = \left(\frac{x^2-x-3}{x^2+1}\right)(x^2+x+1)$

$f'(x) = 2\frac{x^5+2x^3+2x^2-2}{(x^2+1)^2}$ (Form of answer may vary.)

57. $g(\theta) = \frac{\theta}{1-\sin\theta}$

$g'(\theta) = \frac{1-\sin\theta+\theta\cos\theta}{(\sin\theta-1)^2}$ (Form of answer may vary.)

58. $f(\theta) = \frac{\sin\theta}{1-\cos\theta}$

$f'(\theta) = \frac{1}{\cos\theta-1} = \frac{\cos\theta-1}{(1-\cos\theta)^2}$

(Form of answer may vary.)

59. $y = \frac{1+\csc x}{1-\csc x}$

$y' = \frac{(1-\csc x)(-\csc x\cot x)-(1+\csc x)(\csc x\cot x)}{(1-\csc x)^2} = \frac{-2\csc x\cot x}{(1-\csc x)^2}$

$y'\left(\frac{\pi}{6}\right) = \frac{-2(2)(\sqrt{3})}{(1-2)^2} = -4\sqrt{3}$

60. $f(x) = \tan x\ \cot x = 1$

$f'(x) = 0$

$f'(1) = 0$

61. $h(t) = \frac{\sec t}{t}$

$h'(t) = \frac{t(\sec t\tan t)-(\sec t)(1)}{t^2} = \frac{\sec t(t\tan t-1)}{t^2}$

$h'(\pi) = \frac{\sec\pi(\pi\tan\pi-1)}{\pi^2} = \frac{1}{\pi^2}$

62. $f(x) = \sin x(\sin x+\cos x)$

$f'(x) = \sin x(\cos x-\sin x)+(\sin x+\cos x)\cos x$

$= \sin x\cos x-\sin^2 x+\sin x\cos x+\cos^2 x$

$= \sin 2x+\cos 2x$

$f'\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{2}+\cos\frac{\pi}{2} = 1$

63. (a) $f(x) = (x^3-3x+1)(x+2)$, $(1, -3)$

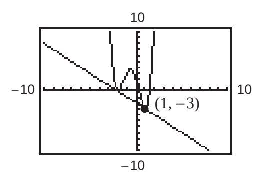
$f'(x) = (x^3-3x+1)(1)+(x+2)(3x^2-3)$

$= 4x^3+6x^2-6x-5$

$f'(1) = -1$; Slope at $(1, -3)$

Tangent line: $y+3 = -1(x-1) \Rightarrow y = -x-2$

(b)



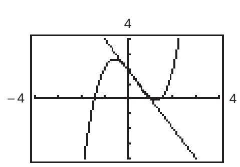
64. (a) $f(x) = (x-1)(x^2-2)$, $(0, 2)$

$f'(x) = (x-1)(2x)+(x^2-2)(1) = 3x^2-2x-2$

$f'(0) = -2$; Slope at $(0, 2)$

Tangent line: $y-2 = -2x \Rightarrow y = -2x+2$

(b)



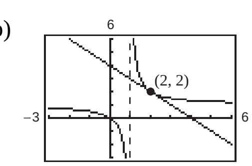
65. (a) $f(x) = \frac{x}{x-1}$, $(2, 2)$

$f'(x) = \frac{(x-1)(1)-x(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$

$f'(2) = \frac{-1}{(2-1)^2} = -1$; Slope at $(2, 2)$

Tangent line: $y-2 = -1(x-2) \Rightarrow y = -x+4$

(b)



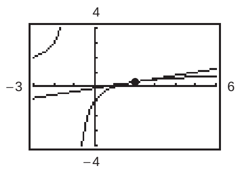
66. (a) $f(x) = \frac{x-1}{x+1}, \quad \left(2, \frac{1}{3}\right)$

$$f'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$f'(2) = \frac{2}{9}; \text{ Slope at } \left(2, \frac{1}{3}\right)$$

Tangent line: $y - \frac{1}{3} = \frac{2}{9}(x-2) \Rightarrow y = \frac{2}{9}x - \frac{1}{9}$

(b)



67. (a) $f(x) = \tan x, \quad \left(\frac{\pi}{4}, 1\right)$

$$f'(x) = \sec^2 x$$

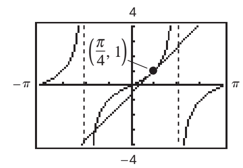
$$f'\left(\frac{\pi}{4}\right) = 2; \text{ Slope at } \left(\frac{\pi}{4}, 1\right)$$

Tangent line: $y - 1 = 2\left(x - \frac{\pi}{4}\right)$

$$y - 1 = 2x - \frac{\pi}{2}$$

$$4x - 2y - \pi + 2 = 0$$

(b)



68. (a) $f(x) = \sec x, \quad \left(\frac{\pi}{3}, 2\right)$

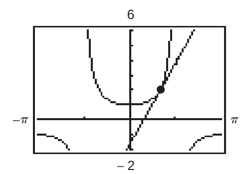
$$f'(x) = \sec x \tan x$$

$$f'\left(\frac{\pi}{3}\right) = 2\sqrt{3}; \text{ Slope at } \left(\frac{\pi}{3}, 2\right)$$

Tangent line: $y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$

$$6\sqrt{3}x - 3y + 6 - 2\sqrt{3}\pi = 0$$

(b)



69. $f(x) = \frac{8}{x^2+4}; \quad (2, 1)$

$$f'(x) = \frac{(x^2+4)(0) - 8(2x)}{(x^2+4)^2} = \frac{-16x}{(x^2+4)^2}$$

$$f'(2) = \frac{-16(2)}{(4+4)^2} = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x-2)$$

$$y = -\frac{1}{2}x + 2$$

$$2y + x - 4 = 0$$

70. $f(x) = \frac{27}{x^2+9}; \quad \left(-3, \frac{3}{2}\right)$

$$f'(x) = \frac{(x^2+9)(0) - 27(2x)}{(x^2+9)^2} = \frac{-54x}{(x^2+9)^2}$$

$$f'(-3) = \frac{-54(-3)}{(9+9)^2} = \frac{1}{2}$$

$$y - \frac{3}{2} = \frac{1}{2}(x+3)$$

$$y = \frac{1}{2}x + 3$$

$$2y - x - 6 = 0$$

71. $f(x) = \frac{16x}{x^2+16}; \quad \left(-2, -\frac{8}{5}\right)$

$$f'(x) = \frac{(x^2+16)(16) - 16x(2x)}{(x^2+16)^2} = \frac{256-16x^2}{(x^2+16)^2}$$

$$f'(-2) = \frac{256 - 16(4)}{20^2} = \frac{12}{25}$$

$$y + \frac{8}{5} = \frac{12}{25}(x+2)$$

$$y = \frac{12}{25}x - \frac{16}{25}$$

$$25y - 12x + 16 = 0$$

72. $f(x) = \frac{4x}{x^2+6}; \quad \left(2, \frac{4}{5}\right)$

$$f'(x) = \frac{(x^2+6)(4) - 4x(2x)}{(x^2+6)^2} = \frac{24-4x^2}{(x^2+6)^2}$$

$$f'(2) = \frac{24 - 16}{10^2} = \frac{2}{25}$$

$$y - \frac{4}{5} = \frac{2}{25}(x-2)$$

$$y = \frac{2}{25}x + \frac{16}{25}$$

$$25y - 2x - 16 = 0$$

73. $f(x) = \dfrac{x^2}{x - 1}$

$$f'(x) = \frac{(x-1)(2x) - x^2(1)}{(x-1)^2}$$

$$= \frac{x^2 - 2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

$f'(x) = 0$ when $x = 0$ or $x = 2$.

Horizontal tangents are at $(0, 0)$ and $(2, 4)$.

74. $f(x) = \dfrac{x^2}{x^2 + 1}$

$$f'(x) = \frac{(x^2+1)(2x) - (x^2)(2x)}{(x^2+1)^2}$$

$$= \frac{2x}{(x^2+1)^2}$$

$f'(x) = 0$ when $x = 0$.

Horizontal tangent is at $(0, 0)$.

75. $f(x) = \dfrac{4x - 2}{x^2}$

$$f'(x) = \frac{x^2(4) - (4x-2)(2x)}{x^4}$$

$$= \frac{4x^2 - 8x^2 + 4x}{x^4}$$

$$= \frac{4 - 4x}{x^3}$$

$f'(x) = 0$ for $4 - 4x = 0 \Rightarrow x = 1$.

$f(1) = 2$

f has a horizontal tangent at $(1, 2)$.

76. $f(x) = \dfrac{x - 4}{x^2 - 7}$

$$f'(x) = \frac{(x^2-7)(1) - (x-4)(2x)}{(x^2-7)^2} = \frac{x^2 - 7 - 2x^2 + 8x}{(x^2-7)^2} = -\frac{x^2 - 8x + 7}{(x^2-7)^2} = -\frac{(x-7)(x-1)}{(x^2-7)^2}$$

$f'(x) = 0$ for $x = 1, 7$; $f(1) = \dfrac{1}{2}$, $f(7) = \dfrac{1}{14}$

f has horizontal tangents at $\left(1, \dfrac{1}{2}\right)$ and $\left(7, \dfrac{1}{14}\right)$.

77. $f(x) = \dfrac{x + 1}{x - 1}$

$$f'(x) = \frac{(x-1) - (x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$2y + x = 6 \Rightarrow y = -\dfrac{1}{2}x + 3$; Slope: $-\dfrac{1}{2}$

$$\frac{-2}{(x-1)^2} = -\frac{1}{2}$$

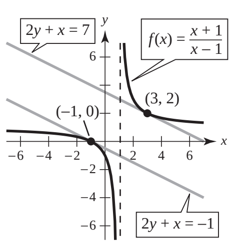
$$(x-1)^2 = 4$$

$$x - 1 = \pm 2$$

$$x = -1, 3; \quad f(-1) = 0, \quad f(3) = 2$$

$$y - 0 = -\frac{1}{2}(x+1) \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$$

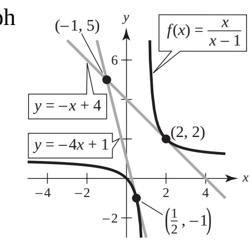
$$y - 2 = -\frac{1}{2}(x-3) \Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$$



78. $f(x) = \dfrac{x}{x - 1}$

$$f'(x) = \frac{(x-1) - x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

Let $(x, y) = (x, x/(x-1))$ be a point of tangency on the graph of f.



$$\frac{5 - (x/(x-1))}{-1 - x} = \frac{-1}{(x-1)^2}$$

$$\frac{4x - 5}{(x-1)(x+1)} = \frac{1}{(x-1)^2}$$

$$(4x - 5)(x - 1) = x + 1$$

$$4x^2 - 10x + 4 = 0$$

$$(x-2)(2x-1) = 0 \Rightarrow x = \frac{1}{2}, 2$$

$$f\left(\frac{1}{2}\right) = -1, f(2) = 2; f'\left(\frac{1}{2}\right) = -4, f'(2) = -1$$

Two tangent lines:

$$y + 1 = -4\left(x - \frac{1}{2}\right) \Rightarrow y = -4x + 1$$

$$y - 2 = -1(x - 2) \Rightarrow y = -x + 4$$

79. $f'(x) = \dfrac{(x+2)3 - 3x(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$g'(x) = \dfrac{(x+2)5 - (5x+4)(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$g(x) = \dfrac{5x+4}{(x+2)} = \dfrac{3x}{(x+2)} + \dfrac{2x+4}{(x+2)} = f(x) + 2$

f and g differ by a constant.

80. $f'(x) = \dfrac{x(\cos x - 3) - (\sin x - 3x)(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$g'(x) = \dfrac{x(\cos x + 2) - (\sin x + 2x)(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$g(x) = \dfrac{\sin x + 2x}{x} = \dfrac{\sin x - 3x + 5x}{x} = f(x) + 5$

f and g differ by a constant.

81. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$p'(1) = f'(1)g(1) + f(1)g'(1) = 1(4) + 6\left(-\dfrac{1}{2}\right) = 1$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$q'(4) = \dfrac{3(-1) - 7(0)}{3^2} = -\dfrac{1}{3}$

82. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$p'(4) = \dfrac{1}{2}(8) + 1(0) = 4$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$q'(7) = \dfrac{4(2) - 4(-1)}{4^2} = \dfrac{12}{16} = \dfrac{3}{4}$

83. Area $= A(t) = (2t+1)\sqrt{t} = 2t^{3/2} + t^{1/2}$

$A'(t) = 2\left(\dfrac{3}{2}t^{1/2}\right) + \dfrac{1}{2}t^{-1/2}$

$= 3t^{1/2} + \dfrac{1}{2}t^{-1/2}$

$= \dfrac{6t+1}{2\sqrt{t}}$ cm²/sec

84. $V = \pi r^2 h = \pi(t+2)\left(\dfrac{1}{2}\sqrt{t}\right)$

$= \dfrac{1}{2}(t^{3/2} + 2t^{1/2})\pi$

$V'(t) = \dfrac{1}{2}\left(\dfrac{3}{2}t^{1/2} + t^{-1/2}\right)\pi = \dfrac{3t+2}{4t^{1/2}}\pi$ cubic inches/sec

85. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x+30}\right),\ 1 \le x$

$\dfrac{dC}{dx} = 100\left(-\dfrac{400}{x^3} + \dfrac{30}{(x+30)^2}\right)$

(a) When $x = 10$: $\dfrac{dC}{dx} = -\$38.13$

(b) When $x = 15$: $\dfrac{dC}{dx} = -\$10.37$

(c) When $x = 20$: $\dfrac{dC}{dx} = -\$3.80$

As the order size increases, the cost per item decreases.

86. $P = \dfrac{k}{V}$

$\dfrac{dP}{dV} = -\dfrac{k}{V^2}$

87. $P(t) = 500\left[1 + \dfrac{4t}{50+t^2}\right]$

$P'(t) = 500\left[\dfrac{(50+t^2)(4) - (4t)(2t)}{(50+t^2)^2}\right]$

$= 500\left[\dfrac{200 - 4t^2}{(50+t^2)^2}\right]$

$= 2000\left[\dfrac{50 - t^2}{(50+t^2)^2}\right]$

$P'(2) \approx 31.55$ bacteria per hour

88. $F = \dfrac{Gm_1m_2}{d^2} = Gm_1m_2d^{-2}$

$\dfrac{dF}{dd} = F'(d) = \dfrac{-2\,Gm_1m_2}{d^3}$

89. (a) $\sec x = \dfrac{1}{\cos x}$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{(\cos x)(0) - (1)(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos x \cos x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$$

(b) $\csc x = \dfrac{1}{\sin x}$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2} = -\frac{\cos x}{\sin x \sin x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cot x$$

(c) $\cot x = \dfrac{\cos x}{\sin x}$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

90. $f(x) = \sec x$

$g(x) = \csc x, \quad [0, 2\pi)$

$f'(x) = g'(x)$

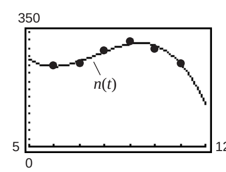
$$\sec x \tan x = -\csc x \cot x \Longrightarrow \frac{\sec x \tan x}{\csc x \cot x} = -1 \Longrightarrow \frac{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}}{\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}} = -1 \Longrightarrow \frac{\sin^3 x}{\cos^3 x} = -1 \Longrightarrow \tan^3 x = -1 \Longrightarrow \tan x = -1$$

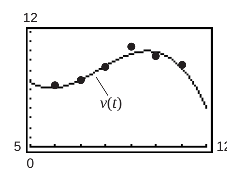
$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

91. (a) $n(t) = -3.5806t^3 + 82.577t^2 - 603.60t + 1667.5$

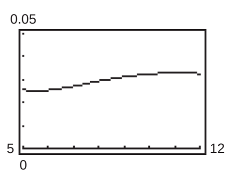
$v(t) = -0.1361t^3 + 3.165t^2 - 23.02t + 59.8$

(b)





(c) $A = \dfrac{v(t)}{n(t)} = \dfrac{-0.1361t^3 + 3.165t^2 - 23.02t + 59.8}{-3.5806t^3 + 82.577t^2 - 603.60t + 1667.5}$



A represents the average retail value (in millions of dollars) per 1000 motor homes.

(d) $A'(t)$ represents the rate of change of the average retail value per 1000 motor homes.

92. (a) $\sin \theta = \dfrac{r}{r + h}$

$r + h = r \csc \theta$

$h = r \csc \theta - r = r(\csc \theta - 1)$

(b) $h'(\theta) = r(-\csc \theta \cdot \cot \theta)$

$$h'(30°) = h'\left(\frac{\pi}{6}\right) = -3960\left(2 \cdot \sqrt{3}\right) = -7920\sqrt{3} \text{ mi/radian}$$

93. $f(x) = 4x^{3/2}$

$f'(x) = 6x^{1/2}$

$f''(x) = 3x^{-1/2} = \dfrac{3}{\sqrt{x}}$

94. $f(x) = x + \dfrac{32}{x^2}$

$f'(x) = 1 - \dfrac{64}{x^3}$

$f''(x) = \dfrac{192}{x^4}$

95. $f(x) = \dfrac{x}{x-1}$

$f'(x) = \dfrac{(x-1)(1) - x(1)}{(x-1)^2} = \dfrac{-1}{(x-1)^2}$

$f''(x) = \dfrac{2}{(x-1)^3}$

96. $f(x) = \dfrac{x^2 + 2x - 1}{x} = x + 2 - \dfrac{1}{x}$

$f'(x) = 1 + \dfrac{1}{x^2}$

$f''(x) = -\dfrac{2}{x^3}$

97. $f(x) = 3 \sin x$

$f'(x) = 3 \cos x$

$f''(x) = -3 \sin x$

98. $f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$

$= \sec x(\sec^2 x + \tan^2 x)$

99. $f'(x) = x^2$

$f''(x) = 2x$

100. $f''(x) = 2 - 2x^{-1}$

$f'''(x) = 2x^{-2} = \dfrac{2}{x^2}$

101. $f'''(x) = 2\sqrt{x}$

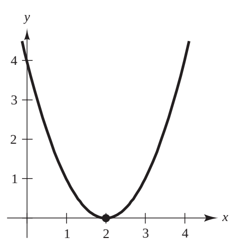
$f^{(4)}(x) = \dfrac{1}{2}(2)x^{-1/2} = \dfrac{1}{\sqrt{x}}$

102. $f^{(4)}(x) = 2x + 1$

$f^{(5)}(x) = 2$

$f^{(6)}(x) = 0$

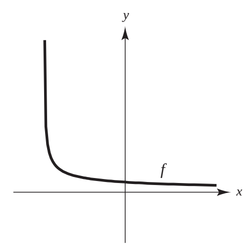
103.



$f(2) = 0$

One such function is $f(x) = (x - 2)^2$.

104. The graph of a differentiable function f such that $f > 0$ and $f' < 0$ for all real numbers x would, in general, look like the graph below.



105. $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2g'(2) + h'(2)$

$= 2(-2) + 4$

$= 0$

106. $f(x) = 4 - h(x)$

$f'(x) = -h'(x)$

$f'(2) = -h'(2) = -4$

107. $f(x) = \dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$

$f'(2) = \dfrac{h(2)g'(2) - g(2)h'(2)}{[h(2)]^2}$

$= \dfrac{(-1)(-2) - (3)(4)}{(-1)^2}$

$= -10$

108. $f(x) = g(x)h(x)$

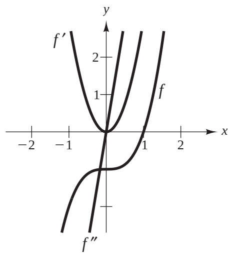
$f'(x) = g(x)h'(x) + h(x)g'(x)$

$f'(2) = g(2)h'(2) + h(2)g'(2)$

$= (3)(4) + (-1)(-2)$

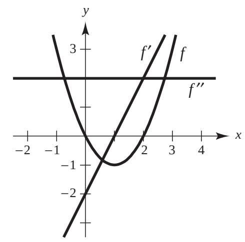
$= 14$

109.



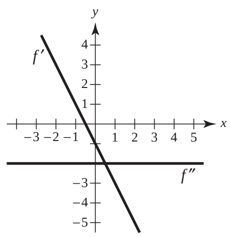
It appears that f is cubic; so f' would be quadratic and f'' would be linear.

110.

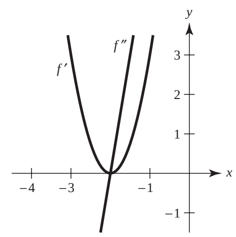


It appears that f is quadratic; so f' would be linear and f'' would be constant.

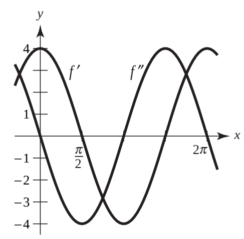
111.



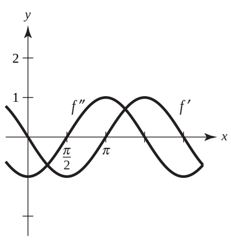
112.



113.



114.



115. $v(t) = 36 - t^2,\ 0 \le t \le 6$

$a(t) = -2t$

$v(3) = 27$ m/sec

$a(3) = -6$ m/sec^2

The speed of the object is decreasing.

116. $v(t) = \dfrac{100t}{2t + 15}$

$$a(t) = \frac{(2t + 15)(100) - (100t)(2)}{(2t + 15)^2}$$

$$= \frac{1500}{(2t + 15)^2}$$

(a) $a(5) = \dfrac{1500}{[2(5) + 15]^2} = 2.4$ ft/sec^2

(b) $a(10) = \dfrac{1500}{[2(10) + 15]^2} \approx 1.2$ ft/sec^2

(c) $a(20) = \dfrac{1500}{[2(20) + 15]^2} \approx 0.5$ ft/sec^2

117. $s(t) = -8.25t^2 + 66t$

$v(t) = -16.50t + 66$

$a(t) = -16.50$

t(sec)	0	1	2	3	4
$s(t)$ (ft)	0	57.75	99	123.75	132
$v(t) = s'(t)$ (ft/sec)	66	49.5	33	16.5	0
$a(t) = v'(t)$ (ft/sec^2)	−16.5	−16.5	−16.5	−16.5	−16.5

Average velocity on:

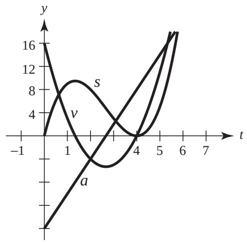
$[0, 1]$ is $\dfrac{57.75 - 0}{1 - 0} = 57.75.$

$[1, 2]$ is $\dfrac{99 - 57.75}{2 - 1} = 41.25.$

$[2, 3]$ is $\dfrac{123.75 - 99}{3 - 2} = 24.75.$

$[3, 4]$ is $\dfrac{132 - 123.75}{4 - 3} = 8.25.$

118. (a)



s position function
v velocity function
a acceleration function

(b) The particle speeds up (accelerates) when $a > 0$ and slows down when $a < 0$.

Answers will vary.

119. $f(x) = x^n$

$f^{(n)}(x) = n(n-1)(n-2)\cdots(2)(1) = n!$

Note: $n! = n(n-1)\cdots 3 \cdot 2 \cdot 1$ (read "n factorial")

120. $f(x) = \dfrac{1}{x}$

$$f^{(n)}(x) = \frac{(-1)^n(n)(n-1)(n-2)\cdots(2)(1)}{x^{n+1}} = \frac{(-1)^n n!}{x^{n+1}}$$

121. $f(x) = g(x)h(x)$

(a) $f'(x) = g(x)h'(x) + h(x)g'(x)$

$$f''(x) = g(x)h''(x) + g'(x)h'(x) + h(x)g''(x) + h'(x)g'(x)$$
$$= g(x)h''(x) + 2g'(x)h'(x) + h(x)g''(x)$$
$$f'''(x) = g(x)h'''(x) + g'(x)h''(x) + 2g'(x)h''(x) + 2g''(x)h'(x) + h(x)g'''(x) + h'(x)g''(x)$$
$$= g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)$$
$$f^{(4)}(x) = g(x)h^{(4)}(x) + g'(x)h'''(x) + 3g'(x)h'''(x) + 3g''(x)h''(x) + 3g''(x)h''(x) + 3g'''(x)h'(x) + g'''(x)h'(x) + g^{(4)}(x)h(x)$$
$$= g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)$$

(b)
$$f^{(n)}(x) = g(x)h^{(n)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{1[(n-1)(n-2)\cdots(2)(1)]}g'(x)h^{(n-1)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{(2)(1)[(n-2)(n-3)\cdots(2)(1)]}g''(x)h^{(n-2)}(x)$$
$$+ \frac{n(n-1)(n-2)\cdots(2)(1)}{(3)(2)(1)[(n-3)(n-4)\cdots(2)(1)]}g'''(x)h^{(n-3)}(x) + \cdots$$
$$+ \frac{n(n-1)(n-2)\cdots(2)(1)}{[(n-1)(n-2)\cdots(2)(1)](1)}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)$$
$$= g(x)h^{(n)}(x) + \frac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \frac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots$$
$$+ \frac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)$$

Note: $n! = n(n-1)\cdots 3 \cdot 2 \cdot 1$ (read "n factorial")

122. $[xf(x)]' = xf'(x) + f(x)$

$[xf(x)]'' = xf''(x) + f'(x) + f'(x) = xf''(x) + 2f'(x)$

$[xf(x)]''' = xf'''(x) + f''(x) + 2f''(x) = xf'''(x) + 3f''(x)$

In general, $[xf(x)]^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x)$.

123. $f(x) = x^n \sin x$

$f'(x) = x^n \cos x + nx^{n-1} \sin x$

$= x^{n-1}(x \cos x + n \sin x)$

When $n = 1$: $f'(x) = x \cos x + \sin x$

When $n = 2$: $f'(x) = x(x \cos x + 2 \sin x)$

When $n = 3$: $f'(x) = x^2(x \cos x + 3 \sin x)$

When $n = 4$: $f'(x) = x^3(x \cos x + 4 \sin x)$

For general n, $f'(x) = x^{n-1}(x \cos x + n \sin x)$.

124. $f(x) = \dfrac{\cos x}{x^n} = x^{-n} \cos x$

$f'(x) = -x^{-n} \sin x - nx^{-n-1} \cos x$

$= -x^{-n-1}(x \sin x + n \cos x)$

$= -\dfrac{x \sin x + n \cos x}{x^{n+1}}$

When $n = 1$: $f'(x) = -\dfrac{x \sin x + \cos x}{x^2}$

When $n = 2$: $f'(x) = -\dfrac{x \sin x + 2 \cos x}{x^3}$

When $n = 3$: $f'(x) = -\dfrac{x \sin x + 3 \cos x}{x^4}$

When $n = 4$: $f'(x) = -\dfrac{x \sin x + 4 \cos x}{x^5}$

For general n, $f'(x) = -\dfrac{x \sin x + n \cos x}{x^{n+1}}$.

125. $y = \dfrac{1}{x}, y' = -\dfrac{1}{x^2}, y'' = \dfrac{2}{x^3}$

$x^3y'' + 2x^2y' = x^3\left[\dfrac{2}{x^3}\right] + 2x^2\left[-\dfrac{1}{x^2}\right] = 2 - 2 = 0$

126.

$y = 2x^3 - 6x + 10$

$y' = 6x^2 - 6$

$y'' = 12x$

$y''' = 12$

$-y''' - xy'' - 2y' = -12 - x(12x) - 2(6x^2 - 6) = -24x^2$

127.

$y = 2 \sin x + 3$

$y' = 2 \cos x$

$y'' = -2 \sin x$

$y'' + y = -2 \sin x + (2 \sin x + 3) = 3$

128.

$y = 3 \cos x + \sin x$

$y' = -3 \sin x + \cos x$

$y'' = -3 \cos x - \sin x$

$y'' + y = (-3 \cos x - \sin x) + (3 \cos x + \sin x) = 0$

129. False. If $y = f(x)g(x)$, then

$\dfrac{dy}{dx} = f(x)g'(x) + g(x)f'(x)$.

130. True. y is a fourth-degree polynomial.

$\dfrac{d^ny}{dx^n} = 0$ when $n > 4$.

131. True

$h'(c) = f(c)g'(c) + g(c)f'(c)$

$= f(c)(0) + g(c)(0)$

$= 0$

132. True

133. True

134. True.

If $v(t) = c$ then $a(t) = v'(t) = 0$.

135. $f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

x-intercept at $(1, 0)$: $0 = a + b + c$

$(2, 7)$ on graph: $7 = 4a + 2b + c$

Slope 10 at $(2, 7)$: $10 = 4a + b$

Subtracting the third equation from the second, $-3 = b + c$. Subtracting this equation from the first, $3 = a$. Then, $10 = 4(3) + b \implies b = -2$. Finally, $-3 = (-2) + c \implies c = -1$.

$f(x) = 3x^2 - 2x - 1$

136. $f(x) = ax^3 + bx^2 + cx + d,\ a \neq 0$

$f'(x) = 3ax^2 + 2bx + c$

$f'(x) = 0 \Rightarrow x = \dfrac{-2b \pm \sqrt{4b^2 - 12ac}}{6a}$

(a) No horizontal tangents: $f'(x) \neq 0$

$4b^2 - 12ac < 0$

Example: $a = c = 1, b = 0$:

$f(x) = x^3 + x$

(b) Exactly one horizontal tangent

$4b^2 - 12ac = 0$

Example: $a = 1, b = 3, c = 3$:

$f(x) = x^3 + 3x^2 + 3x$

(c) Exactly two horizontal tangents

$4b^2 - 12ac > 0$

Example: $b = 1, a = 1, c = 0$:

$f(x) = x^3 + x^2$

137. $f(x) = x|x| = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$

$f'(x) = \left\{\begin{matrix} 2x, & \text{if } x \geq 0 \\ -2x, & \text{if } x < 0 \end{matrix}\right\} = 2|x|$

$f''(x) = \begin{cases} 2, & \text{if } x > 0 \\ -2, & \text{if } x < 0 \end{cases}$

$f''(0)$ does not exist since the left and right derivatives are not equal.

138. (a) $(fg' - f'g)' = fg'' + f'g' - f'g' - f''g$

$= fg'' - f''g$ True

(b) $(fg)'' = (fg' + f'g)'$

$= fg'' + f'g' + f'g' + f''g$

$= fg'' + 2f'g' + f''g$

$\neq fg'' + f''g$ False

Section 2.4 The Chain Rule

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (6x - 5)^4$	$u = 6x - 5$	$y = u^4$
2.	$y = \dfrac{1}{\sqrt{x + 1}}$	$u = x + 1$	$y = u^{-1/2}$
3.	$y = \sqrt{x^2 - 1}$	$u = x^2 - 1$	$y = \sqrt{u}$
4.	$y = 3\tan(\pi x^2)$	$u = \pi x^2$	$y = 3\tan u$
5.	$y = \csc^3 x$	$u = \csc x$	$y = u^3$
6.	$y = \cos\dfrac{3x}{2}$	$u = \dfrac{3x}{2}$	$y = \cos u$

7. $y = (2x - 7)^3$

$y' = 3(2x - 7)^2(2) = 6(2x - 7)^2$

8. $y = 3(4 - x^2)^5$

$y' = 15(4 - x^2)^4(-2x) = -30x(4 - x^2)^4$

9. $g(x) = 3(4 - 9x)^4$

$g'(x) = 12(4 - 9x)^3(-9) = -108(4 - 9x)^3$

10. $f(t) = (9t + 2)^{2/3}$

$f'(t) = \dfrac{2}{3}(9t + 2)^{-1/3}(9) = \dfrac{6}{\sqrt[3]{9t + 2}}$

11. $f(t) = (1 - t)^{1/2}$

$$f'(t) = \frac{1}{2}(1 - t)^{-1/2}(-1) = -\frac{1}{2\sqrt{1 - t}}$$

12. $g(x) = \sqrt{5 - 3x} = (5 - 3x)^{1/2}$

$$g'(x) = \frac{1}{2}(5 - 3x)^{-1/2}(-3) = \frac{-3}{2\sqrt{5 - 3x}}$$

13. $y = (9x^2 + 4)^{1/3}$

$$y' = \frac{1}{3}(9x^2 + 4)^{-2/3}(18x) = \frac{6x}{(9x^2 + 4)^{2/3}}$$

14. $g(x) = \sqrt{x^2 - 2x + 1} = \sqrt{(x - 1)^2} = |x - 1|$

$$g'(x) = \begin{cases} 1, & x > 1 \\ -1, & x < 1 \end{cases}$$

15. $y = 2(4 - x^2)^{1/4}$

$$y' = 2\left(\frac{1}{4}\right)(4 - x^2)^{-3/4}(-2x) = \frac{-x}{\sqrt[4]{(4 - x^2)^3}}$$

16. $f(x) = -3\sqrt[4]{2 - 9x}$

$$f(x) = -3(2 - 9x)^{1/4}$$

$$f'(x) = -\frac{3}{4}(2 - 9x)^{-3/4}(-9) = \frac{27}{4(2 - 9x)^{3/4}}$$

17. $y = (x - 2)^{-1}$

$$y' = -1(x - 2)^{-2}(1) = \frac{-1}{(x - 2)^2}$$

18. $s(t) = \dfrac{1}{t^2 + 3t - 1}$

$$s(t) = (t^2 + 3t - 1)^{-1}$$

$$s'(t) = -1(t^2 + 3t - 1)^{-2}(2t + 3)$$

$$= \frac{-(2t + 3)}{(t^2 + 3t - 1)^2}$$

19. $f(t) = (t - 3)^{-2}$

$$f'(t) = -2(t - 3)^{-3} = \frac{-2}{(t - 3)^3}$$

20. $y = -\dfrac{5}{(t + 3)^3}$

$$y = -5(t + 3)^{-3}$$

$$y' = 15(t + 3)^{-4} = \frac{15}{(t + 3)^4}$$

21. $y = (x + 2)^{-1/2}$

$$\frac{dy}{dx} = -\frac{1}{2}(x + 2)^{-3/2}$$

$$= -\frac{1}{2(x + 2)^{3/2}}$$

22. $g(t) = \sqrt{\dfrac{1}{t^2 - 2}}$

$$g(t) = (t^2 - 2)^{-1/2}$$

$$g'(t) = -\frac{1}{2}(t^2 - 2)^{-3/2}(2t)$$

$$= -\frac{t}{(t^2 - 2)^{3/2}}$$

23. $f(x) = x^2(x - 2)^4$

$$f'(x) = x^2[4(x - 2)^3(1)] + (x - 2)^4(2x)$$

$$= 2x(x - 2)^3[2x + (x - 2)]$$

$$= 2x(x - 2)^3(3x - 2)$$

24. $f(x) = x(3x - 9)^3$

$$f'(x) = x[3(3x - 9)^2(3)] + (3x - 9)^3(1)$$

$$= (3x - 9)^2[9x + 3x - 9]$$

$$= 27(x - 3)^2(4x - 3)$$

25. $y = x\sqrt{1 - x^2} = x(1 - x^2)^{1/2}$

$$y' = x\left[\frac{1}{2}(1 - x^2)^{-1/2}(-2x)\right] + (1 - x^2)^{1/2}(1)$$

$$= -x^2(1 - x^2)^{-1/2} + (1 - x^2)^{1/2}$$

$$= (1 - x^2)^{-1/2}[-x^2 + (1 - x^2)]$$

$$= \frac{1 - 2x^2}{\sqrt{1 - x^2}}$$

26. $y = \frac{1}{2}x^2\sqrt{16 - x^2}$

$$y' = \frac{1}{2}x^2\left(\frac{1}{2}(16 - x^2)^{-1/2}(-2x)\right) + x(16 - x^2)^{1/2}$$

$$= \frac{-x^3}{2\sqrt{16 - x^2}} + x\sqrt{16 - x^2}$$

$$= \frac{-x(3x^2 - 32)}{2\sqrt{16 - x^2}}$$

27. $y = \dfrac{x}{\sqrt{x^2+1}} = x(x^2+1)^{-1/2}$

$y' = x\left[-\dfrac{1}{2}(x^2+1)^{-3/2}(2x)\right] + (x^2+1)^{-1/2}(1)$

$= -x^2(x^2+1)^{-3/2} + (x^2+1)^{-1/2}$

$= (x^2+1)^{-3/2}[-x^2 + (x^2+1)]$

$= \dfrac{1}{(x^2+1)^{3/2}}$

28. $y = \dfrac{x}{\sqrt{x^4+4}}$

$y' = \dfrac{(x^4+4)^{1/2}(1) - x\frac{1}{2}(x^4+4)^{-1/2}(4x^3)}{x^4+4}$

$= \dfrac{x^4+4-2x^4}{(x^4+4)^{3/2}}$

$= \dfrac{4-x^4}{(x^4+4)^{3/2}}$

29. $g(x) = \left(\dfrac{x+5}{x^2+2}\right)^2$

$g'(x) = 2\left(\dfrac{x+5}{x^2+2}\right)\left(\dfrac{(x^2+2)-(x+5)(2x)}{(x^2+2)^2}\right)$

$= \dfrac{2(x+5)(2-10x-x^2)}{(x^2+2)^3}$

30. $h(t) = \left(\dfrac{t^2}{t^3+2}\right)^2$

$h'(t) = 2\left(\dfrac{t^2}{t^3+2}\right)\left(\dfrac{(t^3+2)(2t)-t^2(3t^2)}{(t^3+2)^2}\right)$

$= \dfrac{2t^2(4t-t^4)}{(t^3+2)^3} = \dfrac{2t^3(4-t^3)}{(t^3+2)^3}$

31. $f(v) = \left(\dfrac{1-2v}{1+v}\right)^3$

$f'(v) = 3\left(\dfrac{1-2v}{1+v}\right)^2\left(\dfrac{(1+v)(-2)-(1-2v)}{(1+v)^2}\right)$

$= \dfrac{-9(1-2v)^2}{(1+v)^4}$

32. $g(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^3$

$g'(x) = 3\left(\dfrac{3x^2-2}{2x+3}\right)^2\left(\dfrac{(2x+3)(6x)-(3x^2-2)(2)}{(2x+3)^2}\right)$

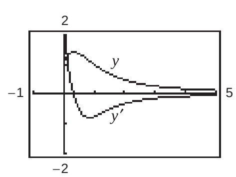
$= \dfrac{3(3x^2-2)^2(6x^2+18x+4)}{(2x+3)^4}$

$= \dfrac{6(3x^2-2)^2(3x^2+9x+2)}{(2x+3)^4}$

33. $y = \dfrac{\sqrt{x}+1}{x^2+1}$

$y' = \dfrac{1-3x^2-4x^{3/2}}{2\sqrt{x}(x^2+1)^2}$

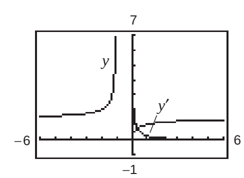
The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



34. $y = \sqrt{\dfrac{2x}{x+1}}$

$y' = \dfrac{1}{\sqrt{2x}(x+1)^{3/2}}$

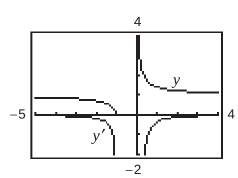
y' has no zeros.



35. $y = \sqrt{\dfrac{x+1}{x}}$

$y' = -\dfrac{\sqrt{(x+1)/x}}{2x(x+1)}$

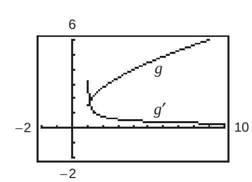
y' has no zeros.



36. $g(x) = \sqrt{x-1} + \sqrt{x+1}$

$g'(x) = \dfrac{1}{2\sqrt{x-1}} + \dfrac{1}{2\sqrt{x+1}}$

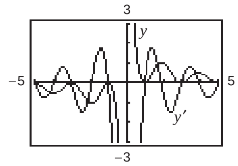
g' has no zeros.



37. $y = \dfrac{\cos \pi x + 1}{x}$

$\dfrac{dy}{dx} = \dfrac{-\pi x \sin \pi x - \cos \pi x - 1}{x^2}$

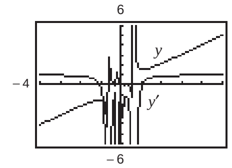
$= -\dfrac{\pi x \sin \pi x + \cos \pi x + 1}{x^2}$



The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.

38. $y = x^2 \tan \dfrac{1}{x}$

$\dfrac{dy}{dx} = 2x \tan \dfrac{1}{x} - \sec^2 \dfrac{1}{x}$



The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.

39. (a) $y = \sin x$

$y' = \cos x$

$y'(0) = 1$

1 cycle in $[0, 2\pi]$

(b) $y = \sin 2x$

$y' = 2 \cos 2x$

$y'(0) = 2$

2 cycles in $[0, 2\pi]$

The slope of $\sin ax$ at the origin is a.

40. (a) $y = \sin 3x$

$y' = 3 \cos 3x$

$y'(0) = 3$

3 cycles in $[0, 2\pi]$

(b) $y = \sin\left(\dfrac{x}{2}\right)$

$y' = \left(\dfrac{1}{2}\right)\cos\left(\dfrac{x}{2}\right)$

$y'(0) = \dfrac{1}{2}$

Half cycle in $[0, 2\pi]$

The slope of $\sin ax$ at the origin is a.

41. $y = \cos 3x$

$\dfrac{dy}{dx} = -3 \sin 3x$

42. $y = \sin \pi x$

$\dfrac{dy}{dx} = \pi \cos \pi x$

43. $g(x) = 3 \tan 4x$

$g'(x) = 12 \sec^2 4x$

44. $h(x) = \sec(x^2)$

$h'(x) = 2x \sec(x^2) \tan(x^2)$

45. $y = \sin(\pi x)^2 = \sin(\pi^2 x^2)$

$y' = \cos(\pi^2 x^2)[2\pi^2 x] = 2\pi^2 x \cos(\pi^2 x^2)$

46. $y = \cos(1 - 2x)^2 = \cos((1 - 2x)^2)$

$y' = -\sin(1 - 2x)^2(2(1 - 2x)(-2))$

$= 4(1 - 2x) \sin(1 - 2x)^2$

47. $h(x) = \sin 2x \cos 2x$

$h'(x) = \sin 2x(-2 \sin 2x) + \cos 2x(2 \cos 2x)$

$= 2 \cos^2 2x - 2 \sin^2 2x$

$= 2 \cos 4x$

Alternate solution: $h(x) = \frac{1}{2} \sin 4x$

$h'(x) = \frac{1}{2} \cos 4x(4) = 2 \cos 4x$

48. $g(\theta) = \sec\left(\frac{1}{2}\theta\right) \tan\left(\frac{1}{2}\theta\right)$

$g'(\theta) = \sec\left(\frac{1}{2}\theta\right) \sec^2\left(\frac{1}{2}\theta\right)\frac{1}{2} + \tan\left(\frac{1}{2}\theta\right) \sec\left(\frac{1}{2}\theta\right) \tan\left(\frac{1}{2}\theta\right)\frac{1}{2}$

$= \frac{1}{2} \sec\left(\frac{1}{2}\theta\right)\left[\sec^2\left(\frac{1}{2}\theta\right) + \tan^2\left(\frac{1}{2}\theta\right)\right]$

49. $f(x) = \dfrac{\cot x}{\sin x} = \dfrac{\cos x}{\sin^2 x}$

$f'(x) = \dfrac{\sin^2 x(-\sin x) - \cos x(2 \sin x \cos x)}{\sin^4 x}$

$= \dfrac{-\sin^2 x - 2 \cos^2 x}{\sin^3 x} = \dfrac{-1 - \cos^2 x}{\sin^3 x}$

50. $g(v) = \dfrac{\cos v}{\csc v} = \cos v \cdot \sin v$

$g'(v) = \cos v(\cos v) + \sin v(-\sin v)$

$= \cos^2 v - \sin^2 v = \cos 2v$

51. $y = 4 \sec^2 x$

$y' = 8 \sec x \cdot \sec x \tan x$

$= 8 \sec^2 x \tan x$

52. $g(t) = 5 \cos^2 \pi t = 5(\cos \pi t)^2$

$g'(t) = 10 \cos \pi t(-\sin \pi t)(\pi)$

$= -10\pi(\sin \pi t)(\cos \pi t)$

$= -5\pi \sin 2\pi t$

53. $f(\theta) = \frac{1}{4} \sin^2 2\theta = \frac{1}{4}(\sin 2\theta)^2$

$f'(\theta) = 2\left(\frac{1}{4}\right)(\sin 2\theta)(\cos 2\theta)(2)$

$= \sin 2\theta \cos 2\theta = \frac{1}{2} \sin 4\theta$

54. $h(t) = 2\cot^2(\pi t + 2)$

$h'(t) = 4\cot(\pi t + 2)(-\csc^2(\pi t + 2)(\pi))$

$= -4\pi\cot(\pi t + 2)\csc^2(\pi t + 2)$

55. $f(t) = 3\sec^2(\pi t - 1)$

$f'(t) = 6\sec(\pi t - 1)\sec(\pi t - 1)\tan(\pi t - 1)(\pi)$

$= 6\pi\sec^2(\pi t - 1)\tan(\pi t - 1) = \dfrac{6\pi\sin(\pi t - 1)}{\cos^3(\pi t - 1)}$

56. $y = 3x - 5\cos(\pi x)^2$

$= 3x - 5\cos(\pi^2 x^2)$

$\dfrac{dy}{dx} = 3 + 5\sin(\pi^2 x^2)(2\pi^2 x)$

$= 3 + 10\pi^2 x\sin(\pi x)^2$

57. $y = \sqrt{x} + \dfrac{1}{4}\sin(2x)^2$

$= \sqrt{x} + \dfrac{1}{4}\sin(4x^2)$

$\dfrac{dy}{dx} = \dfrac{1}{2}x^{-1/2} + \dfrac{1}{4}\cos(4x^2)(8x)$

$= \dfrac{1}{2\sqrt{x}} + 2x\cos(2x)^2$

58. $y = \sin x^{1/3} + (\sin x)^{1/3}$

$y' = \cos x^{1/3}\left(\dfrac{1}{3}x^{-2/3}\right) + \dfrac{1}{3}(\sin x)^{-2/3}\cos x$

$= \dfrac{1}{3}\left[\dfrac{\cos x^{1/3}}{x^{2/3}} + \dfrac{\cos x}{(\sin x)^{2/3}}\right]$

59. $s(t) = (t^2 + 2t + 8)^{1/2}, \quad (2, 4)$

$s'(t) = \dfrac{1}{2}(t^2 + 2t + 8)^{-1/2}(2t + 2)$

$= \dfrac{t + 1}{\sqrt{t^2 + 2t + 8}}$

$s'(2) = \dfrac{3}{4}$

60. $y = (3x^3 + 4x)^{1/5}, \quad (2, 2)$

$y' = \dfrac{1}{5}(3x^3 + 4x)^{-4/5}(9x^2 + 4)$

$= \dfrac{9x^2 + 4}{5(3x^3 + 4x)^{4/5}}$

$y'(2) = \dfrac{1}{2}$

61. $f(x) = \dfrac{3}{x^3 - 4} = 3(x^3 - 4)^{-1}, \quad \left(-1, -\dfrac{3}{5}\right)$

$f'(x) = -3(x^3 - 4)^{-2}(3x^2) = -\dfrac{9x^2}{(x^3 - 4)^2}$

$f'(-1) = -\dfrac{9}{25}$

62. $f(x) = \dfrac{1}{(x^2 - 3x)^2} = (x^2 - 3x)^{-2}, \quad \left(4, \dfrac{1}{16}\right)$

$f'(x) = -2(x^2 - 3x)^{-3}(2x - 3) = \dfrac{-2(2x - 3)}{(x^2 - 3x)^3}$

$f'(4) = -\dfrac{5}{32}$

63. $f(t) = \dfrac{3t + 2}{t - 1}, \quad (0, -2)$

$f'(t) = \dfrac{(t - 1)(3) - (3t + 2)(1)}{(t - 1)^2} = \dfrac{-5}{(t - 1)^2}$

$f'(0) = -5$

64. $f(x) = \dfrac{x + 1}{2x - 3}, \quad (2, 3)$

$f'(x) = \dfrac{(2x - 3)(1) - (x + 1)(2)}{(2x - 3)^2} = \dfrac{-5}{(2x - 3)^2}$

$f'(2) = -5$

65. $y = 37 - \sec^3(2x), \quad (0, 36)$

$y' = -3\sec^2(2x)[2\sec(2x)\tan(2x)]$

$= -6\sec^3(2x)\tan(2x)$

$y'(0) = 0$

66. $y = \frac{1}{x} + \sqrt{\cos x}, \quad \left(\frac{\pi}{2}, \frac{2}{\pi}\right)$

$y' = -\frac{1}{x^2} - \frac{\sin x}{2\sqrt{\cos x}}$

$y'(\pi/2)$ is undefined.

67. (a) $f(x) = \sqrt{3x^2 - 2}, \quad (3, 5)$

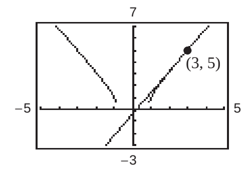
$f'(x) = \frac{1}{2}(3x^2 - 2)^{-1/2}(6x) = \frac{3x}{\sqrt{3x^2 - 2}}$

$f'(3) = \frac{9}{5}$

Tangent line:

$y - 5 = \frac{9}{5}(x - 3) \Longrightarrow 9x - 5y - 2 = 0$

(b)



68. (a) $f(x) = \frac{1}{3}x\sqrt{x^2 + 5}, \quad (2, 2)$

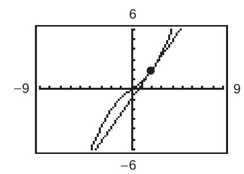
$f'(x) = \frac{1}{3}x\left[\frac{1}{2}(x^2 + 5)^{-1/2}(2x)\right] + \frac{1}{3}(x^2 + 5)^{1/2}$

$= \frac{x^2}{3\sqrt{x^2 + 5}} + \frac{1}{3}\sqrt{x^2 + 5}$

$f'(2) = \frac{4}{3(3)} + \frac{1}{3}(3) = \frac{13}{9}$

Tangent line: $y - 2 = \frac{13}{9}(x - 2) \Longrightarrow 13x - 9y - 8 = 0$

(b)



69. (a) $y = (2x^3 + 1)^2, \quad (-1, 1)$

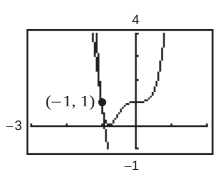
$y' = 2(2x^3 + 1)(6x^2) = 12x^2(2x^3 + 1)$

$y'(-1) = 12(-1) = -12$

Tangent line: $y - 1 = -12(x + 1)$

$y = -12x - 11$

(b)



70. (a) $f(x) = (9 - x^2)^{2/3}, \quad (1, 4)$

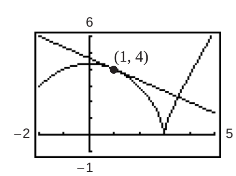
$f'(x) = \frac{2}{3}(9 - x^2)^{-1/3}(-2x) = \frac{-4x}{3(9 - x^2)^{1/3}}$

$f'(1) = \frac{-4}{3(8)^{1/3}} = -\frac{2}{3}$

Tangent line: $y - 4 = -\frac{2}{3}(x - 1)$

$y = -\frac{2}{3}x + \frac{14}{3}$

(b)



71. (a) $f(x) = \sin 2x, \quad (\pi, 0)$

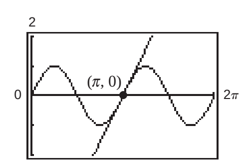
$f'(x) = 2\cos 2x$

$f'(\pi) = 2$

Tangent line:

$y = 2(x - \pi) \Longrightarrow 2x - y - 2\pi = 0$

(b)



72. (a) $y = \cos 3x, \quad \left(\frac{\pi}{4}, -\frac{\sqrt{2}}{2}\right)$

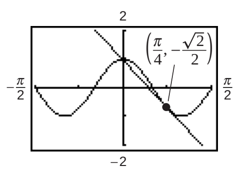
$y' = -3 \sin 3x$

$y'\left(\frac{\pi}{4}\right) = -3 \sin\left(\frac{3\pi}{4}\right) = \frac{-3\sqrt{2}}{2}$

Tangent line: $y + \frac{\sqrt{2}}{2} = \frac{-3\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$y = \frac{-3\sqrt{2}}{2}x + \frac{3\sqrt{2}\pi}{8} - \frac{\sqrt{2}}{2}$

(b)



73. (a) $f(x) = \tan^2 x, \quad \left(\frac{\pi}{4}, 1\right)$

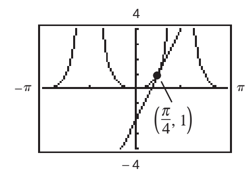
$f'(x) = 2 \tan x \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2(1)(2) = 4$

Tangent line:

$y - 1 = 4\left(x - \frac{\pi}{4}\right) \Rightarrow 4x - y + (1 - \pi) = 0$

(b)



74. (a) $y = 2 \tan^3 x, \quad \left(\frac{\pi}{4}, 2\right)$

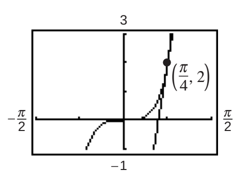
$y' = 6 \tan^2 x \cdot \sec^2 x$

$y'\left(\frac{\pi}{4}\right) = 6(1)(2) = 12$

Tangent line: $y - 2 = 12\left(x - \frac{\pi}{4}\right)$

$y = 12x + 2 - 3\pi$

(b)



75. (a) $g(t) = \frac{3t^2}{\sqrt{t^2 + 2t - 1}}, \quad \left(\frac{1}{2}, \frac{3}{2}\right)$

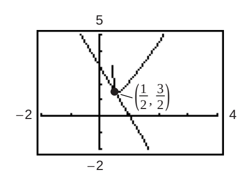
$g'(t) = \frac{3t(t^2 + 3t - 2)}{(t^2 + 2t - 1)^{3/2}}$

$g'\left(\frac{1}{2}\right) = -3$

(b) $y - \frac{3}{2} = -3\left(x - \frac{1}{2}\right)$

$y = -3x + 3$

(c)



76. (a) $f(x) = \sqrt{x}(2 - x)^2, \quad (4, 8)$

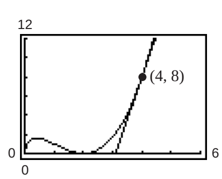
$f'(x) = \frac{(x - 2)(5x - 2)}{2\sqrt{x}}$

$f'(4) = 9$

(b) $y - 8 = 9(x - 4)$

$y = 9x - 28$

(c)



77. (a) $s(t) = \frac{(4 - 2t)\sqrt{1 + t}}{3}, \quad \left(0, \frac{4}{3}\right)$

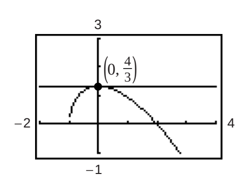
$s'(t) = \frac{-2\sqrt{1 + t}}{3} + \frac{2 - t}{3\sqrt{1 + t}}$

$s'(0) = 0$

(b) $y - \frac{4}{3} = 0(x - 0)$

$y = \frac{4}{3}$

(c)



78. (a) $y = (t^2 - 9)\sqrt{t+2}, \quad (2, -10)$

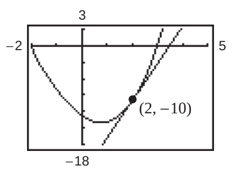
$$y' = \frac{5t^2 + 8t - 9}{2\sqrt{t+2}}$$

$$y'(2) = \frac{27}{4}$$

(b) $y + 10 = \frac{27}{4}(t - 2)$

$$y = \frac{27}{4}t - \frac{47}{2}$$

(c)



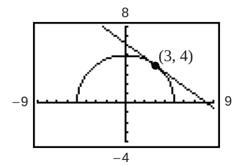
79. $f(x) = \sqrt{25 - x^2}, \quad (3, 4)$

$$f'(x) = \frac{-x}{\sqrt{25 - x^2}}$$

$$f'(3) = -\frac{3}{4}$$

$$y - 4 = -\frac{3}{4}(x - 3)$$

$$y = -\frac{3}{4}x + \frac{25}{4}; \quad \text{Tangent line}$$



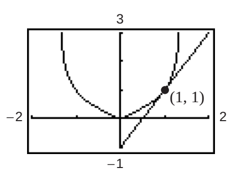
80. $f(x) = \frac{|x|}{\sqrt{2 - x^2}}, \quad (1, 1)$

$$f'(x) = \frac{2}{(2 - x^2)^{3/2}} \quad \text{for } x > 0$$

$$f'(1) = 2$$

$$y - 1 = 2(x - 1)$$

$$y = 2x - 1; \quad \text{Tangent line}$$



81. $f(x) = 2\cos x + \sin 2x, \quad 0 < x < 2\pi$

$$f'(x) = -2\sin x + 2\cos 2x$$

$$= -2\sin x + 2 - 4\sin^2 x = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(\sin x + 1)(2\sin x - 1) = 0$$

$$\sin x = -1 \implies x = \frac{3\pi}{2}$$

$$\sin x = \frac{1}{2} \implies x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Horizontal tangents at $x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{5\pi}{6}$

Horizontal tangent at the points $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right), \left(\frac{3\pi}{2}, 0\right)$, and $\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right)$

82. $f(x) = \frac{x}{\sqrt{2x - 1}}$

$$f'(x) = \frac{(2x - 1)^{1/2} - x(2x - 1)^{-1/2}}{2x - 1}$$

$$= \frac{2x - 1 - x}{(2x - 1)^{3/2}}$$

$$= \frac{x - 1}{(2x - 1)^{3/2}}$$

$$\frac{x - 1}{(2x - 1)^{3/2}} = 0 \implies x = 1$$

Horizontal tangent at $(1, 1)$

83. $f(x) = 2(x^2 - 1)^3$

$$f'(x) = 6(x^2 - 1)^2(2x)$$

$$= 12x(x^4 - 2x^2 + 1)$$

$$= 12x^5 - 24x^3 + 12x$$

$$f''(x) = 60x^4 - 72x^2 + 12$$

$$= 12(5x^2 - 1)(x^2 - 1)$$

84. $f(x) = (x - 2)^{-1}$

$f'(x) = -(x - 2)^{-2} = \dfrac{-1}{(x - 2)^2}$

$f''(x) = 2(x - 2)^{-3} = \dfrac{2}{(x - 2)^3}$

85. $f(x) = \sin x^2$

$f'(x) = 2x \cos x^2$

$f''(x) = 2x[2x(-\sin x^2)] + 2 \cos x^2$

$= 2[\cos x^2 - 2x^2 \sin x^2]$

86. $f(x) = \sec^2 \pi x$

$f'(x) = 2 \sec \pi x(\pi \sec \pi x \tan \pi x)$

$= 2\pi \sec^2 \pi x \tan \pi x$

$f''(x) = 2\pi \sec^2 \pi x(\sec^2 \pi x)(\pi) + 2\pi \tan \pi x(2\pi \sec^2 \pi x \tan \pi x)$

$= 2\pi^2 \sec^4 \pi x + 4\pi^2 \sec^2 \pi x \tan^2 \pi x$

$= 2\pi^2 \sec^2 \pi x(\sec^2 \pi x + 2 \tan^2 \pi x)$

$= 2\pi^2 \sec^2 \pi x(3 \sec^2 \pi x - 2)$

87. $h(x) = \dfrac{1}{9}(3x + 1)^3, \quad \left(1, \dfrac{64}{9}\right)$

$h'(x) = \dfrac{1}{9}3(3x + 1)^2(3) = (3x + 1)^2$

$h''(x) = 2(3x + 1)(3) = 6(3x + 1)$

$h''(1) = 24$

88. $f(x) = \dfrac{1}{\sqrt{x + 4}} = (x + 4)^{-1/2}, \quad \left(0, \dfrac{1}{2}\right)$

$f'(x) = -\dfrac{1}{2}(x + 4)^{-3/2}$

$f''(x) = \dfrac{3}{4}(x + 4)^{-5/2} = \dfrac{3}{4(x + 4)^{5/2}}$

$f''(0) = \dfrac{3}{128}$

89. $f(x) = \cos(x^2), \quad (0, 1)$

$f'(x) = -\sin(x^2)(2x) = -2x \sin(x^2)$

$f''(x) = -2x \cos(x^2)(2x) - 2 \sin(x^2)$

$= -4x^2 \cos(x^2) - 2 \sin(x^2)$

$f''(0) = 0$

90. $g(t) = \tan(2t), \quad \left(\dfrac{\pi}{6}, \sqrt{3}\right)$

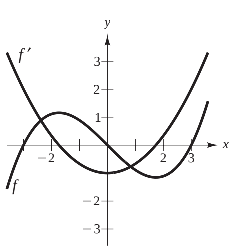
$g'(t) = 2 \sec^2(2t)$

$g''(t) = 4 \sec(2t) \cdot \sec(2t) \tan(2t)2$

$= 8 \sec^2(2t) \tan(2t)$

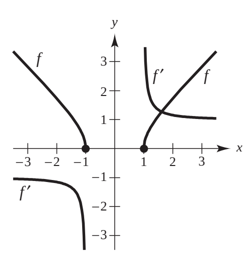
$g''\left(\dfrac{\pi}{6}\right) = 32\sqrt{3}$

91.



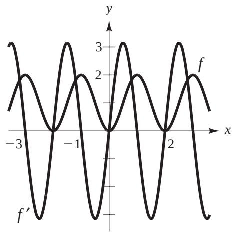
The zeros of f' correspond to the points where the graph of f has horizontal tangents.

92.



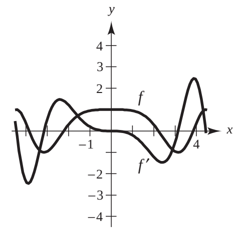
f is decreasing on $(-\infty, -1)$ so f' must be negative there.
f is increasing on $(1, \infty)$ so f' must be positive there.

93.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

94.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

95. $g(x) = f(3x)$

$g'(x) = f'(3x)(3) \implies g'(x) = 3f'(3x)$

96. $g(x) = f(x^2)$

$g'(x) = f'(x^2)(2x) \implies g'(x) = 2xf'(x^2)$

97. (a) $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + g'(x)h(x)$

$f'(5) = (-3)(-2) + (6)(3) = 24$

(b) $f(x) = g(h(x))$

$f'(x) = g'(h(x))h'(x)$

$f'(5) = g'(3)(-2) = -2g'(3)$

Need $g'(3)$ to find $f'(5)$.

(c) $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

$$f'(5) = \frac{(3)(6) - (-3)(-2)}{(3)^2} = \frac{12}{9} = \frac{4}{3}$$

(d) $f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2 g'(x)$

$f'(5) = 3(-3)^2(6) = 162$

98. (a) $g(x) = f(x) - 2 \implies g'(x) = f'(x)$

(b) $h(x) = 2f(x) \implies h'(x) = 2f'(x)$

(c) $r(x) = f(-3x) \implies r'(x) = f'(-3x)(-3) = -3f'(-3x)$

Hence, you need to know $f'(-3x)$.

$r'(0) = -3f'(0) = (-3)\left(-\frac{1}{3}\right) = 1$

$r'(-1) = -3f'(3) = (-3)(-4) = 12$

(d) $s(x) = f(x + 2) \implies s'(x) = f'(x + 2)$

Hence, you need to know $f'(x + 2)$.

$s'(-2) = f'(0) = -\frac{1}{3}$, etc.

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	-2	-4	-8
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$	-1	-2	-4		

99. (a) $h(x) = f(g(x)),\ g(1) = 4,\ g'(1) = -\frac{1}{2},\ f'(4) = -1,\ h'(x) = f'(g(x))g'(x)$

$h'(1) = f'(g(1))g'(1)$

$= f'(4)g'(1)$

$= (-1)\left(-\frac{1}{2}\right) = \frac{1}{2}$

(b) $s(x) = g(f(x)),\ f(5) = 6,\ f'(5) = -1$, $g'(6)$ does not exist.

$s'(x) = g'(f(x))f'(x)$

$s'(5) = g'(f(5))f'(5) = g'(6)(-1)$

Since $g'(6)$ does not exist, $s'(5)$ is not defined.

100. (a) $h(x) = f(g(x))$

$h'(x) = f'(g(x))g'(x)$

$h'(3) = f'(g(3))g'(3)$

$= f'(5)(1)$

$= \frac{1}{2}$

(b) $s(x) = g(f(x))$

$s'(x) = g'(f(x))f'(x)$

$s'(9) = g'(f(9))f'(9)$

$= g'(8)(2)$

$= (-1)(2)$

$= -2$

101. (a) $F = 132{,}400(331 - v)^{-1}$

$F' = (-1)(132{,}400)(331 - v)^{-2}(-1)$

$= \dfrac{132{,}400}{(331 - v)^2}$

When $v = 30$, $F' \approx 1.461$.

(b) $F = 132{,}400(331 + v)^{-1}$

$F' = (-1)(132{,}400)(331 + v)^{-2}(1)$

$= \dfrac{-132{,}400}{(331 + v)^2}$

When $v = 30$, $F' \approx -1.016$.

102. $y = \dfrac{1}{3}\cos 12t - \dfrac{1}{4}\sin 12t$

$v = y' = \dfrac{1}{3}[-12\sin 12t] - \dfrac{1}{4}[12\cos 12t]$

$= -4\sin 12t - 3\cos 12t$

When $t = \pi/8$, $y = 0.25$ feet and $v = 4$ feet per second.

103. $\theta = 0.2\cos 8t$

The maximum angular displacement is $\theta = 0.2$ (since $-1 \le \cos 8t \le 1$).

$\dfrac{d\theta}{dt} = 0.2[-8\sin 8\text{t}] = -1.6\sin 8t$

When $t = 3$, $d\theta/dt = -1.6\sin 24 \approx 1.4489$ radians per second.

104. $y = A\cos\omega t$

(a) Amplitude: $A = \dfrac{3.5}{2} = 1.75$

$y = 1.75\cos\omega t$

Period: $10 \Longrightarrow \omega = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$

$y = 1.75\cos\dfrac{\pi t}{5}$

(b) $v = y' = 1.75\left[-\dfrac{\pi}{5}\sin\dfrac{\pi t}{5}\right] = -0.35\pi\sin\dfrac{\pi t}{5}$

105. $S = C(R^2 - r^2)$

$\dfrac{dS}{dt} = C\left(2R\dfrac{dR}{dt} - 2r\dfrac{dr}{dt}\right)$

Since r is constant, we have $dr/dt = 0$ and

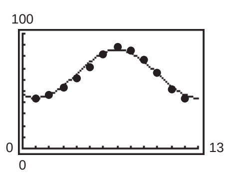
$\dfrac{dS}{dt} = (1.76 \times 10^5)(2)(1.2 \times 10^{-2})(10^{-5})$

$= 4.224 \times 10^{-2} = 0.04224.$

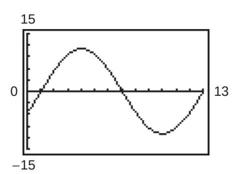
106. (a) Using a graphing utility, or trial and error, you obtain a model similar to

$T(t) = 65 + 21\sin\left(\dfrac{\pi t}{6} - 2.1\right).$

(b)



(c) $T'(t) \approx 11\cos\left(\dfrac{\pi t}{6} - 2.1\right)$



(d) The temperature changes most rapidly in the spring (April–June) and fall (September–November).

107. (a) $x = -1.6372t^3 + 19.3120t^2 - 0.5082t - 0.6162$

(b) $C = 60x + 1350$

$= 60(-1.6372t^3 + 19.3120t^2 - 0.5082t - 0.6162) + 1350$

$\frac{dC}{dt} = 60(-4.9116t^2 + 38.624t - 0.5082)$

$= -294.696t^2 + 2317.44t - 30.492$

The function dC/dt is quadratic, not linear. The cost function levels off at the end of the day, perhaps due to fatigue.

108. $f(x) = \sin \beta x$

(a) $f'(x) = \beta \cos \beta x$

$f''(x) = -\beta^2 \sin \beta x$

$f'''(x) = -\beta^3 \cos \beta x$

$f^{(4)} = \beta^4 \sin \beta x$

(b) $f''(x) + \beta^2 f(x) = -\beta^2 \sin \beta x + \beta^2(\sin \beta x) = 0$

(c) $f^{(2k)}(x) = (-1)^k \beta^{2k} \sin \beta x$

$f^{(2k-1)}(x) = (-1)^{k+1} \beta^{2k-1} \cos \beta x$

109. $f(x + p) = f(x)$ for all x.

(a) Yes, $f'(x + p) = f'(x)$, which shows that f' is periodic as well.

(b) Yes, let $g(x) = f(2x)$, so $g'(x) = 2f'(2x)$. Since f' is periodic, so is g'.

110. (a) $r'(x) = f'(g(x))g'(x)$

$r'(1) = f'(g(1))g'(1)$

Note that $g(1) = 4$ and $f'(4) = \frac{5 - 0}{6 - 2} = \frac{5}{4}$.

Also, $g'(1) = 0$. Thus, $r'(1) = 0$.

(b) $s'(x) = g'(f(x))f'(x)$

$s'(4) = g'(f(4))f'(4)$

Note that $f(4) = \frac{5}{2}$, $g'\left(\frac{5}{2}\right) = \frac{6 - 4}{6 - 2} = \frac{1}{2}$ and

$f'(4) = \frac{5}{4}$. Thus, $s'(4) = \frac{1}{2}\left(\frac{5}{4}\right) = \frac{5}{8}$.

111. (a) $g(x) = \sin^2 x + \cos^2 x = 1 \Longrightarrow g'(x) = 0$

$g'(x) = 2 \sin x \cos x + 2 \cos x(-\sin x) = 0$

(b) $\tan^2 x + 1 = \sec^2 x$

$g(x) + 1 = f(x)$

Taking derivatives of both sides, $g'(x) = f'(x)$. Equivalently, $f'(x) = 2 \sec x \cdot \sec x \cdot \tan x$ and $g'(x) = 2 \tan x \cdot \sec^2 x$, which are the same.

112. (a) If $f(-x) = -f(x)$, then

$\frac{d}{dx}[f(-x)] = \frac{d}{dx}[-f(x)]$

$f'(-x)(-1) = -f'(x)$

$f'(-x) = f'(x)$.

Thus, $f'(x)$ is even.

(b) If $f(-x) = f(x)$, then

$\frac{d}{dx}[f(-x)] = \frac{d}{dx}[f(x)]$

$f'(-x)(-1) = f'(x)$

$f'(-x) = -f'(x)$.

Thus, f' is odd.

113. $|u| = \sqrt{u^2}$

$\frac{d}{dx}[|u|] = \frac{d}{dx}\left[\sqrt{u^2}\right] = \frac{1}{2}(u^2)^{-1/2}(2uu')$

$= \frac{uu'}{\sqrt{u^2}} = u'\frac{u}{|u|}, \quad u \neq 0$

114. $g(x) = |2x - 3|$

$g'(x) = 2\left(\frac{2x - 3}{|2x - 3|}\right), \quad x \neq \frac{3}{2}$

115. $f(x) = |x^2 - 4|$

$$f'(x) = 2x\left(\frac{x^2-4}{|x^2-4|}\right), \quad x \neq \pm 2$$

116. $h(x) = |x| \cos x$

$$h'(x) = -|x| \sin x + \frac{x}{|x|} \cos x, \quad x \neq 0$$

117. $f(x) = |\sin x|$

$$f'(x) = \cos x\left(\frac{\sin x}{|\sin x|}\right), \quad x \neq k\pi$$

118. (a) $f(x) = \tan \dfrac{\pi x}{4}$ $\qquad f(1) = 1$

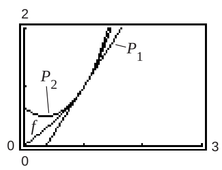
$$f'(x) = \frac{\pi}{4} \sec^2 \frac{\pi x}{4} \qquad f'(1) = \frac{\pi}{4}(2) = \frac{\pi}{2}$$

$$f''(x) = \frac{\pi}{2} \sec^2 \frac{\pi x}{4} \cdot \tan \frac{\pi x}{4}\left(\frac{\pi}{4}\right) \qquad f''(1) = \frac{\pi^2}{8}(2)(1) = \frac{\pi^2}{4}$$

$$P_1(x) = f'(1)(x-1) + f(1) = \frac{\pi}{2}(x-1) + 1$$

$$P_2(x) = \frac{1}{2}\left(\frac{\pi^2}{4}\right)(x-1)^2 + f'(1)(x-1) + f(1) = \frac{\pi^2}{8}(x-1)^2 + \frac{\pi}{2}(x-1) + 1$$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = c = 1$.

119. (a)

$$f(x) = \sec(2x)$$

$$f'(x) = 2(\sec 2x)(\tan 2x)$$

$$f''(x) = 2[2(\sec 2x)(\tan 2x)] \tan 2x + 2(\sec 2x)(\sec^2 2x)(2)$$

$$= 4[(\sec 2x)(\tan^2 2x) + \sec^3 2x]$$

$$f\left(\frac{\pi}{6}\right) = \sec\left(\frac{\pi}{3}\right) = 2$$

$$f'\left(\frac{\pi}{6}\right) = 2 \sec\left(\frac{\pi}{3}\right) \tan\left(\frac{\pi}{3}\right) = 4\sqrt{3}$$

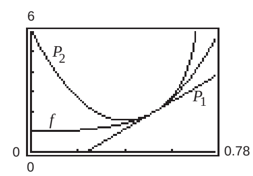
$$f''\left(\frac{\pi}{6}\right) = 4[2(3) + 2^3] = 56$$

$$P_1(x) = 4\sqrt{3}\left(x - \frac{\pi}{6}\right) + 2$$

$$P_2(x) = \frac{1}{2}(56)\left(x - \frac{\pi}{6}\right)^2 + 4\sqrt{3}\left(x - \frac{\pi}{6}\right) + 2$$

$$= 28\left(x - \frac{\pi}{6}\right)^2 + 4\sqrt{3}\left(x - \frac{\pi}{6}\right) + 2$$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/6$.

120. False. If $y = (1 - x)^{1/2}$, then $y' = \frac{1}{2}(1 - x)^{-1/2}(-1)$.

121. False. If $f(x) = \sin^2 2x$, then $f'(x) = 2(\sin 2x)(2 \cos 2x)$.

122. True

123.

$$\begin{aligned}
f(x) &= a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx \\
f'(x) &= a_1 \cos x + 2a_2 \cos 2x + \cdots + na_n \cos nx \\
f'(0) &= a_1 + 2a_2 + \cdots + na_n \\
|a_1 + 2a_2 + \cdots + na_n| &= |f'(0)| \\
&= \lim_{x\to 0}\left|\frac{f(x) - f(0)}{x - 0}\right| \\
&= \lim_{x\to 0}\left|\frac{f(x)}{\sin x}\right| \cdot \left|\frac{\sin x}{x}\right| \\
&= \lim_{x\to 0}\left|\frac{f(x)}{\sin x}\right| \le 1
\end{aligned}$$

124. $\dfrac{d}{dx}\left[\dfrac{P_n(x)}{(x^k - 1)^{n+1}}\right] = \dfrac{(x^k - 1)^{n+1}P_n'(x) - P_n(x)(n + 1)(x^k - 1)^n kx^{k-1}}{(x^k - 1)^{2n+2}}$

$$= \frac{(x^k - 1)P_n'(x) - (n + 1)kx^{k-1}P_n(x)}{(x^k - 1)^{n+2}}$$

$$P_n(x) = (x^k - 1)^{n+1}\frac{d^n}{dx^n}\left[\frac{1}{x^k - 1}\right] \Rightarrow$$

$$P_{n+1}(x) = (x^k - 1)^{n+2}\frac{d}{dx}\left[\frac{d^n}{dx^n}\left[\frac{1}{x^k - 1}\right]\right] = (x^k - 1)P_n'(x) - (n + 1)kx^{k-1}P_n(x)$$

$$P_{n+1}(1) = -(n + 1)kP_n(1)$$

For $n = 1$, $\dfrac{d}{dx}\left[\dfrac{1}{x^k - 1}\right] = \dfrac{-kx^{k-1}}{(x^k - 1)^2} = \dfrac{P_1(x)}{(x^k - 1)^2} \Rightarrow P_1(1) = -k$. Also, $P_0(1) = 1$.

We now use mathematical induction to verify that $P_n(1) = (-k)^n n!$ for $n \ge 0$. Assume true for n. Then

$$\begin{aligned}
P_{n+1}(1) &= -(n + 1)k\,P_n(1) \\
&= -(n + 1)k(-k)^n n! \\
&= (-k)^{n+1}(n + 1)!.
\end{aligned}$$

Section 2.5 Implicit Differentiation

1.
$$\begin{aligned}
x^2 + y^2 &= 36 \\
2x + 2yy' &= 0 \\
y' &= \frac{-x}{y}
\end{aligned}$$

2.
$$\begin{aligned}
x^2 - y^2 &= 16 \\
2x - 2yy' &= 0 \\
y' &= \frac{x}{y}
\end{aligned}$$

3.
$$\begin{aligned}
x^{1/2} + y^{1/2} &= 9 \\
\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' &= 0 \\
y' &= -\frac{x^{-1/2}}{y^{-1/2}} \\
&= -\sqrt{\frac{y}{x}}
\end{aligned}$$

4.
$$\begin{aligned}
x^3 + y^3 &= 8 \\
3x^2 + 3y^2y' &= 0 \\
y' &= -\frac{x^2}{y^2}
\end{aligned}$$

5.
$$\begin{aligned}
x^3 - xy + y^2 &= 4 \\
3x^2 - xy' - y + 2yy' &= 0 \\
(2y - x)y' &= y - 3x^2 \\
y' &= \frac{y - 3x^2}{2y - x}
\end{aligned}$$

6.
$$\begin{aligned} x^2y + y^2x &= -2 \\ x^2y' + 2xy + y^2 + 2yxy' &= 0 \\ (x^2 + 2xy)y' &= -(y^2 + 2xy) \\ y' &= \frac{-y(y+2x)}{x(x+2y)} \end{aligned}$$

7.
$$\begin{aligned} x^3y^3 - y - x &= 0 \\ 3x^3y^2y' + 3x^2y^3 - y' - 1 &= 0 \\ (3x^3y^2 - 1)y' &= 1 - 3x^2y^3 \\ y' &= \frac{1 - 3x^2y^3}{3x^3y^2 - 1} \end{aligned}$$

8.
$$\begin{aligned} (xy)^{1/2} - x + 2y &= 0 \\ \frac{1}{2}(xy)^{-1/2}(xy' + y) - 1 + 2y' &= 0 \\ \frac{x}{2\sqrt{xy}}y' + \frac{y}{2\sqrt{xy}} - 1 + 2y' &= 0 \\ xy' + y - 2\sqrt{xy} + 4\sqrt{xy}\,y' &= 0 \\ y' &= \frac{2\sqrt{xy} - y}{4\sqrt{xy} + x} \end{aligned}$$

9.
$$\begin{aligned} x^3 - 3x^2y + 2xy^2 &= 12 \\ 3x^2 - 3x^2y' - 6xy + 4xyy' + 2y^2 &= 0 \\ (4xy - 3x^2)y' &= 6xy - 3x^2 - 2y^2 \\ y' &= \frac{6xy - 3x^2 - 2y^2}{4xy - 3x^2} \end{aligned}$$

10.
$$\begin{aligned} 2\sin x \cos y &= 1 \\ 2[\sin x(-\sin y)y' + \cos y(\cos x)] &= 0 \\ y' &= \frac{\cos x \cos y}{\sin x \sin y} \\ &= \cot x \cot y \end{aligned}$$

11.
$$\begin{aligned} \sin x + 2\cos 2y &= 1 \\ \cos x - 4(\sin 2y)y' &= 0 \\ y' &= \frac{\cos x}{4\sin 2y} \end{aligned}$$

12.
$$\begin{aligned} (\sin \pi x + \cos \pi y)^2 &= 2 \\ 2(\sin \pi x + \cos \pi y)[\pi \cos \pi x - \pi(\sin \pi y)y'] &= 0 \\ \pi \cos \pi x - \pi(\sin \pi y)y' &= 0 \\ y' &= \frac{\cos \pi x}{\sin \pi y} \end{aligned}$$

13.
$$\begin{aligned} \sin x &= x(1 + \tan y) \\ \cos x &= x(\sec^2 y)y' + (1 + \tan y)(1) \\ y' &= \frac{\cos x - \tan y - 1}{x\sec^2 y} \end{aligned}$$

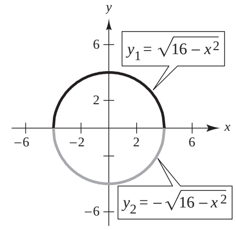
14.
$$\begin{aligned} \cot y &= x - y \\ (-\csc^2 y)y' &= 1 - y' \\ y' &= \frac{1}{1 - \csc^2 y} \\ &= \frac{1}{-\cot^2 y} = -\tan^2 y \end{aligned}$$

15.
$$\begin{aligned} y &= \sin(xy) \\ y' &= [xy' + y]\cos(xy) \\ y' - x\cos(xy)y' &= y\cos(xy) \\ y' &= \frac{y\cos(xy)}{1 - x\cos(xy)} \end{aligned}$$

16.
$$\begin{aligned} x &= \sec\frac{1}{y} \\ 1 &= -\frac{y'}{y^2}\sec\frac{1}{y}\tan\frac{1}{y} \\ y' &= \frac{-y^2}{\sec(1/y)\tan(1/y)} \\ &= -y^2\cos\left(\frac{1}{y}\right)\cot\left(\frac{1}{y}\right) \end{aligned}$$

17. (a)
$$\begin{aligned} x^2 + y^2 &= 16 \\ y^2 &= 16 - x^2 \\ y &= \pm\sqrt{16 - x^2} \end{aligned}$$

(b)



(c) Explicitly:
$$\begin{aligned} \frac{dy}{dx} &= \pm\frac{1}{2}(16 - x^2)^{-1/2}(-2x) \\ &= \frac{\mp x}{\sqrt{16 - x^2}} = \frac{-x}{\pm\sqrt{16 - x^2}} = \frac{-x}{y} \end{aligned}$$

(d) Implicitly: $2x + 2yy' = 0$
$$y' = -\frac{x}{y}$$

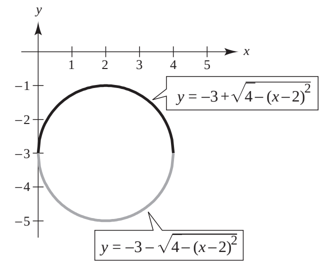
18. (a) $(x^2 - 4x + 4) + (y^2 + 6y + 9) = -9 + 4 + 9$

$$(x-2)^2 + (y+3)^2 = 4, \quad \text{Circle}$$

$$(y+3)^2 = 4 - (x-2)^2$$

$$y = -3 \pm \sqrt{4 - (x-2)^2}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \pm\frac{1}{2}[4 - (x-2)^2]^{-1/2}(-2)(x-2)$$

$$= \frac{\mp(x-2)}{\sqrt{4-(x-2)^2}}$$

$$= \frac{-(x-2)}{\pm\sqrt{4-(x-2)^2}}$$

$$= \frac{-(x-2)}{-3 \pm \sqrt{4-(x-2)^2} + 3}$$

$$= \frac{-(x-2)}{y+3}$$

(d) Implicitly:

$$2x + 2yy' - 4 + 6y' = 0$$

$$(2y + 6)y' = -2(x-2)$$

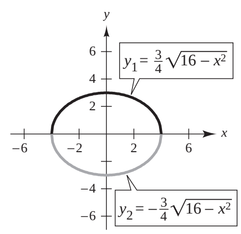
$$y' = \frac{-(x-2)}{y+3}$$

19. (a) $16y^2 = 144 - 9x^2$

$$y^2 = \frac{1}{16}(144 - 9x^2) = \frac{9}{16}(16 - x^2)$$

$$y = \pm\frac{3}{4}\sqrt{16 - x^2}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \pm\frac{3}{8}(16 - x^2)^{-1/2}(-2x)$$

$$= \mp\frac{3x}{4\sqrt{16 - x^2}} = \frac{-3x}{4(4/3)y} = \frac{-9x}{16y}$$

(d) Implicitly: $18x + 32yy' = 0$

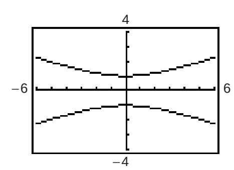
$$y' = \frac{-9x}{16y}$$

20. (a) $9y^2 = x^2 + 9$

$$y^2 = \frac{x^2}{9} + 1 = \frac{x^2 + 9}{9}$$

$$y = \frac{\pm\sqrt{x^2 + 9}}{3}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \frac{\pm\frac{1}{2}(x^2 + 9)^{-1/2}(2x)}{3}$$

$$= \frac{\pm x}{3\sqrt{x^2 + 9}} = \frac{\pm x}{3(\pm 3y)} = \frac{x}{9y}$$

(d) Implicitly: $9y^2 - x^2 = 9$

$$18yy' - 2x = 0$$

$$18yy' = 2x$$

$$y' = \frac{2x}{18y} = \frac{x}{9y}$$

21.

$$xy = 4$$

$$xy' + y(1) = 0$$

$$xy' = -y$$

$$y' = \frac{-y}{x}$$

At $(-4, -1)$: $y' = -\frac{1}{4}$

22.

$$x^2 - y^3 = 0$$

$$2x - 3y^2y' = 0$$

$$y' = \frac{2x}{3y^2}$$

At $(1, 1)$: $y' = \frac{2}{3}$

23.
$$y^2 = \frac{x^2 - 4}{x^2 + 4}$$
$$2yy' = \frac{(x^2 + 4)(2x) - (x^2 - 4)(2x)}{(x^2 + 4)^2}$$
$$2yy' = \frac{16x}{(x^2 + 4)^2}$$
$$y' = \frac{8x}{y(x^2 + 4)^2}$$
At $(2, 0)$: y' is undefined.

24.
$$(x + y)^3 = x^3 + y^3$$
$$x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3$$
$$3x^2y + 3xy^2 = 0$$
$$x^2y + xy^2 = 0$$
$$x^2y' + 2xy + 2xyy' + y^2 = 0$$
$$(x^2 + 2xy)y' = -(y^2 + 2xy)$$
$$y' = -\frac{y(y + 2x)}{x(x + 2y)}$$
At $(-1, 1)$: $y' = -1$

25.
$$x^{2/3} + y^{2/3} = 5$$
$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$$
$$y' = \frac{-x^{-1/3}}{y^{-1/3}} = -\sqrt[3]{\frac{y}{x}}$$
At $(8, 1)$: $y' = -\frac{1}{2}$

26.
$$x^3 + y^3 = 4xy + 1$$
$$3x^2 + 3y^2y' = 4xy' + 4y$$
$$(3y^2 - 4x)y' = 4y - 3x^2$$
$$y' = \frac{4y - 3x^2}{(3y^2 - 4x)}$$
At $(2, 1)$: $y' = \frac{4 - 12}{3 - 8} = \frac{8}{5}$

27.
$$\tan(x + y) = x$$
$$(1 + y')\sec^2(x + y) = 1$$
$$y' = \frac{1 - \sec^2(x + y)}{\sec^2(x + y)}$$
$$= \frac{-\tan^2(x + y)}{\tan^2(x + y) + 1}$$
$$= -\sin^2(x + y)$$
$$= -\frac{x^2}{x^2 + 1}$$
At $(0, 0)$: $y' = 0$

28.
$$x\cos y = 1$$
$$x[-y'\sin y] + \cos y = 0$$
$$y' = \frac{\cos y}{x \sin y}$$
$$= \frac{1}{x}\cot y$$
$$= \frac{\cot y}{x}$$
At $\left(2, \frac{\pi}{3}\right)$: $y' = \frac{1}{2\sqrt{3}}$

29.
$$(x^2 + 4)y = 8$$
$$(x^2 + 4)y' + y(2x) = 0$$
$$y' = \frac{-2xy}{x^2 + 4}$$
$$= \frac{-2x[8/(x^2 + 4)]}{x^2 + 4}$$
$$= \frac{-16x}{(x^2 + 4)^2}$$
At $(2, 1)$: $y' = \frac{-32}{64} = -\frac{1}{2}$

$\left(\text{Or, you could just solve for } y\text{: } y = \frac{8}{x^2 + 4}\right)$

30.
$$(4 - x)y^2 = x^3$$
$$(4 - x)(2yy') + y^2(-1) = 3x^2$$
$$y' = \frac{3x^2 + y^2}{2y(4 - x)}$$
At $(2, 2)$: $y' = 2$

31. $(x^2 + y^2)^2 = 4x^2y$

$$2(x^2 + y^2)(2x + 2yy') = 4x^2y' + y(8x)$$

$$4x^3 + 4x^2yy' + 4xy^2 + 4y^3y' = 4x^2y' + 8xy$$

$$4x^2yy' + 4y^3y' - 4x^2y' = 8xy - 4x^3 - 4xy^2$$

$$4y'(x^2y + y^3 - x^2) = 4(2xy - x^3 - xy^2)$$

$$y' = \frac{2xy - x^3 - xy^2}{x^2y + y^3 - x^2}$$

At $(1, 1)$: $y' = 0$

32. $x^3 + y^3 - 6xy = 0$

$$3x^2 + 3y^2y' - 6xy' - 6y = 0$$

$$y'(3y^2 - 6x) = 6y - 3x^2$$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{2y - x^2}{y^2 - 2x}$$

At $\left(\frac{4}{3}, \frac{8}{3}\right)$: $y' = \frac{(16/3) - (16/9)}{(64/9) - (8/3)} = \frac{32}{40} = \frac{4}{5}$

33. $(y - 2)^2 = 4(x - 3)$, $(4, 0)$

$$2(y - 2)y' = 4$$

$$y' = \frac{2}{y - 2}$$

At $(4, 0)$: $y' = -1$

Tangent line: $y - 0 = -1(x - 4)$

$$y = -x + 4$$

34. $(x + 1)^2 + (y - 2)^2 = 20$, $(3, 4)$

$$2(x + 1) + 2(y - 2)y' = 0$$

$$2(y - 2)y' = -2(x + 1)$$

$$y' = \frac{-(x + 1)}{y - 2}$$

At $(3, 4)$: $y' = -2$

Tangent line: $y - 4 = -2(x - 3)$

$$y = -2x + 10$$

35. $xy = 1$, $(1, 1)$

$$xy' + y = 0$$

$$y' = \frac{-y}{x}$$

At $(1, 1)$: $y' = -1$

Tangent line: $y - 1 = -1(x - 1)$

$$y = -x + 2$$

36. $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$, $\left(\sqrt{3}, 1\right)$

$$14x - 6\sqrt{3}xy' - 6\sqrt{3}y + 26yy' = 0$$

$$y' = \frac{6\sqrt{3}y - 14x}{26y - 6\sqrt{3}x}$$

At $\left(\sqrt{3}, 1\right)$: $y' = \frac{6\sqrt{3} - 14\sqrt{3}}{26 - 6\sqrt{3}\sqrt{3}} = \frac{-8\sqrt{3}}{8} = -\sqrt{3}$

Tangent line: $y - 1 = -\sqrt{3}\left(x - \sqrt{3}\right)$

$$y = -\sqrt{3}x + 4$$

37. $x^2y^2 - 9x^2 - 4y^2 = 0$, $\left(-4, 2\sqrt{3}\right)$

$$x^2\,2yy' + 2xy^2 - 18x - 8yy' = 0$$

$$y' = \frac{18x - 2xy^2}{2x^2y - 8y}$$

At $\left(-4, 2\sqrt{3}\right)$: $y' = \frac{18(-4) - 2(-4)(12)}{2(16)\left(2\sqrt{3}\right) - 16\sqrt{3}}$

$$= \frac{24}{48\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$$

Tangent line: $y - 2\sqrt{3} = \frac{\sqrt{3}}{6}(x + 4)$

$$y = \frac{\sqrt{3}}{6}x + \frac{8}{3}\sqrt{3}$$

38. $x^{2/3} + y^{2/3} = 5$, $(8, 1)$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$$

$$y' = \frac{-x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}$$

At $(8, 1)$: $y' = -\frac{1}{2}$

Tangent line: $y - 1 = -\frac{1}{2}(x - 8)$

$$y = -\frac{1}{2}x + 5$$

39. $3(x^2 + y^2)^2 = 100(x^2 - y^2), \quad (4, 2)$

$6(x^2 + y^2)(2x + 2yy') = 100(2x - 2yy')$

At (4, 2):

$$6(16 + 4)(8 + 4y') = 100(8 - 4y')$$
$$960 + 480y' = 800 - 400y'$$
$$880y' = -160$$
$$y' = -\tfrac{2}{11}$$

Tangent line: $y - 2 = -\tfrac{2}{11}(x - 4)$

$$11y + 2x - 30 = 0$$
$$y = -\tfrac{2}{11}x + \tfrac{30}{11}$$

40. $y^2(x^2 + y^2) = 2x^2, \quad (1, 1)$

$$y^2x^2 + y^4 = 2x^2$$
$$2yy'x^2 + 2xy^2 + 4y^3y' = 4x$$

At (1, 1):

$$2y' + 2 + 4y' = 4$$
$$6y' = 2$$
$$y' = \tfrac{1}{3}$$

Tangent line: $y - 1 = \tfrac{1}{3}(x - 1)$

$$y = \tfrac{1}{3}x + \tfrac{2}{3}$$

41. (a) $\dfrac{x^2}{2} + \dfrac{y^2}{8} = 1, \quad (1, 2)$

$$x + \frac{yy'}{4} = 0$$
$$y' = -\frac{4x}{y}$$

At (1, 2): $y' = -2$

Tangent line: $y - 2 = -2(x - 1)$

$$y = -2x + 4$$

(b) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \implies \dfrac{2x}{a^2} + \dfrac{2yy'}{b^2} = 0 \implies y' = \dfrac{-b^2x}{a^2y}$

$y - y_0 = \dfrac{-b^2x_0}{a^2y_0}(x - x_0)$, Tangent line at (x_0, y_0)

$$\frac{y_0y}{b^2} - \frac{y_0^2}{b^2} = \frac{-x_0x}{a^2} + \frac{x_0^2}{a^2}$$

Since $\dfrac{x_0^2}{a^2} + \dfrac{y_0^2}{b^2} = 1$, you have $\dfrac{y_0y}{b^2} + \dfrac{x_0x}{a^2} = 1$.

Note: From part (a), $\dfrac{1(x)}{2} + \dfrac{2(y)}{8} = 1 \implies \dfrac{1}{4}y = -\dfrac{1}{2}x + 1 \implies y = -2x + 4$, Tangent line.

42. (a) $\dfrac{x^2}{6} - \dfrac{y^2}{8} = 1, \quad (3, -2)$

$$\frac{x}{3} - \frac{y}{4}y' = 0$$
$$\frac{y}{4}y' = \frac{x}{3}$$
$$y' = \frac{4x}{3y}$$

At (3, −2): $y' = \dfrac{4(3)}{3(-2)} = -2$

Tangent line: $y + 2 = -2(x - 3)$

$$y = -2x + 4$$

(b) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1 \implies \dfrac{2x}{a^2} - \dfrac{2yy'}{b^2} = 0 \implies y' = \dfrac{xb^2}{ya^2}$

$y - y_0 = \dfrac{x_0b^2}{y_0a^2}(x - x_0)$, Tangent line at (x_0, y_0)

$$\frac{yy_0}{b^2} - \frac{y_0^2}{b^2} = \frac{x_0x}{a^2} - \frac{x_0^2}{a^2}$$

Since $\dfrac{x_0^2}{a^2} - \dfrac{y_0^2}{b^2} = 1$, you have $\dfrac{x_0x}{a^2} - \dfrac{yy_0}{b^2} = 1$.

Note: From part (a), $\dfrac{3x}{6} - \dfrac{(-2)y}{8} = 1 \implies \dfrac{1}{2}x + \dfrac{y}{4} = 1 \implies y = -2x + 4$, Tangent line.

43.
$$\tan y = x$$
$$y'\sec^2 y = 1$$
$$y' = \frac{1}{\sec^2 y} = \cos^2 y, \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$
$$\sec^2 y = 1 + \tan^2 y = 1 + x^2$$
$$y' = \frac{1}{1 + x^2}$$

44.
$$\cos y = x$$
$$-\sin y \cdot y' = 1$$
$$y' = \frac{-1}{\sin y}, \quad 0 < y < \pi$$
$$\sin^2 y + \cos^2 y = 1$$
$$\sin^2 y = 1 - \cos^2 y$$
$$\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$$
$$y' = \frac{-1}{\sqrt{1 - x^2}}, \quad -1 < x < 1$$

45.
$$x^2 + y^2 = 36$$
$$2x + 2yy' = 0$$
$$y' = \frac{-x}{y}$$
$$y'' = \frac{y(-1) + xy'}{y^2} = \frac{-y + x(-x/y)}{y^2} = \frac{-y^2 - x^2}{y^3} = \frac{-36}{y^3}$$

46.
$$x^2y^2 - 2x = 3$$
$$2x^2yy' + 2xy^2 - 2 = 0$$
$$x^2yy' + xy^2 - 1 = 0$$
$$y' = \frac{1 - xy^2}{x^2y}$$
$$2xyy' + x^2(y')^2 + x^2yy'' + 2xyy' + y^2 = 0$$
$$4xyy' + x^2(y')^2 + x^2yy'' + y^2 = 0$$
$$\frac{4 - 4xy^2}{x} + \frac{(1 - xy^2)^2}{x^2y^2} + x^2yy'' + y^2 = 0$$
$$4xy^2 - 4x^2y^4 + 1 - 2xy^2 + x^2y^4 + x^4y^3y'' + x^2y^4 = 0$$
$$x^4y^3y'' = 2x^2y^4 - 2xy^2 - 1$$
$$y'' = \frac{2x^2y^4 - 2xy^2 - 1}{x^4y^3}$$

47.
$$x^2 - y^2 = 16$$
$$2x - 2yy' = 0$$
$$y' = \frac{x}{y}$$
$$x - yy' = 0$$
$$1 - yy'' - (y')^2 = 0$$
$$1 - yy'' - \left(\frac{x}{y}\right)^2 = 0$$
$$y^2 - y^3y'' = x^2$$
$$y'' = \frac{y^2 - x^2}{y^3} = \frac{-16}{y^3}$$

48.
$$1 - xy = x - y$$
$$y - xy = x - 1$$
$$y = \frac{x - 1}{1 - x} = -1$$
$$y' = 0$$
$$y'' = 0$$

49.
$$y^2 = x^3$$
$$2yy' = 3x^2$$
$$y' = \frac{3x^2}{2y} = \frac{3x^2}{2y} \cdot \frac{xy}{xy}$$
$$= \frac{3y}{2x} \cdot \frac{x^3}{y^2} = \frac{3y}{2x}$$
$$y'' = \frac{2x(3y') - 3y(2)}{4x^2}$$
$$= \frac{2x[3 \cdot (3y/2x)] - 6y}{4x^2}$$
$$= \frac{3y}{4x^2} = \frac{3x}{4y}$$

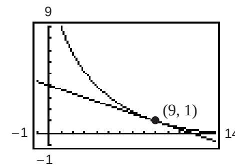
50. $y^2 = 4x$

$$2yy' = 4$$

$$y' = \frac{2}{y}$$

$$y'' = -2y^{-2}y' = \left[\frac{-2}{y^2}\right] \cdot \frac{2}{y} = \frac{-4}{y^3}$$

51. $\sqrt{x} + \sqrt{y} = 4$



$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$$

$$y' = \frac{-\sqrt{y}}{\sqrt{x}}$$

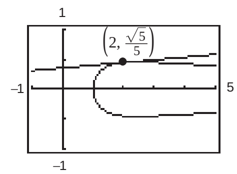
At $(9, 1)$: $y' = -\frac{1}{3}$

Tangent line: $y - 1 = -\frac{1}{3}(x - 9)$

$$y = -\frac{1}{3}x + 4$$

$$x + 3y - 12 = 0$$

52. $y^2 = \dfrac{x - 1}{x^2 + 1}$



$$2yy' = \frac{(x^2 + 1)(1) - (x - 1)(2x)}{(x^2 + 1)^2}$$

$$= \frac{x^2 + 1 - 2x^2 + 2x}{(x^2 + 1)^2}$$

$$y' = \frac{1 + 2x - x^2}{2y(x^2 + 1)^2}$$

At $\left(2, \dfrac{\sqrt{5}}{5}\right)$: $y' = \dfrac{1 + 4 - 4}{\left[\left(2\sqrt{5}\right)/5\right](4 + 1)^2} = \dfrac{1}{10\sqrt{5}}$

Tangent line: $y - \dfrac{\sqrt{5}}{5} = \dfrac{1}{10\sqrt{5}}(x - 2)$

$$10\sqrt{5}y - 10 = x - 2$$

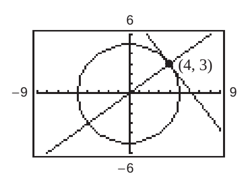
$$x - 10\sqrt{5}y + 8 = 0$$

53. $x^2 + y^2 = 25$

$$2x + 2yy' = 0$$

$$y' = \frac{-x}{y}$$

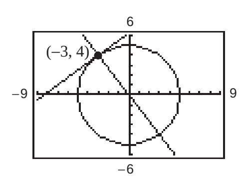
At $(4, 3)$:



Tangent line: $y - 3 = \dfrac{-4}{3}(x - 4) \Longrightarrow 4x + 3y - 25 = 0$

Normal line: $y - 3 = \dfrac{3}{4}(x - 4) \Longrightarrow 3x - 4y = 0$

At $(-3, 4)$:



Tangent line: $y - 4 = \dfrac{3}{4}(x + 3) \Longrightarrow 3x - 4y + 25 = 0$

Normal line: $y - 4 = \dfrac{-4}{3}(x + 3) \Longrightarrow 4x + 3y = 0$

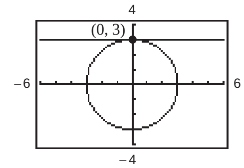
54. $x^2 + y^2 = 9$

$$y' = \frac{-x}{y}$$

At $(0, 3)$:

Tangent line: $y = 3$

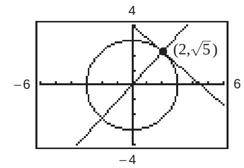
Normal line: $x = 0$



At $(2, \sqrt{5})$:

Tangent line: $y - \sqrt{5} = \dfrac{-2}{\sqrt{5}}(x - 2) \Longrightarrow 2x + \sqrt{5}y - 9 = 0$

Normal line: $y - \sqrt{5} = \dfrac{\sqrt{5}}{2}(x - 2) \Longrightarrow \sqrt{5}x - 2y = 0$



55. $x^2 + y^2 = r^2$

$2x + 2yy' = 0$

$$y' = \frac{-x}{y} = \text{slope of tangent line}$$

$$\frac{y}{x} = \text{slope of normal line}$$

Let (x_0, y_0) be a point on the circle. If $x_0 = 0$, then the tangent line is horizontal, the normal line is vertical and, hence, passes through the origin. If $x_0 \neq 0$, then the equation of the normal line is

$$y - y_0 = \frac{y_0}{x_0}(x - x_0)$$

$$y = \frac{y_0}{x_0}x$$

which passes through the origin.

56. $y^2 = 4x$

$2yy' = 4$

$$y' = \frac{2}{y} = 1 \text{ at } (1, 2)$$

Equation of normal line at $(1, 2)$ is $y - 2 = -1(x - 1)$, $y = 3 - x$. The centers of the circles must be on the normal line and at a distance of 4 units from $(1, 2)$. Therefore,

$$(x - 1)^2 + [(3 - x) - 2]^2 = 16$$

$$2(x - 1)^2 = 16$$

$$x = 1 \pm 2\sqrt{2}.$$

Centers of the circles: $(1 + 2\sqrt{2}, 2 - 2\sqrt{2})$ and $(1 - 2\sqrt{2}, 2 + 2\sqrt{2})$

Equations: $(x - 1 - 2\sqrt{2})^2 + (y - 2 + 2\sqrt{2})^2 = 16$

$(x - 1 + 2\sqrt{2})^2 + (y - 2 - 2\sqrt{2})^2 = 16$

57. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

$$50x + 32yy' + 200 - 160y' = 0$$

$$y' = \frac{200 + 50x}{160 - 32y}$$

Horizontal tangents occur when $x = -4$:

$$25(16) + 16y^2 + 200(-4) - 160y + 400 = 0$$

$$y(y - 10) = 0 \Longrightarrow y = 0, 10$$

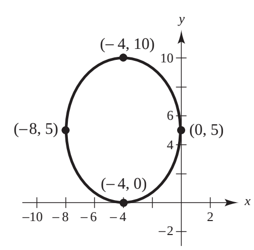
Horizontal tangents: $(-4, 0), (-4, 10)$

Vertical tangents occur when $y = 5$:

$$25x^2 + 400 + 200x - 800 + 400 = 0$$

$$25x(x + 8) = 0 \Longrightarrow x = 0, -8$$

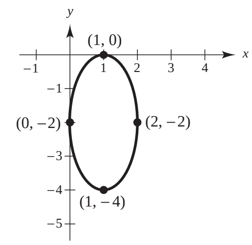
Vertical tangents: $(0, 5), (-8, 5)$



58. $4x^2 + y^2 - 8x + 4y + 4 = 0$

$$8x + 2yy' - 8 + 4y' = 0$$

$$y' = \frac{8 - 8x}{2y + 4} = \frac{4 - 4x}{y + 2}$$



Horizontal tangents occur when $x = 1$:

$$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$$

$$y^2 + 4y = y(y + 4) = 0 \Longrightarrow y = 0, -4$$

Horizontal tangents: $(1, 0), (1, -4)$

Vertical tangents occur when $y = -2$:

$$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0$$

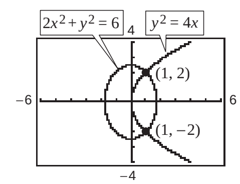
$$4x^2 - 8x = 4x(x - 2) = 0 \Longrightarrow x = 0, 2$$

Vertical tangents: $(0, -2), (2, -2)$

59. Find the points of intersection by letting $y^2 = 4x$ in the equation $2x^2 + y^2 = 6$.

$2x^2 + 4x = 6$ and $(x + 3)(x - 1) = 0$

The curves intersect at $(1, \pm 2)$.



Ellipse:	*Parabola*:
$4x + 2yy' = 0$	$2yy' = 4$
$y' = -\dfrac{2x}{y}$	$y' = \dfrac{2}{y}$

At $(1, 2)$, the slopes are:

$y' = -1$ $\qquad$ $y' = 1$

At $(1, -2)$, the slopes are:

$y' = 1$ $\qquad$ $y' = -1$

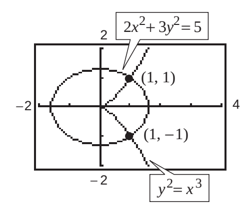
Tangents are perpendicular.

60. Find the points of intersection by letting $y^2 = x^3$ in the equation $2x^2 + 3y^2 = 5$.

$2x^2 + 3x^3 = 5$ and $3x^3 + 2x^2 - 5 = 0$

Intersect when $x = 1$.

Points of intersection: $(1, \pm 1)$



$y^2 = x^3$:	$2x^2 + 3y^2 = 5$:
$2yy' = 3x^2$	$4x + 6yy' = 0$
$y' = \dfrac{3x^2}{2y}$	$y' = -\dfrac{2x}{3y}$

At $(1, 1)$, the slopes are:

$y' = \dfrac{3}{2}$ $\qquad$ $y' = -\dfrac{2}{3}$

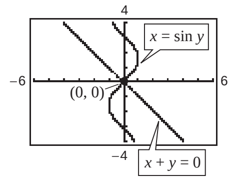
At $(1, -1)$, the slopes are:

$y' = -\dfrac{3}{2}$ $\qquad$ $y' = \dfrac{2}{3}$

Tangents are perpendicular.

61. $y = -x$ and $x = \sin y$

Point of intersection: $(0, 0)$



$\underline{y = -x:}$ $\qquad$ $\underline{x = \sin y:}$

$y' = -1$ $\qquad$ $1 = y' \cos y$

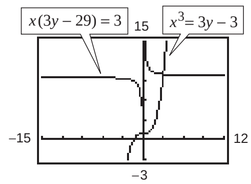
$\qquad\qquad y' = \sec y$

At $(0, 0)$, the slopes are:

$y' = -1$ $\qquad$ $y' = 1$

Tangents are perpendicular.

62. Rewriting each equation and differentiating:



$x^3 = 3(y - 1)$ $\qquad$ $x(3y - 29) = 3$

$y = \dfrac{x^3}{3} + 1$ $\qquad$ $y = \dfrac{1}{3}\left(\dfrac{3}{x} + 29\right)$

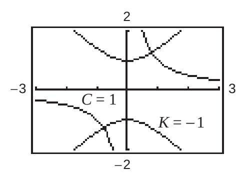
$y' = x^2$ $\qquad$ $y' = -\dfrac{1}{x^2}$

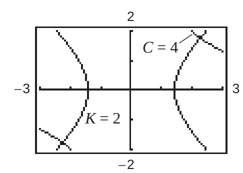
For each value of x, the derivatives are negative reciprocals of each other. Thus, the tangent lines are orthogonal at both points of intersection.

63. $xy = C$ $\qquad$ $x^2 - y^2 = K$

$xy' + y = 0$ $\qquad$ $2x - 2yy' = 0$

$y' = -\dfrac{y}{x}$ $\qquad$ $y' = \dfrac{x}{y}$



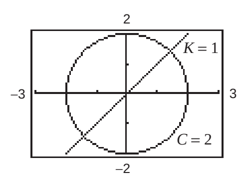


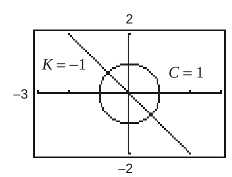
At any point of intersection (x, y) the product of the slopes is $(-y/x)(x/y) = -1$. The curves are orthogonal.

64. $x^2 + y^2 = C^2$ $\qquad$ $y = Kx$

$2x + 2yy' = 0$ $\qquad$ $y' = K$

$y' = -\dfrac{x}{y}$





At the point of intersection (x, y), the product of the slopes is $(-x/y)(K) = (-x/Kx)(K) = -1$. The curves are orthogonal.

65. $2y^2 - 3x^4 = 0$

(a) $4yy' - 12x^3 = 0$

$$4yy' = 12x^3$$

$$y' = \frac{12x^3}{4y} = \frac{3x^3}{y}$$

(b) $4y\dfrac{dy}{dt} - 12x^3\dfrac{dx}{dt} = 0$

$$y\frac{dy}{dt} = 3x^3\frac{dx}{dt}$$

66. $x^2 - 3xy^2 + y^3 = 10$

(a) $2x - 3y^2 - 6xyy' + 3y^2y' = 0$

$$(-6xy + 3y^2)y' = 3y^2 - 2x$$

$$y' = \frac{3y^2 - 2x}{3y^2 - 6xy}$$

(b) $2x\dfrac{dx}{dt} - 3y^2\dfrac{dx}{dt} - 6xy\dfrac{dy}{dt} + 3y^2\dfrac{dy}{dt} = 0$

$$(2x - 3y^2)\frac{dx}{dt} = (6xy - 3y^2)\frac{dy}{dt}$$

67. $\cos \pi y - 3 \sin \pi x = 1$

(a) $-\pi \sin(\pi y)y' - 3\pi \cos \pi x = 0$

$$y' = \frac{-3 \cos \pi x}{\sin \pi y}$$

(b) $-\pi \sin(\pi y) \dfrac{dy}{dt} - 3\pi \cos(\pi x) \dfrac{dx}{dt} = 0$

$$-\sin(\pi y) \frac{dy}{dt} = 3 \cos(\pi x) \frac{dx}{dt}$$

68. $4 \sin x \cos y = 1$

(a) $4 \sin x(-\sin y)y' + 4 \cos x \cos y = 0$

$$y' = \frac{\cos x \cos y}{\sin x \sin y}$$

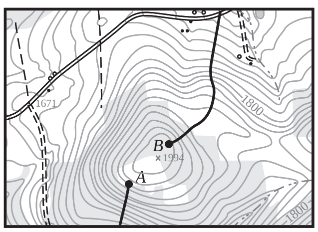
(b) $4 \sin x(-\sin y) \dfrac{dy}{dt} + 4 \cos x \dfrac{dx}{dt} \cos y = 0$

$$\cos x \cos y \frac{dx}{dt} = \sin x \sin y \frac{dy}{dt}$$

69. A function is in explicit form if y is written as a function of x: $y = f(x)$. For example, $y = x^3$. An implicit equation is not in the form $y = f(x)$. For example, $x^2 + y^2 = 5$.

70. Given an implicit equation, first differentiate both sides with respect to x. Collect all terms involving y' on the left, and all other terms to the right. Factor out y' on the left side. Finally, divide both sides by the left-hand factor that does not contain y'.

71.



Use starting point B.

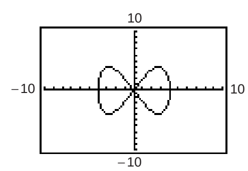
72. Highest wind speed near L

73. (a) $x^4 = 4(4x^2 - y^2)$

$$4y^2 = 16x^2 - x^4$$

$$y^2 = 4x^2 - \frac{1}{4}x^4$$

$$y = \pm\sqrt{4x^2 - \frac{1}{4}x^4}$$

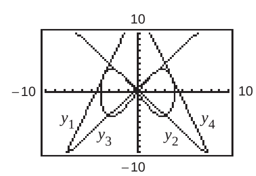


(b) $y = 3 \Longrightarrow 9 = 4x^2 - \dfrac{1}{4}x^4$

$$36 = 16x^2 - x^4$$

$$x^4 - 16x^2 + 36 = 0$$

$$x^2 = \frac{16 \pm \sqrt{256 - 144}}{2} = 8 \pm \sqrt{28}$$



Note that $x^2 = 8 \pm \sqrt{28} = 8 \pm 2\sqrt{7} = \left(1 \pm \sqrt{7}\right)^2$. Hence, there are four values of x:

$-1 - \sqrt{7}, 1 - \sqrt{7}, -1 + \sqrt{7}, 1 + \sqrt{7}$

To find the slope, $2yy' = 8x - x^3 \Longrightarrow y' = \dfrac{x(8 - x^2)}{2(3)}$.

—CONTINUED—

73. —CONTINUED—

For $x = -1 - \sqrt{7}$, $y' = \frac{1}{3}\left(\sqrt{7} + 7\right)$, and the line is

$$y_1 = \frac{1}{3}\left(\sqrt{7} + 7\right)\left(x + 1 + \sqrt{7}\right) + 3 = \frac{1}{3}\left[\left(\sqrt{7} + 7\right)x + 8\sqrt{7} + 23\right].$$

For $x = 1 - \sqrt{7}$, $y' = \frac{1}{3}\left(\sqrt{7} - 7\right)$, and the line is

$$y_2 = \frac{1}{3}\left(\sqrt{7} - 7\right)\left(x - 1 + \sqrt{7}\right) + 3 = \frac{1}{3}\left[\left(\sqrt{7} - 7\right)x + 23 - 8\sqrt{7}\right].$$

For $x = -1 + \sqrt{7}$, $y' = -\frac{1}{3}\left(\sqrt{7} - 7\right)$, and the line is

$$y_3 = -\frac{1}{3}\left(\sqrt{7} - 7\right)\left(x + 1 - \sqrt{7}\right) + 3 = -\frac{1}{3}\left[\left(\sqrt{7} - 7\right)x - \left(23 - 8\sqrt{7}\right)\right].$$

For $x = 1 + \sqrt{7}$, $y' = -\frac{1}{3}\left(\sqrt{7} + 7\right)$, and the line is

$$y_4 = -\frac{1}{3}\left(\sqrt{7} + 7\right)\left(x - 1 - \sqrt{7}\right) + 3 = -\frac{1}{3}\left[\left(\sqrt{7} + 7\right)x - \left(8\sqrt{7} + 23\right)\right].$$

(c) Equating y_3 and y_4:

$$-\frac{1}{3}\left(\sqrt{7} - 7\right)\left(x + 1 - \sqrt{7}\right) + 3 = -\frac{1}{3}\left(\sqrt{7} + 7\right)\left(x - 1 - \sqrt{7}\right) + 3$$

$$\left(\sqrt{7} - 7\right)\left(x + 1 - \sqrt{7}\right) = \left(\sqrt{7} + 7\right)\left(x - 1 - \sqrt{7}\right)$$

$$\sqrt{7}x + \sqrt{7} - 7 - 7x - 7 + 7\sqrt{7} = \sqrt{7}x - \sqrt{7} - 7 + 7x - 7 - 7\sqrt{7}$$

$$16\sqrt{7} = 14x$$

$$x = \frac{8\sqrt{7}}{7}$$

If $x = \frac{8\sqrt{7}}{7}$, then $y = 5$ and the lines intersect at $\left(\frac{8\sqrt{7}}{7}, 5\right)$.

74. $\sqrt{x} + \sqrt{y} = \sqrt{c}$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

Tangent line at (x_0, y_0): $y - y_0 = -\frac{\sqrt{y_0}}{\sqrt{x_0}}(x - x_0)$

x-intercept: $\left(x_0 + \sqrt{x_0}\sqrt{y_0}, 0\right)$

y-intercept: $\left(0, y_0 + \sqrt{x_0}\sqrt{y_0}\right)$

Sum of intercepts:

$$\left(x_0 + \sqrt{x_0}\sqrt{y_0}\right) + \left(y_0 + \sqrt{x_0}\sqrt{y_0}\right) = x_0 + 2\sqrt{x_0}\sqrt{y_0} + y_0$$

$$= \left(\sqrt{x_0} + \sqrt{y_0}\right)^2$$

$$= \left(\sqrt{c}\right)^2 = c$$

75. $y = x^{p/q}$; p, q integers and $q > 0$

$$y^q = x^p$$

$$qy^{q-1}y' = px^{p-1}$$

$$y' = \frac{p}{q} \cdot \frac{x^{p-1}}{y^{q-1}} = \frac{p}{q} \cdot \frac{x^{p-1}y}{y^q}$$

$$= \frac{p}{q} \cdot \frac{x^{p-1}}{x^p}x^{p/q} = \frac{p}{q}x^{p/q-1}$$

Thus, if $y = x^n$, $n = p/q$, then $y' = nx^{n-1}$.

76. $x^2 + y^2 = 25, \text{ slope} = \frac{3}{4}$

$2x + 2yy' = 0$

$y' = -\frac{x}{y} = \frac{3}{4} \Rightarrow y = -\frac{4}{3}x$

$x^2 + \left(\frac{16}{9}x^2\right) = 25$

$\frac{25}{9}x^2 = 25$

$x = \pm 3$

Points: $(3, -4)$ and $(-3, 4)$

77. $y^4 = y^2 - x^2$

$4y^3y' = 2yy' - 2x$

$2x = (2y - 4y^3)y'$

$y' = \frac{2x}{2y - 4y^3} = 0 \Rightarrow x = 0$

Horizontal tangents at $(0, 1)$ and $(0, -1)$

Note: $y^4 - y^2 + x^2 = 0$

$$y^2 = \frac{1 \pm \sqrt{1 - 4x^2}}{2}$$

If you graph these four equations, you will see that these are horizontal tangents at $(0, \pm 1)$, but not at $(0, 0)$.

78. $\frac{x^2}{4} + \frac{y^2}{9} = 1, \quad (4, 0)$

$\frac{2x}{4} + \frac{2yy'}{9} = 0$

$y' = \frac{-9x}{4y}$

$\frac{-9x}{4y} = \frac{y - 0}{x - 4}$

$-9x(x - 4) = 4y^2$

But, $9x^2 + 4y^2 = 36 \Rightarrow 4y^2 = 36 - 9x^2$. Hence, $-9x^2 + 36x = 4y^2 = 36 - 9x^2 \Rightarrow x = 1$.

Points on ellipse: $\left(1, \pm\frac{3}{2}\sqrt{3}\right)$

At $\left(1, \frac{3}{2}\sqrt{3}\right)$: $y' = \frac{-9x}{4y} = \frac{-9}{4\left[(3/2)\sqrt{3}\right]} = -\frac{\sqrt{3}}{2}$

At $\left(1, -\frac{3}{2}\sqrt{3}\right)$: $y' = \frac{\sqrt{3}}{2}$

Tangent lines: $y = -\frac{\sqrt{3}}{2}(x - 4) = -\frac{\sqrt{3}}{2}x + 2\sqrt{3}$

$y = \frac{\sqrt{3}}{2}(x - 4) = \frac{\sqrt{3}}{2}x - 2\sqrt{3}$

79. $x = y^2$

$1 = 2yy'$

$y' = \frac{1}{2y}$, slope of tangent line

Consider the slope of the normal line joining $(x_0, 0)$ and $(x, y) = (y^2, y)$ on the parabola.

$-2y = \frac{y - 0}{y^2 - x_0}$

$y^2 - x_0 = -\frac{1}{2}$

$y^2 = x_0 - \frac{1}{2}$

—CONTINUED—

79. —CONTINUED—

(a) If $x_0 = \frac{1}{4}$, then $y^2 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$, which is impossible. Thus, the only normal line is the x-axis $(y = 0)$.

(b) If $x_0 = \frac{1}{2}$, then $y^2 = 0 \implies y = 0$. Same as part (a).

(c) If $x_0 = 1$, then $y^2 = \frac{1}{2} = x$ and there are three normal lines:

The x-axis, the line joining $(x_0, 0)$ and $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$, and the line joining $(x_0, 0)$ and $\left(\frac{1}{2}, -\frac{1}{\sqrt{2}}\right)$

If two normals are perpendicular, then their slopes are -1 and 1. Thus,

$$-2y = -1 = \frac{y - 0}{y^2 - x_0} \implies y = \frac{1}{2} \quad \text{and} \quad \frac{1/2}{(1/4) - x_0} = -1 \implies \frac{1}{4} - x_0 = -\frac{1}{2} \implies x_0 = \frac{3}{4}.$$

The perpendicular normal lines are $y = -x + \frac{3}{4}$ and $y = x - \frac{3}{4}$.

80. (a) $\dfrac{x^2}{32} + \dfrac{y^2}{8} = 1$

$$\frac{2x}{32} + \frac{2yy'}{8} = 0 \implies y' = \frac{-x}{4y}$$

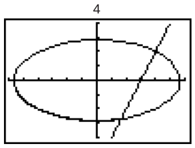
At $(4, 2)$: $y' = \dfrac{-4}{4(2)} = -\dfrac{1}{2}$

Slope of normal line is 2.

$$y - 2 = 2(x - 4)$$

$$y = 2x - 6$$

(b)



(c) $$\frac{x^2}{32} + \frac{(2x - 6)^2}{8} = 1$$

$$x^2 + 4(4x^2 - 24x + 36) = 32$$

$$17x^2 - 96x + 112 = 0$$

$$(17x - 28)(x - 4) = 0 \implies x = 4, \frac{28}{17}$$

Second point: $\left(\dfrac{28}{17}, -\dfrac{46}{17}\right)$

Section 2.6 Related Rates

1. $y = \sqrt{x}$

$$\frac{dy}{dt} = \left(\frac{1}{2\sqrt{x}}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = 2\sqrt{x}\,\frac{dy}{dt}$$

(a) When $x = 4$ and $dx/dt = 3$,

$$\frac{dy}{dt} = \frac{1}{2\sqrt{4}}(3) = \frac{3}{4}.$$

(b) When $x = 25$ and $dy/dt = 2$,

$$\frac{dx}{dt} = 2\sqrt{25}\,(2) = 20.$$

2. $y = 2(x^2 - 3x)$

$$\frac{dy}{dt} = (4x - 6)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{4x - 6}\,\frac{dy}{dt}$$

(a) When $x = 3$ and

$$\frac{dx}{dt} = 2, \frac{dy}{dt} = [4(3) - 6](2) = 12.$$

(b) When $x = 1$ and

$$\frac{dy}{dt} = 5, \frac{dx}{dt} = \frac{1}{4(1) - 6}(5) = -\frac{5}{2}.$$

3. $xy = 4$

$$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = \left(-\frac{y}{x}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(-\frac{x}{y}\right)\frac{dy}{dt}$$

(a) When $x = 8$, $y = 1/2$, and $dx/dt = 10$,

$$\frac{dy}{dt} = -\frac{1/2}{8}(10) = -\frac{5}{8}.$$

(b) When $x = 1$, $y = 4$, and $dy/dt = -6$,

$$\frac{dx}{dt} = -\frac{1}{4}(-6) = \frac{3}{2}.$$

4. $x^2 + y^2 = 25$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \left(-\frac{x}{y}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(-\frac{y}{x}\right)\frac{dy}{dt}$$

(a) When $x = 3$, $y = 4$, and $dx/dt = 8$,

$$\frac{dy}{dt} = -\frac{3}{4}(8) = -6.$$

(b) When $x = 4$, $y = 3$, and $dy/dt = -2$,

$$\frac{dx}{dt} = -\frac{3}{4}(-2) = \frac{3}{2}.$$

5. $y = x^2 + 1$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = 2x\,\frac{dx}{dt}$$

(a) When $x = -1$,

$$\frac{dy}{dt} = 2(-1)(2) = -4 \text{ cm/sec}.$$

(b) When $x = 0$,

$$\frac{dy}{dt} = 2(0)(2) = 0 \text{ cm/sec}.$$

(c) When $x = 1$,

$$\frac{dy}{dt} = 2(1)(2) = 4 \text{ cm/sec}.$$

6. $y = \dfrac{1}{1 + x^2}$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \left[\frac{-2x}{(1 + x^2)^2}\right]\frac{dx}{dt}$$

(a) When $x = -2$,

$$\frac{dy}{dt} = \frac{-2(-2)(2)}{25} = \frac{8}{25} \text{ cm/sec}.$$

(b) When $x = 0$,

$$\frac{dy}{dt} = 0 \text{ cm/sec}.$$

(c) When $x = 2$,

$$\frac{dy}{dt} = \frac{-2(2)(2)}{25} = \frac{-8}{25} \text{ cm/sec}.$$

7. $y = \tan x$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \sec^2 x\,\frac{dx}{dt}$$

(a) When $x = -\pi/3$,

$$\frac{dy}{dt} = (2)^2(2) = 8 \text{ cm/sec}.$$

(b) When $x = -\pi/4$,

$$\frac{dy}{dt} = \left(\sqrt{2}\right)^2(2) = 4 \text{ cm/sec}.$$

(c) When $x = 0$,

$$\frac{dy}{dt} = (1)^2(2) = 2 \text{ cm/sec}.$$

8. $y = \sin x$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \cos x\,\frac{dx}{dt}$$

(a) When $x = \pi/6$,

$$\frac{dy}{dt} = \left(\cos\frac{\pi}{6}\right)(2) = \sqrt{3} \text{ cm/sec}.$$

(b) When $x = \pi/4$,

$$\frac{dy}{dt} = \left(\cos\frac{\pi}{4}\right)(2) = \sqrt{2} \text{ cm/sec}.$$

(c) When $x = \pi/3$,

$$\frac{dy}{dt} = \left(\cos\frac{\pi}{3}\right)(2) = 1 \text{ cm/sec}.$$

9. (a) $\frac{dx}{dt}$ negative $\Rightarrow \frac{dy}{dt}$ positive

(b) $\frac{dy}{dt}$ positive $\Rightarrow \frac{dx}{dt}$ negative

10. (a) $\frac{dx}{dt}$ negative $\Rightarrow \frac{dy}{dt}$ negative

(b) $\frac{dy}{dt}$ positive $\Rightarrow \frac{dx}{dt}$ positive

11. Yes, y changes at a constant rate.

$$\frac{dy}{dt} = a \cdot \frac{dx}{dt}$$

No, the rate dy/dt is a multiple of dx/dt.

12. Answers will vary. See page 149.

13. $D = \sqrt{x^2 + y^2} = \sqrt{x^2 + (x^2 + 1)^2} = \sqrt{x^4 + 3x^2 + 1}$

$$\frac{dx}{dt} = 2$$

$$\frac{dD}{dt} = \frac{1}{2}(x^4 + 3x^2 + 1)^{-1/2}(4x^3 + 6x)\frac{dx}{dt}$$

$$= \frac{2x^3 + 3x}{\sqrt{x^4 + 3x^2 + 1}}\frac{dx}{dt}$$

$$= \frac{4x^3 + 6x}{\sqrt{x^4 + 3x^2 + 1}}$$

14. $D = \sqrt{x^2 + y^2} = \sqrt{x^2 + \sin^2 x}$

$$\frac{dx}{dt} = 2$$

$$\frac{dD}{dt} = \frac{1}{2}(x^2 + \sin^2 x)^{-1/2}(2x + 2\sin x \cos x)\frac{dx}{dt}$$

$$= \frac{x + \sin x \cos x}{\sqrt{x^2 + \sin^2 x}}\frac{dx}{dt}$$

$$= \frac{2x + 2\sin x \cos x}{\sqrt{x^2 + \sin^2 x}}$$

15. $A = \pi r^2$

$$\frac{dr}{dt} = 3$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

(a) When $r = 6$, $\frac{dA}{dt} = 2\pi(6)(3) = 36\pi$ cm²/min.

(b) When $r = 24$, $\frac{dA}{dt} = 2\pi(24)(3) = 144\pi$ cm²/min.

16. $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

If dr/dt is constant, dA/dt is not constant.

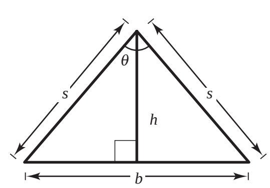
dA/dt depends on r and dr/dt.

17. (a) $\sin\frac{\theta}{2} = \frac{(1/2)b}{s} \Rightarrow b = 2s\sin\frac{\theta}{2}$

$$\cos\frac{\theta}{2} = \frac{h}{s} \Rightarrow h = s\cos\frac{\theta}{2}$$

$$A = \frac{1}{2}bh = \frac{1}{2}\left(2s\sin\frac{\theta}{2}\right)\left(s\cos\frac{\theta}{2}\right)$$

$$= \frac{s^2}{2}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{s^2}{2}\sin\theta$$



(b) $\frac{dA}{dt} = \frac{s^2}{2}\cos\theta\,\frac{d\theta}{dt}$ where $\frac{d\theta}{dt} = \frac{1}{2}$ rad/min.

When $\theta = \frac{\pi}{6}$, $\frac{dA}{dt} = \frac{s^2}{2}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}s^2}{8}$.

When $\theta = \frac{\pi}{3}$, $\frac{dA}{dt} = \frac{s^2}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{s^2}{8}$.

(c) If $\frac{d\theta}{dt}$ is constant, $\frac{dA}{dt}$ is proportional to $\cos\theta$.

18. $V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = 2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

(a) When $r = 6$,

$\frac{dV}{dt} = 4\pi(6)^2(2) = 288\pi \text{ in}^3/\text{min}.$

When $r = 24$,

$\frac{dV}{dt} = 4\pi(24)^2(2) = 4608\pi \text{ in}^3/\text{min}.$

(b) If dr/dt is constant, dV/dt is proportional to r^2.

19. $V = \frac{4}{3}\pi r^3, \frac{dV}{dt} = 800$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{4\pi r^2}\left(\frac{dV}{dt}\right) = \frac{1}{4\pi r^2}(800)$

(a) When $r = 30$,

$\frac{dr}{dt} = \frac{1}{4\pi(30)^2}(800) = \frac{2}{9\pi} \text{ cm/min}.$

(b) When $r = 60$,

$\frac{dr}{dt} = \frac{1}{4\pi(60)^2}(800) = \frac{1}{18\pi} \text{ cm/min}.$

20. $V = x^3$

$\frac{dx}{dt} = 3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

(a) When $x = 1$,

$\frac{dV}{dt} = 3(1)^2(3) = 9 \text{ cm}^3/\text{sec}.$

(b) When $x = 10$,

$\frac{dV}{dt} = 3(10)^2(3) = 900 \text{ cm}^3/\text{sec}.$

21. $s = 6x^2$

$\frac{dx}{dt} = 3$

$\frac{ds}{dt} = 12x \frac{dx}{dt}$

(a) When $x = 1$,

$\frac{ds}{dt} = 12(1)(3) = 36 \text{ cm}^2/\text{sec}.$

(b) When $x = 10$,

$\frac{ds}{dt} = 12(10)(3) = 360 \text{ cm}^2/\text{sec}.$

22. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(3r) = \pi r^3$

$\frac{dr}{dt} = 2$

$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$

(a) When $r = 6$,

$\frac{dV}{dt} = 3\pi(6)^2(2) = 216\pi \text{ in}^3/\text{min}.$

(b) When $r = 24$,

$\frac{dV}{dt} = 3\pi(24)^2(2) = 3456\pi \text{ in}^3/\text{min}.$

23. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{9}{4}h^2\right)h \quad [\text{since } 2r = 3h]$

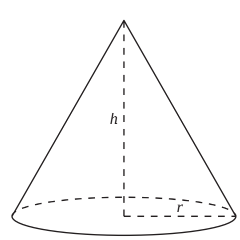
$= \frac{3\pi}{4}h^3$

$\frac{dV}{dt} = 10$

$\frac{dV}{dt} = \frac{9\pi}{4}h^2 \frac{dh}{dt} \Longrightarrow \frac{dh}{dt} = \frac{4\,(dV/dt)}{9\pi h^2}$

When $h = 15$,

$\frac{dh}{dt} = \frac{4(10)}{9\pi(15)^2} = \frac{8}{405\pi} \text{ ft/min}.$

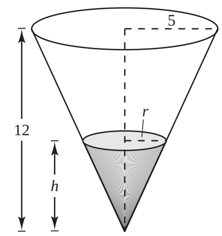


24. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{25}{144}h^3 = \frac{25\pi}{3(144)}h^3$ $\left(\text{By similar triangles, } \frac{r}{5} = \frac{h}{12} \Rightarrow r = \frac{5}{12}h.\right)$

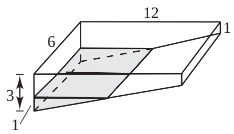
$\frac{dV}{dt} = 10$

$\frac{dV}{dt} = \frac{25\pi}{144}h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \left(\frac{144}{25\pi h^2}\right)\frac{dV}{dt}$

When $h = 8$, $\frac{dh}{dt} = \frac{144}{25\pi(64)}(10) = \frac{9}{10\pi}$ ft/min.



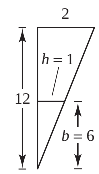
25.



(a) Total volume of pool $= \frac{1}{2}(2)(12)(6) + (1)(6)(12) = 144 \text{ m}^3$

Volume of 1 m of water $= \frac{1}{2}(1)(6)(6) = 18 \text{ m}^3$ (see similar triangle diagram)

% pool filled $= \frac{18}{144}(100\%) = 12.5\%$



(b) Since for $0 \le h \le 2$, $b = 6h$, you have

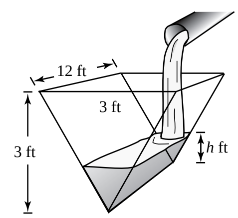
$V = \frac{1}{2}bh(6) = 3bh = 3(6h)h = 18h^2$

$\frac{dV}{dt} = 36h\frac{dh}{dt} = \frac{1}{4} \Rightarrow \frac{dh}{dt} = \frac{1}{144h} = \frac{1}{144(1)} = \frac{1}{144}$ m/min.

26. $V = \frac{1}{2}bh(12) = 6bh = 6h^2$ (since $b = h$)

(a) $\frac{dV}{dt} = 12h\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{12h}\frac{dV}{dt}$

When $h = 1$ and $\frac{dV}{dt} = 2$, $\frac{dh}{dt} = \frac{1}{12(1)}(2) = \frac{1}{6}$ ft/min.

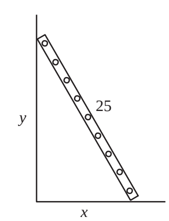


(b) If $\frac{dh}{dt} = \frac{3}{8}$ in./min $= \frac{1}{32}$ ft/min and $h = 2$ feet, then $\frac{dV}{dt} = (12)(2)\left(\frac{1}{32}\right) = \frac{3}{4}$ ft³/min.

27. $x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-x}{y} \cdot \frac{dx}{dt} = \frac{-2x}{y}$ since $\frac{dx}{dt} = 2$.



(a) When $x = 7$, $y = \sqrt{576} = 24$, $\frac{dy}{dt} = \frac{-2(7)}{24} = \frac{-7}{12}$ ft/sec.

When $x = 15$, $y = \sqrt{400} = 20$, $\frac{dy}{dt} = \frac{-2(15)}{20} = \frac{-3}{2}$ ft/sec.

When $x = 24$, $y = 7$, $\frac{dy}{dt} = \frac{-2(24)}{7} = \frac{-48}{7}$ ft/sec.

—CONTINUED—

27. —CONTINUED—

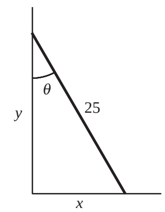
(b) $A = \frac{1}{2}xy$

$$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$$

From part (a) we have $x = 7$, $y = 24$, $\frac{dx}{dt} = 2$, and $\frac{dy}{dt} = -\frac{7}{12}$. Thus,

$$\frac{dA}{dt} = \frac{1}{2}\left[7\left(-\frac{7}{12}\right) + 24(2)\right] = \frac{527}{24} \approx 21.96 \text{ ft}^2/\text{sec}.$$

(c) $\tan\theta = \frac{x}{y}$



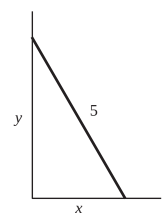
$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta\left[\frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}\right]$$

Using $x = 7$, $y = 24$, $\frac{dx}{dt} = 2$, $\frac{dy}{dt} = -\frac{7}{12}$ and $\cos\theta = \frac{24}{25}$, we have

$$\frac{d\theta}{dt} = \left(\frac{24}{25}\right)^2\left[\frac{1}{24}(2) - \frac{7}{(24)^2}\left(-\frac{7}{12}\right)\right] = \frac{1}{12} \text{ rad/sec.}$$

28. $x^2 + y^2 = 25$



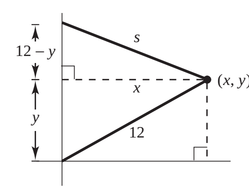
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{y}{x}\cdot\frac{dy}{dt} = -\frac{0.15y}{x} \quad \left(\text{since } \frac{dy}{dt} = 0.15\right)$$

When $x = 2.5$, $y = \sqrt{18.75}$, $\frac{dx}{dt} = -\frac{\sqrt{18.75}}{2.5}0.15 \approx -0.26$ m/sec.

29. When $y = 6$, $x = \sqrt{12^2 - 6^2} = 6\sqrt{3}$, and $s = \sqrt{x^2 + (12 - y)^2}$

$$= \sqrt{108 + 36} = 12.$$



$$x^2 + (12 - y)^2 = s^2$$

$$2x\frac{dx}{dt} + 2(12 - y)(-1)\frac{dy}{dt} = 2s\frac{ds}{dt}$$

$$x\frac{dx}{dt} + (y - 12)\frac{dy}{dt} = s\frac{ds}{dt}$$

Also, $x^2 + y^2 = 12^2$.

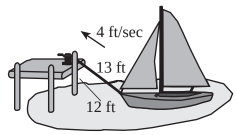
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

Thus, $x\frac{dx}{dt} + (y - 12)\left(\frac{-x}{y}\frac{dx}{dt}\right) = s\frac{ds}{dt}$.

$$\frac{dx}{dt}\left[x - x + \frac{12x}{y}\right] = s\frac{ds}{dt} \Rightarrow \frac{dx}{dt} = \frac{sy}{12x}\cdot\frac{ds}{dt} = \frac{(12)(6)}{(12)(6\sqrt{3})}(-0.2) = \frac{-1}{5\sqrt{3}} = \frac{-\sqrt{3}}{15} \text{ m/sec (horizontal)}$$

$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt} = \frac{-6\sqrt{3}}{6}\cdot\frac{(-\sqrt{3})}{15} = \frac{1}{5} \text{ m/sec (vertical)}$$

30. Let L be the length of the rope.



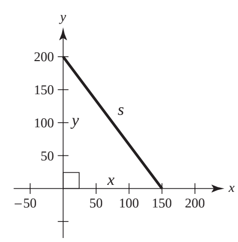
(a)
$$L^2 = 144 + x^2$$
$$2L\frac{dL}{dt} = 2x\frac{dx}{dt}$$
$$\frac{dx}{dt} = \frac{L}{x}\cdot\frac{dL}{dt} = -\frac{4L}{x} \quad \left(\text{since } \frac{dL}{dt} = -4 \text{ ft/sec}\right)$$

When $L = 13$:
$$x = \sqrt{L^2 - 144} = \sqrt{169 - 144} = 5$$
$$\frac{dx}{dt} = -\frac{4(13)}{5} = -\frac{52}{5} = -10.4 \text{ ft/sec}$$

Speed of the boat increases as it approaches the dock.

(b) If $\frac{dx}{dt} = -4$, and $L = 13$:
$$\frac{dL}{dt} = \frac{x}{L}\frac{dx}{dt} = \frac{5}{13}(-4) = \frac{-20}{13} \text{ ft/sec}$$
$$\frac{dL}{dt} = \frac{x}{L}\frac{dx}{dt} = \frac{\sqrt{L^2 - 144}}{L}(-4)$$
$$\lim_{L\to 12^+}\frac{dL}{dt} = \lim_{L\to 12^+}\frac{-4}{L}\sqrt{L^2 - 144} = 0$$

31. (a)
$$s^2 = x^2 + y^2$$
$$\frac{dx}{dt} = -450$$
$$\frac{dy}{dt} = -600$$
$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$
$$\frac{ds}{dt} = \frac{x(dx/dt) + y(dy/dt)}{s}$$

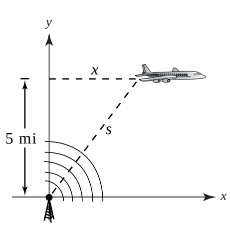


When $x = 150$ and $y = 200$, $s = 250$ and $\frac{ds}{dt} = \frac{150(-450) + 200(-600)}{250} = -750$ mph.

(b) $t = \frac{250}{750} = \frac{1}{3}$ hr $= 20$ min

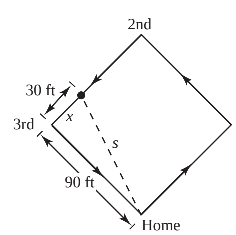
32.
$$x^2 + y^2 = s^2$$
$$2x\frac{dx}{dt} + 0 = 2s\frac{ds}{dt} \quad \left(\text{since } \frac{dy}{dt} = 0\right)$$
$$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$$

When $s = 10$, $x = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$,
$$\frac{dx}{dt} = \frac{10}{5\sqrt{3}}(240) = \frac{480}{\sqrt{3}} = 160\sqrt{3} \approx 277.13 \text{ mph.}$$



33.
$$s^2 = 90^2 + x^2$$
$$x = 30$$
$$\frac{dx}{dt} = -28$$
$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} \Longrightarrow \frac{ds}{dt} = \frac{x}{s}\cdot\frac{dx}{dt}$$

When $x = 30$, $s = \sqrt{90^2 + 30^2} = 30\sqrt{10}$,
$$\frac{ds}{dt} = \frac{30}{30\sqrt{10}}(-28) = \frac{-28}{\sqrt{10}} \approx -8.85 \text{ ft/sec.}$$



34. $s^2 = 90^2 + x^2$

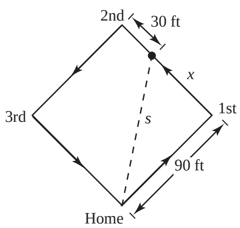
$x = 60$

$\frac{dx}{dt} = 28$

$\frac{ds}{dt} = \frac{x}{s} \cdot \frac{dx}{dt}$

When $x = 60$, $s = \sqrt{90^2 + 60^2} = 30\sqrt{13}$,

$\frac{ds}{dt} = \frac{60}{30\sqrt{13}}(28) = \frac{56}{\sqrt{13}} \approx 15.53$ ft/sec.

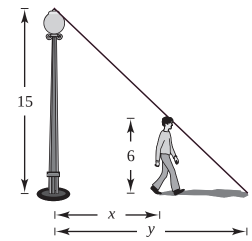


35. (a) $\frac{15}{6} = \frac{y}{y - x} \Longrightarrow 15y - 15x = 6y$

$y = \frac{5}{3}x$

$\frac{dx}{dt} = 5$

$\frac{dy}{dt} = \frac{5}{3} \cdot \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$ ft/sec



(b) $\frac{d(y - x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = \frac{25}{3} - 5 = \frac{10}{3}$ ft/sec

36. (a) $\frac{20}{6} = \frac{y}{y - x}$

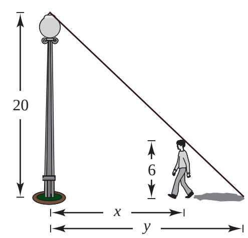
$20y - 20x = 6y$

$14y = 20x$

$y = \frac{10}{7}x$

$\frac{dx}{dt} = -5$

$\frac{dy}{dt} = \frac{10}{7}\frac{dx}{dt} = \frac{10}{7}(-5) = \frac{-50}{7}$ ft/sec



(b) $\frac{d(y - x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = \frac{-50}{7} - (-5) = \frac{-50}{7} + \frac{35}{7} = \frac{-15}{7}$ ft/sec

37. $x(t) = \frac{1}{2}\sin\frac{\pi t}{6}, x^2 + y^2 = 1$

(a) Period: $\frac{2\pi}{\pi/6} = 12$ seconds

(b) When $x = \frac{1}{2}$, $y = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ m.

Lowest point: $\left(0, \frac{\sqrt{3}}{2}\right)$

(c) When $x = \frac{1}{4}$, $y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$ and $t = 1$:

$\frac{dx}{dt} = \frac{1}{2}\left(\frac{\pi}{6}\right)\cos\frac{\pi t}{6} = \frac{\pi}{12}\cos\frac{\pi t}{6}$

$x^2 + y^2 = 1$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Longrightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$

Thus, $\frac{dy}{dt} = -\frac{1/4}{\sqrt{15}/4} \cdot \frac{\pi}{12}\cos\left(\frac{\pi}{6}\right)$

$= \frac{-\pi}{\sqrt{15}}\left(\frac{1}{12}\right)\frac{\sqrt{3}}{2} = \frac{-\pi}{24}\frac{1}{\sqrt{5}} = \frac{-\sqrt{5}\pi}{120}.$

Speed $= \left|\frac{-\sqrt{5}\pi}{120}\right| = \frac{\sqrt{5}\pi}{120}$ m/sec

38. $x(t) = \frac{3}{5}\sin \pi t,\ x^2 + y^2 = 1$

(a) Period: $\frac{2\pi}{\pi} = 2$ seconds

(b) When $x = \frac{3}{5}$, $y = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$ m.

Lowest point: $\left(0, \frac{4}{5}\right)$

(c) When $x = \frac{3}{10}$, $y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$ and $\frac{3}{10} = \frac{3}{5}\sin \pi t \implies \sin \pi t = \frac{1}{2} \implies t = \frac{1}{6}$:

$$\frac{dx}{dt} = \frac{3}{5}\pi \cos \pi t$$

$$x^2 + y^2 = 1$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \implies \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

$$\text{Thus, } \frac{dy}{dt} = \frac{-3/10}{\sqrt{15}/4} \cdot \frac{3}{5}\pi\cos\left(\frac{\pi}{6}\right)$$

$$= \frac{-9\pi}{25\sqrt{5}} = \frac{-9\sqrt{5}\pi}{125}.$$

$$\text{Speed} = \left|\frac{-9\sqrt{5}\pi}{125}\right| \approx 0.5058 \text{ m/sec}$$

39. Since the evaporation rate is proportional to the surface area, $dV/dt = k(4\pi r^2)$. However, since $V = (4/3)\pi r^3$, we have

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}.$$

Therefore, $k(4\pi r^2) = 4\pi r^2\frac{dr}{dt} \implies k = \frac{dr}{dt}$.

40.
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{dR_1}{dt} = 1$$

$$\frac{dR_2}{dt} = 1.5$$

$$\frac{1}{R^2}\cdot\frac{dR}{dt} = \frac{1}{R_1^2}\cdot\frac{dR_1}{dt} + \frac{1}{R_2^2}\cdot\frac{dR_2}{dt}$$

When $R_1 = 50$ and $R_2 = 75$:

$$R = 30$$

$$\frac{dR}{dt} = (30)^2\left[\frac{1}{(50)^2}(1) + \frac{1}{(75)^2}(1.5)\right] = 0.6 \text{ ohms/sec}$$

41.
$$pV^{1.3} = k$$

$$1.3pV^{0.3}\frac{dV}{dt} + V^{1.3}\frac{dp}{dt} = 0$$

$$V^{0.3}\left(1.3p\frac{dV}{dt} + V\frac{dp}{dt}\right) = 0$$

$$1.3p\frac{dV}{dt} = -V\frac{dp}{dt}$$

42.
$$rg\tan\theta = v^2$$

$32r\tan\theta = v^2$, r is a constant.

$$32r\sec^2\theta\frac{d\theta}{dt} = 2v\frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{16r}{v}\sec^2\theta\frac{d\theta}{dt}$$

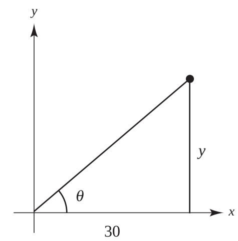
Likewise, $\frac{d\theta}{dt} = \frac{v}{16r}\cos^2\theta\frac{dv}{dt}$.

43. $\tan\theta = \dfrac{y}{30}$

$$\frac{dy}{dt} = 3 \text{ m/sec}$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{30}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{30}\cos^2\theta \cdot \frac{dy}{dt}$$



When $y = 30$, $\theta = \dfrac{\pi}{4}$, and $\cos\theta = \dfrac{\sqrt{2}}{2}$. Thus, $\dfrac{d\theta}{dt} = \dfrac{1}{30}\left(\dfrac{1}{2}\right)(3) = \dfrac{1}{20}$ rad/sec.

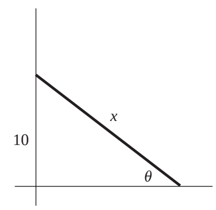
44. $\sin\theta = \dfrac{10}{x}$

$$\frac{dx}{dt} = (-1) \text{ ft/sec}$$

$$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{-10}{x^2}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-10}{x^2}\frac{dx}{dt}(\sec\theta)$$

$$= \frac{-10}{25^2}(-1)\frac{25}{\sqrt{25^2 - 10^2}} = \frac{10}{25}\frac{1}{5\sqrt{21}} = \frac{2}{25\sqrt{21}} = \frac{2\sqrt{21}}{525} \approx 0.017 \text{ rad/sec}$$



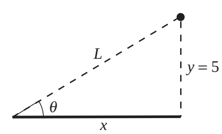
45. $\tan\theta = \dfrac{y}{x}, y = 5$

$$\frac{dx}{dt} = -600 \text{ mi/hr}$$

$$(\sec^2\theta)\frac{d\theta}{dt} = -\frac{5}{x^2}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta\left(-\frac{5}{x^2}\right)\frac{dx}{dt} = \frac{x^2}{L^2}\left(-\frac{5}{x^2}\right)\frac{dx}{dt}$$

$$= \left(-\frac{5^2}{L^2}\right)\left(\frac{1}{5}\right)\frac{dx}{dt} = (-\sin^2\theta)\left(\frac{1}{5}\right)(-600) = 120\sin^2\theta$$



(a) When $\theta = 30°$,

$$\frac{d\theta}{dt} = \frac{120}{4}$$

$$= 30 \text{ rad/hr} = \frac{1}{2} \text{ rad/min.}$$

(b) When $\theta = 60°$,

$$\frac{d\theta}{dt} = 120\left(\frac{3}{4}\right)$$

$$= 90 \text{ rad/hr} = \frac{3}{2} \text{ rad/min.}$$

(c) When $\theta = 75°$,

$$\frac{d\theta}{dt} = 120\sin^2 75°$$

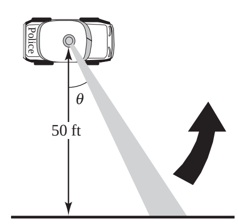
$$\approx 111.96 \text{ rad/hr} \approx 1.87 \text{ rad/min.}$$

46. $\tan\theta = \dfrac{x}{50}$

$$\frac{d\theta}{dt} = 30(2\pi) = 60\pi \text{ rad/min} = \pi \text{ rad/sec}$$

$$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{50}\left(\frac{dx}{dt}\right)$$

$$\frac{dx}{dt} = 50\sec^2\theta\left(\frac{d\theta}{dt}\right)$$



(a) When $\theta = 30°$, $\dfrac{dx}{dt} = \dfrac{200\pi}{3}$ ft/sec.

(b) When $\theta = 60°$, $\dfrac{dx}{dt} = 200\pi$ ft/sec.

(c) When $\theta = 70°$, $\dfrac{dx}{dt} \approx 427.43\pi$ ft/sec.

47. $\frac{d\theta}{dt} = (10 \text{ rev/sec})(2\pi \text{ rad/rev}) = 20\pi \text{ rad/sec}$

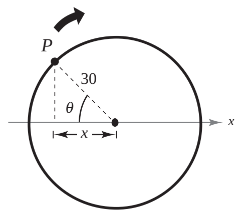
(a) $\cos\theta = \frac{x}{30}$

$$-\sin\theta \frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt}$$

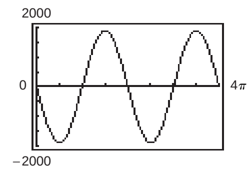
$$\frac{dx}{dt} = -30\sin\theta\frac{d\theta}{dt}$$

$$= -30\sin\theta(20\pi)$$

$$= -600\pi\sin\theta$$



(b)



(c) $|dx/dt| = |-600\pi\sin\theta|$ is greatest when

$|\sin\theta| = 1 \implies \theta = \frac{\pi}{2} + n\pi$ (or $90° + n \cdot 180°$).

$|dx/dt|$ is least when $\theta = n\pi$ (or $n \cdot 180°$).

(d) For $\theta = 30°$,

$$\frac{dx}{dt} = -600\pi\sin(30°)$$

$$= -600\pi\frac{1}{2} = -300\pi \text{ cm/sec.}$$

For $\theta = 60°$,

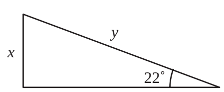
$$\frac{dx}{dt} = -600\pi\sin(60°)$$

$$= -600\pi\frac{\sqrt{3}}{2} = -300\sqrt{3}\pi \text{ cm/sec.}$$

48. $\sin 22° = \frac{x}{y}$

$$0 = -\frac{x}{y^2}\cdot\frac{dy}{dt} + \frac{1}{y}\cdot\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x}{y}\cdot\frac{dy}{dt} = (\sin 22°)(240) \approx 89.9056 \text{ mi/hr}$$



49. $\tan\theta = \frac{x}{50} \implies x = 50\tan\theta$

$$\frac{dx}{dt} = 50\sec^2\theta\ \frac{d\theta}{dt}$$

$$2 = 50\sec^2\theta\ \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{25}\cos^2\theta, \quad -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

50. (a) $dy/dt = 3(dx/dt)$ means that y changes three times as fast as x changes.

(b) y changes slowly when $x \approx 0$ or $x \approx L$. y changes more rapidly when x is near the middle of the interval.

51. $x^2 + y^2 = 25$; acceleration of the top of the ladder $= \frac{d^2y}{dt^2}$

First derivative: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

Second derivative: $x\frac{d^2x}{dt^2} + \frac{dx}{dt}\cdot\frac{dx}{dt} + y\frac{d^2y}{dt^2} + \frac{dy}{dt}\cdot\frac{dy}{dt} = 0$

$$\frac{d^2y}{dt^2} = \left(\frac{1}{y}\right)\left[-x\frac{d^2x}{dt^2} - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2\right]$$

When $x = 7$, $y = 24$, $\frac{dy}{dt} = -\frac{7}{12}$, and $\frac{dx}{dt} = 2$ (see Exercise 27). Since $\frac{dx}{dt}$ is constant, $\frac{d^2x}{dt^2} = 0$.

$$\frac{d^2y}{dt^2} = \frac{1}{24}\left[-7(0) - (2)^2 - \left(-\frac{7}{12}\right)^2\right] = \frac{1}{24}\left[-4 - \frac{49}{144}\right] = \frac{1}{24}\left[-\frac{625}{144}\right] \approx -0.1808 \text{ ft/sec}^2$$

52. $L^2 = 144 + x^2$; acceleration of the boat $= \dfrac{d^2x}{dt^2}$

First derivative: $2L\dfrac{dL}{dt} = 2x\dfrac{dx}{dt}$

$$L\frac{dL}{dt} = x\frac{dx}{dt}$$

Second derivative: $L\dfrac{d^2L}{dt^2} + \dfrac{dL}{dt}\cdot\dfrac{dL}{dt} = x\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt}\cdot\dfrac{dx}{dt}$

$$\frac{d^2x}{dt^2} = \left(\frac{1}{x}\right)\left[L\frac{d^2L}{dt^2} + \left(\frac{dL}{dt}\right)^2 - \left(\frac{dx}{dt}\right)^2\right]$$

When $L = 13$, $x = 5$, $\dfrac{dx}{dt} = -10.4$, and $\dfrac{dL}{dt} = -4$ (see Exercise 30). Since $\dfrac{dL}{dt}$ is constant, $\dfrac{d^2L}{dt^2} = 0$.

$$\frac{d^2x}{dt^2} = \frac{1}{5}[13(0) + (-4)^2 - (-10.4)^2]$$

$$= \frac{1}{5}[16 - 108.16] = \frac{1}{5}[-92.16] = -18.432 \text{ ft/sec}^2$$

53. (a) $m(s) = 0.3754s^3 - 18.780s^2 + 313.23s - 1707.8$

(b) $\dfrac{dm}{dt} = (1.1262s^2 - 37.560s + 313.23)\dfrac{ds}{dt}$

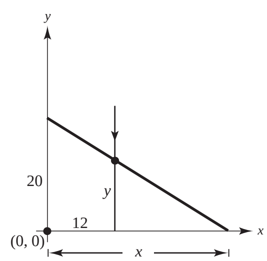
If $t = 10$ and $\dfrac{ds}{dt} = 0.75$, then $s = 17.8$ and $\dfrac{dm}{dt} \approx 1.1154$ million/year.

54. $y(t) = -4.9t^2 + 20$

$$\frac{dy}{dt} = -9.8t$$

$$y(1) = -4.9 + 20 = 15.1$$

$$y'(1) = -9.8$$



By similar triangles: $\dfrac{20}{x} = \dfrac{y}{x - 12}$

$$20x - 240 = xy$$

When $y = 15.1$: $20x - 240 = x(15.1)$

$$(20 - 15.1)x = 240$$

$$x = \frac{240}{4.9}$$

$$20x - 240 = xy$$

$$20\frac{dx}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x}{20 - y}\frac{dy}{dt}$$

At $t = 1$, $\dfrac{dx}{dt} = \dfrac{240/4.9}{20 - 15.1}(-9.8) \approx -97.96$ m/sec.

Review Exercises for Chapter 2

1. $f(x) = x^2 - 2x + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[(x + \Delta x)^2 - 2(x + \Delta x) + 3] - [x^2 - 2x + 3]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x^2 + 2x(\Delta x) + (\Delta x)^2 - 2x - 2(\Delta x) + 3) - (x^2 - 2x + 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 - 2(\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 2) = 2x - 2$$

2. $f(x) = \sqrt{x} + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(\sqrt{x + \Delta x} + 1) - (\sqrt{x} + 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

3. $f(x) = \dfrac{x + 1}{x - 1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\dfrac{x + \Delta x + 1}{x + \Delta x - 1} - \dfrac{x + 1}{x - 1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x + 1)(x - 1) - (x + \Delta x - 1)(x + 1)}{\Delta x(x + \Delta x - 1)(x - 1)}$$

$$= \lim_{\Delta x \to 0} \frac{(x^2 + x\,\Delta x + x - x - \Delta x - 1) - (x^2 + x\,\Delta x - x + x + \Delta x - 1)}{\Delta x(x + \Delta x - 1)(x - 1)}$$

$$= \lim_{\Delta x \to 0} \frac{-2\,\Delta x}{\Delta x(x + \Delta x - 1)(x - 1)} = \lim_{\Delta x \to 0} \frac{-2}{(x + \Delta x - 1)(x - 1)} = \frac{-2}{(x - 1)^2}$$

4. $f(x) = \dfrac{2}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\dfrac{2}{x + \Delta x} - \dfrac{2}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x - (2x + 2\,\Delta x)}{\Delta x(x + \Delta x)x}$$

$$= \lim_{\Delta x \to 0} \frac{-2\,\Delta x}{\Delta x(x + \Delta x)x}$$

$$= \lim_{\Delta x \to 0} \frac{-2}{(x + \Delta x)x} = \frac{-2}{x^2}$$

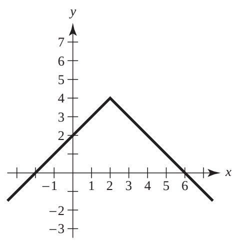
5. f is differentiable for all $x \neq -1$.

6. f is differentiable for all $x \neq -3$.

7. $f(x) = 4 - |x - 2|$

(a) Continuous at $x = 2$

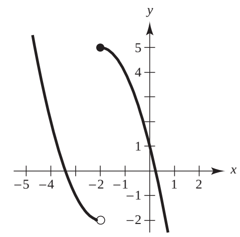
(b) Not differentiable at $x = 2$ because of the sharp turn in the graph.



8. $f(x) = \begin{cases} x^2 + 4x + 2, & \text{if } x < -2 \\ 1 - 4x - x^2, & \text{if } x \geq -2 \end{cases}$

(a) Nonremovable discontinuity at $x = -2$

(b) Not differentiable at $x = -2$ because the function is discontinuous there.



9. Using the limit definition, you obtain $g'(x) = \frac{4}{3}x - \frac{1}{6}$. At $x = -1$,

$$g'(-1) = -\frac{4}{3} - \frac{1}{6} = \frac{-3}{2}.$$

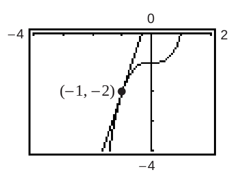
10. Using the limit definition, you obtain $h'(x) = \frac{3}{8} - 4x$. At $x = -2$,

$$h'(-2) = \frac{3}{8} - 4(-2) = \frac{67}{8}.$$

11. (a) Using the limit definition, $f'(x) = 3x^2$. At $x = -1$, $f'(-1) = 3$. The tangent line is

$$y - (-2) = 3(x - (-1))$$
$$y = 3x + 1.$$

(b)



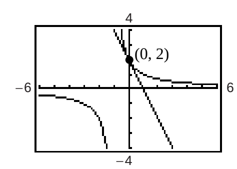
12. (a) Using the limit definition,

$$f'(x) = \frac{-2}{(x + 1)^2}.$$

At $x = 0$, $f'(0) = -2$. The tangent line is

$$y - 2 = -2(x - 0)$$
$$y = -2x + 2.$$

(b)



13.
$$\begin{aligned} g'(2) &= \lim_{x\to 2} \frac{g(x) - g(2)}{x - 2} \\ &= \lim_{x\to 2} \frac{x^2(x - 1) - 4}{x - 2} \\ &= \lim_{x\to 2} \frac{x^3 - x^2 - 4}{x - 2} \\ &= \lim_{x\to 2} \frac{(x - 2)(x^2 + x + 2)}{x - 2} \\ &= \lim_{x\to 2} (x^2 + x + 2) = 8 \end{aligned}$$

14.
$$\begin{aligned} f'(2) &= \lim_{x\to 2} \frac{f(x) - f(2)}{x - 2} \\ &= \lim_{x\to 2} \frac{\frac{1}{x + 1} - \frac{1}{3}}{x - 2} \\ &= \lim_{x\to 2} \frac{3 - x - 1}{(x - 2)(x + 1)3} \\ &= \lim_{x\to 2} \frac{-1}{(x + 1)3} = \frac{-1}{9} \end{aligned}$$

15. $y = 25$

$y' = 0$

16. $y = -12$

$y' = 0$

17. $f(x) = x^8$

$f'(x) = 8x^7$

18. $g(x) = x^{12}$

$g'(x) = 12x^{11}$

19. $h(t) = 3t^4$

$h'(t) = 12t^3$

20. $f(t) = -8t^5$

$f'(t) = -40t^4$

21. $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$= 3x(x - 2)$

22. $g(s) = 4s^4 - 5s^2$

$g'(s) = 16s^3 - 10s$

23. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x} = 6x^{1/2} + 3x^{1/3}$

$h'(x) = 3x^{-1/2} + x^{-2/3} = \dfrac{3}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^2}}$

24. $f(x) = x^{1/2} - x^{-1/2}$

$f'(x) = \dfrac{1}{2}x^{-1/2} + \dfrac{1}{2}x^{-3/2} = \dfrac{x + 1}{2x^{3/2}}$

25. $g(t) = \dfrac{2}{3}t^{-2}$

$g'(t) = \dfrac{-4}{3}t^{-3} = \dfrac{-4}{3t^3}$

26. $h(x) = \dfrac{2}{9}x^{-2}$

$h'(x) = \dfrac{-4}{9}x^{-3} = \dfrac{-4}{9x^3}$

27. $f(\theta) = 2\theta - 3\sin\theta$

$f'(\theta) = 2 - 3\cos\theta$

28. $g(\alpha) = 4\cos\alpha + 6$

$g'(\alpha) = -4\sin\alpha$

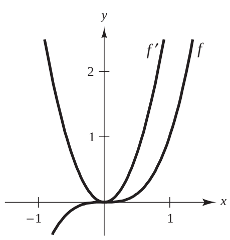
29. $f(\theta) = 3\cos\theta - \dfrac{\sin\theta}{4}$

$f'(\theta) = -3\sin\theta - \dfrac{\cos\theta}{4}$

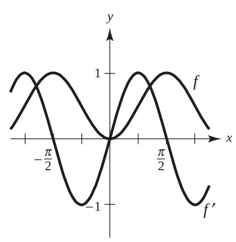
30. $g(\alpha) = \dfrac{5\sin\alpha}{3} - 2\alpha$

$g'(\alpha) = \dfrac{5\cos\alpha}{3} - 2$

31.



32.



33. $F = 200\sqrt{T}$

$F'(t) = \dfrac{100}{\sqrt{T}}$

(a) When $T = 4$,
$F'(4) = 50$ vibrations/sec/lb.

(b) When $T = 9$,
$F'(9) = 33\frac{1}{3}$ vibrations/sec/lb.

34. $s = -16t^2 + s_0$

First ball:

$-16t^2 + 100 = 0$

$t = \sqrt{\dfrac{100}{16}} = \dfrac{10}{4} = 2.5$ seconds to hit ground

Second ball:

$-16t^2 + 75 = 0$

$t^2 = \sqrt{\dfrac{75}{16}} = \dfrac{5\sqrt{3}}{4} \approx 2.165$ seconds to hit ground

Since the second ball was released one second after the first ball, the first ball will hit the ground first. The second ball will hit the ground $3.165 - 2.5 = 0.665$ second later.

35. $s(t) = -16t^2 + s_0$

$s(9.2) = -16(9.2)^2 + s_0 = 0$

$s_0 = 1354.24$

The building is approximately 1354 feet high (or 415 m).

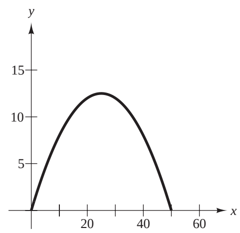
36. $s(t) = -16t^2 + 14{,}400 = 0$

$16t^2 = 14{,}400$

$t = 30$ sec

Since 600 mph $= \frac{1}{6}$ mi/sec, in 30 seconds the bomb will move horizontally $\left(\frac{1}{6}\right)(30) = 5$ miles.

37. (a)



Total horizontal distance: 50

(b) $0 = x - 0.02x^2$

$0 = x\left(1 - \dfrac{x}{50}\right)$ implies $x = 50$.

(c) Ball reaches maximum height when $x = 25$.

(d) $y = x - 0.02x^2$

$y' = 1 - 0.04x$

$y'(0) = 1$

$y'(10) = 0.6$

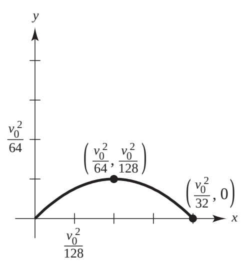
$y'(25) = 0$

$y'(30) = -0.2$

$y'(50) = -1$

(e) $y'(25) = 0$

38.



(a) $y = x - \dfrac{32}{{v_0}^2}x^2 = x\left(1 - \dfrac{32}{{v_0}^2}x\right)$

$= 0$ if $x = 0$ or $x = \dfrac{{v_0}^2}{32}$

Projectile strikes the ground when $x = {v_0}^2/32$.

Projectile reaches its maximum height at $x = {v_0}^2/64$ (one-half the distance).

(b) $y' = 1 - \dfrac{64}{{v_0}^2}x$

When $x = \dfrac{{v_0}^2}{64}$, $y' = 1 - \dfrac{64}{{v_0}^2}\left(\dfrac{{v_0}^2}{64}\right) = 0$.

(c) $y = x - \dfrac{32}{{v_0}^2}x^2 = x\left(1 - \dfrac{32}{{v_0}^2}x\right) = 0$

when $x = 0$ and $x = {s_0}^2/32$. Therefore, the range is $x = {v_0}^2/32$. When the initial velocity is doubled the range is

$$x = \frac{(2v_0)^2}{32} = \frac{4{v_0}^2}{32}$$

or four times the initial range. From part (a), the maximum height occurs when $x = {v_0}^2/64$. The maximum height is

$$y\left(\frac{{v_0}^2}{64}\right) = \frac{{v_0}^2}{64} - \frac{32}{{v_0}^2}\left(\frac{{v_0}^2}{64}\right)^2 = \frac{{v_0}^2}{64} - \frac{{v_0}^2}{128} = \frac{{v_0}^2}{128}.$$

If the initial velocity is doubled, the maximum height is

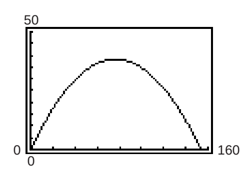
$$y\left[\frac{(2v_0)^2}{64}\right] = \frac{(2v_0)^2}{128} = 4\left(\frac{{v_0}^2}{128}\right)$$

or four times the original maximum height.

(d) $v_0 = 70$ ft/sec

Range: $x = \dfrac{{v_0}^2}{32} = \dfrac{(70)^2}{32} = 153.125$ ft

Maximum height: $y = \dfrac{{v_0}^2}{128} = \dfrac{(70)^2}{128} \approx 38.28$ ft



39. $x(t) = t^2 - 3t + 2 = (t - 2)(t - 1)$

(a) $v(t) = x'(t) = 2t - 3$

(b) $v(t) < 0$ for $t < \frac{3}{2}$

(c) $v(t) = 0$ for $t = \frac{3}{2}$

$x = \left(\frac{3}{2} - 2\right)\left(\frac{3}{2} - 1\right) = \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{1}{4}$

(d) $x(t) = 0$ for $t = 1, 2$

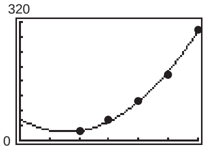
$|v(1)| = |2(1) - 3| = 1$

$|v(2)| = |2(2) - 3| = 1$

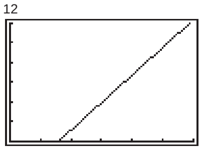
The speed is 1 when the position is 0.

40. (a) $y = 0.14x^2 - 4.43x + 58.4$

(b)



(c)



(d) If $x = 65$, $y \approx 362$ feet.

(e) As the speed increases, the stopping distance increases at an increasing rate.

41. $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

$f'(x) = (3x^2 + 7)(2x - 2) + (x^2 - 2x + 3)(6x)$

$= 2(6x^3 - 9x^2 + 16x - 7)$

42. $g(x) = (x^3 - 3x)(x + 2)$

$g'(x) = (x^3 - 3x)(1) + (x + 2)(3x^2 - 3)$

$= x^3 - 3x + 3x^3 + 6x^2 - 3x - 6$

$= 4x^3 + 6x^2 - 6x - 6$

43. $h(x) = \sqrt{x}\sin x = x^{1/2}\sin x$

$$h'(x) = \frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x$$

44. $f(t) = t^3\cos t$

$f'(t) = t^3(-\sin t) + \cos t(3t^2)$

$= -t^3\sin t + 3t^2\cos t$

45. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2}$$

$$= \frac{-(x^2 + 1)}{(x^2 - 1)^2}$$

46. $f(x) = \dfrac{6x - 5}{x^2 + 1}$

$$f'(x) = \frac{(x^2 + 1)(6) - (6x - 5)(2x)}{(x^2 + 1)^2}$$

$$= \frac{2(3 + 5x - 3x^2)}{(x^2 + 1)^2}$$

47. $f(x) = (4 - 3x^2)^{-1}$

$$f'(x) = -(4 - 3x^2)^{-2}(-6x) = \frac{6x}{(4 - 3x^2)^2}$$

48. $f(x) = 9(3x^2 - 2x)^{-1}$

$$f'(x) = -9(3x^2 - 2x)^{-2}(6x - 2) = \frac{18(1 - 3x)}{(3x^2 - 2x)^2}$$

49. $y = \dfrac{x^2}{\cos x}$

$$y' = \frac{\cos x(2x) - x^2(-\sin x)}{\cos^2 x} = \frac{2x\cos x + x^2\sin x}{\cos^2 x}$$

50. $y = \dfrac{\sin x}{x^2}$

$$y' = \frac{(x^2)\cos x - (\sin x)(2x)}{x^4} = \frac{x\cos x - 2\sin x}{x^3}$$

51. $y = 3x^2\sec x$

$y' = 3x^2\sec x\tan x + 6x\sec x$

52. $y = 2x - x^2\tan x$

$y' = 2 - x^2\sec^2 x - 2x\tan x$

53. $y = x\cos x - \sin x$

$y' = -x\sin x + \cos x - \cos x = -x\sin x$

54. $g(x) = 3x\sin x + x^2\cos x$

$g'(x) = 3x\cos x + 3\sin x - x^2\sin x + 2x\cos x$

$= 5x\cos x + (3 - x^2)\sin x$

55. $f(x) = \dfrac{2x^3 - 1}{x^2} = 2x - x^{-2}, \quad (1, 1)$

$f'(x) = 2 + 2x^{-3}$

$f'(1) = 4$

Tangent line: $y - 1 = 4(x - 1)$

$y = 4x - 3$

56. $f(x) = \dfrac{x + 1}{x - 1}, \quad \left(\dfrac{1}{2}, -3\right)$

$f'(x) = \dfrac{(x - 1) - (x + 1)}{(x - 1)^2} = \dfrac{-2}{(x - 1)^2}$

$f'\left(\dfrac{1}{2}\right) = \dfrac{-2}{(1/4)} = -8$

Tangent line: $y + 3 = -8\left(x - \dfrac{1}{2}\right)$

$y = -8x + 1$

57. $f(x) = -x\tan x, \quad (0, 0)$

$f'(x) = -x\sec^2 x - \tan x$

$f'(0) = 0$

Tangent line: $y - 0 = 0(x - 0)$

$y = 0$

58. $f(x) = \dfrac{1 + \sin x}{1 - \sin x}, \quad (\pi, 1)$

$f'(x) = \dfrac{(1 - \sin x)\cos x - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2}$

$f'(\pi) = \dfrac{-1 - 1}{1} = -2$

Tangent line: $y - 1 = -2(x - \pi)$

$y = -2x + 2\pi + 1$

59. $v(t) = 36 - t^2, \quad 0 \le t \le 6$

$a(t) = v'(t) = -2t$

$v(4) = 36 - 16 = 20$ m/sec

$a(4) = -8$ m/sec^2

60. $v(t) = \dfrac{90t}{4t + 10}$

$a(t) = \dfrac{(4t + 10)90 - 90t(4)}{(4t + 10)^2}$

$= \dfrac{900}{(4t + 10)^2} = \dfrac{225}{(2t + 5)^2}$

(a) $v(1) = \dfrac{90}{14} \approx 6.43$ ft/sec

$a(1) = \dfrac{225}{49} \approx 4.59$ ft/sec^2

(b) $v(5) = \dfrac{90(5)}{30} = 15$ ft/sec

$a(5) = \dfrac{225}{15^2} = 1$ ft/sec^2

(c) $v(10) = \dfrac{90(10)}{50} = 18$ ft/sec

$a(10) = \dfrac{225}{25^2} = 0.36$ ft/sec^2

61. $g(t) = t^3 - 3t + 2$

$g'(t) = 3t^2 - 3$

$g''(t) = 6t$

62. $f(x) = 12x^{1/4}$

$f'(x) = 3x^{-3/4}$

$f''(x) = \dfrac{-9}{4}x^{-7/4} = \dfrac{-9}{4x^{7/4}}$

63. $f(\theta) = 3\tan\theta$

$f'(\theta) = 3\sec^2\theta$

$f''(\theta) = 6\sec\theta(\sec\theta\tan\theta)$

$= 6\sec^2\theta\tan\theta$

64.
$$h(t) = 4\sin t - 5\cos t$$
$$h'(t) = 4\cos t + 5\sin t$$
$$h''(t) = -4\sin t + 5\cos t$$

65.
$$y = 2\sin x + 3\cos x$$
$$y' = 2\cos x - 3\sin x$$
$$y'' = -2\sin x - 3\cos x$$
$$y'' + y = -(2\sin x + 3\cos x) + (2\sin x + 3\cos x) = 0$$

66.
$$y = \frac{(10 - \cos x)}{x}$$
$$xy + \cos x = 10$$
$$xy' + y - \sin x = 0$$
$$xy' + y = \sin x$$

67.
$$h(x) = \left(\frac{x-3}{x^2+1}\right)^2$$
$$h'(x) = 2\left(\frac{x-3}{x^2+1}\right)\left(\frac{(x^2+1)(1) - (x-3)(2x)}{(x^2+1)^2}\right) = \frac{2(x-3)(-x^2+6x+1)}{(x^2+1)^3}$$

68.
$$f(x) = \left(x^2 + \frac{1}{x}\right)^5$$
$$f'(x) = 5\left(x^2 + \frac{1}{x}\right)^4\left(2x - \frac{1}{x^2}\right)$$

69.
$$f(s) = (s^2-1)^{5/2}(s^3+5)$$
$$f'(s) = (s^2-1)^{5/2}(3s^2) + (s^3+5)\left(\frac{5}{2}\right)(s^2-1)^{3/2}(2s)$$
$$= s(s^2-1)^{3/2}[3s(s^2-1) + 5(s^3+5)]$$
$$= s(s^2-1)^{3/2}(8s^3 - 3s + 25)$$

70.
$$h(\theta) = \frac{\theta}{(1-\theta)^3}$$
$$h'(\theta) = \frac{(1-\theta)^3 - \theta[3(1-\theta)^2(-1)]}{(1-\theta)^6}$$
$$= \frac{(1-\theta)^2(1-\theta+3\theta)}{(1-\theta)^6} = \frac{2\theta+1}{(1-\theta)^4}$$

71.
$$y = 3\cos(3x+1)$$
$$y' = -3\sin(3x+1)(3)$$
$$y' = -9\sin(3x+1)$$

72.
$$y = 1 - \cos 2x + 2\cos^2 x$$
$$y' = 2\sin 2x - 4\cos x\sin x$$
$$= 2[2\sin x\cos x] - 4\sin x\cos x$$
$$= 0$$

73.
$$y = \frac{x}{2} - \frac{\sin 2x}{4}$$
$$y' = \frac{1}{2} - \frac{1}{4}\cos 2x(2) = \frac{1}{2}(1 - \cos 2x) = \sin^2 x$$

74.
$$y = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5}$$
$$y' = \sec^6 x(\sec x\tan x) - \sec^4 x(\sec x\tan x)$$
$$= \sec^5 x\tan x(\sec^2 x - 1)$$
$$= \sec^5 x\tan^3 x$$

75.
$$y = \frac{2}{3}\sin^{3/2} x - \frac{2}{7}\sin^{7/2} x$$
$$y' = \sin^{1/2} x\cos x - \sin^{5/2} x\cos x$$
$$= (\cos x)\sqrt{\sin x}(1 - \sin^2 x)$$
$$= (\cos^3 x)\sqrt{\sin x}$$

76.
$$f(x) = \frac{3x}{\sqrt{x^2+1}}$$
$$f'(x) = \frac{3(x^2+1)^{1/2} - 3x\frac{1}{2}(x^2+1)^{-1/2}(2x)}{x^2+1}$$
$$= \frac{3(x^2+1) - 3x^2}{(x^2+1)^{3/2}} = \frac{3}{(x^2+1)^{3/2}}$$

77.
$$y = \frac{\sin \pi x}{x+2}$$
$$y' = \frac{(x+2)\pi\cos\pi x - \sin\pi x}{(x+2)^2}$$

78. $y = \dfrac{\cos(x-1)}{x-1}$

$y' = \dfrac{-(x-1)\sin(x-1) - \cos(x-1)(1)}{(x-1)^2}$

$= -\dfrac{1}{(x-1)^2}[(x-1)\sin(x-1) + \cos(x-1)]$

79. $f(x) = \sqrt{1-x^3}$

$f'(x) = \dfrac{1}{2}(1-x^3)^{-1/2}(-3x^2) = \dfrac{-3x^2}{2\sqrt{1-x^3}}$

$f'(-2) = \dfrac{-12}{2(3)} = -2$

80. $f(x) = \sqrt[3]{x^2-1}$

$f'(x) = \dfrac{1}{3}(x^2-1)^{-2/3}(2x) = \dfrac{2x}{3(x^2-1)^{2/3}}$

$f'(3) = \dfrac{2(3)}{3(4)} = \dfrac{1}{2}$

81. $y = \dfrac{1}{2}\csc 2x$

$y' = -\csc 2x \cot 2x$

$y'\left(\dfrac{\pi}{4}\right) = 0$

82. $y = \csc 3x + \cot 3x$

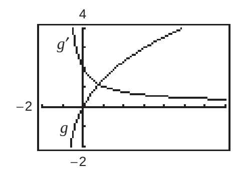
$y' = -3\csc 3x \cot 3x - 3\csc^2 3x$

$y'\left(\dfrac{\pi}{6}\right) = 0 - 3 = -3$

83. $g(x) = 2x(x+1)^{-1/2}$

$g'(x) = \dfrac{x+2}{(x+1)^{3/2}}$

g' does not equal zero for any value of x in the domain. The graph of g has no horizontal tangent lines.

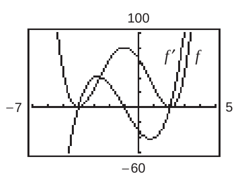


84. $f(x) = [(x-2)(x+4)]^2 = (x^2+2x-8)^2$

$f'(x) = 4(x^3+3x^2-6x-8)$

$= 4(x-2)(x+1)(x+4)$

The zeros of f' correspond to the points on the graph of f where the tangent line is horizontal.

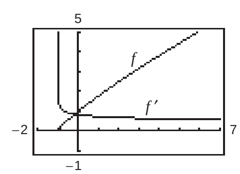


85. $f(t) = \sqrt{t+1}\,\sqrt[3]{t+1}$

$f(t) = (t+1)^{1/2}(t+1)^{1/3} = (t+1)^{5/6}$

$f'(t) = \dfrac{5}{6(t+1)^{1/6}}$

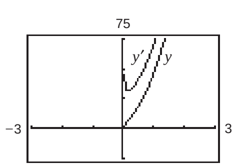
f' does not equal zero for any x in the domain. The graph of f has no horizontal tangent lines.



86. $y = \sqrt{3x}(x+2)^3$

$y' = \dfrac{3(x+2)^2(7x+2)}{2\sqrt{3x}}$

y' does not equal zero for any x in the domain. The graph has no horizontal tangent lines.



87. $f(t) = t^2(t-1)^5$, $(2, 4)$

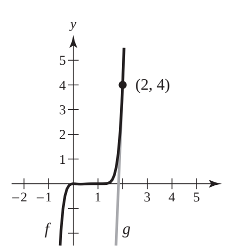
(a) $f'(t) = t(t-1)^4(7t-2)$

$f'(2) = 24$

(b) $y - 4 = 24(t-2)$

$y = 24t - 44$

(c)



88. $g(x) = x\sqrt{x^2+1}, \quad \left(3, 3\sqrt{10}\right)$

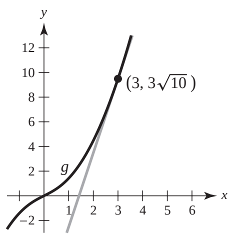
(a) $g'(x) = \sqrt{x^2+1} + \dfrac{x^2}{\sqrt{x^2+1}}$

$g'(3) = \dfrac{19\sqrt{10}}{10}$

(b) $y - 3\sqrt{10} = \dfrac{19\sqrt{10}}{10}(x-3)$

$y = \dfrac{19\sqrt{10}}{10}x - \dfrac{27\sqrt{10}}{10}$

(c)



89. $y = \tan\sqrt{1-x}, \quad \left(-2, \tan\sqrt{3}\right)$

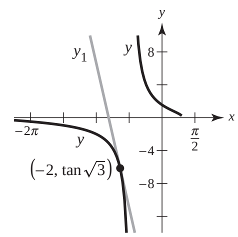
(a) $y' = \dfrac{-1}{2\sqrt{1-x}\cos^2\sqrt{1-x}}$

$y'(-2) = \dfrac{-\sqrt{3}}{6\cos^2\sqrt{3}} \approx -11.1983$

(b) $y - \tan\sqrt{3} = \dfrac{-\sqrt{3}}{6\cos^2\sqrt{3}}(x+2)$

$y = \dfrac{-\sqrt{3}}{6\cos^2\sqrt{3}}x + \tan\sqrt{3} - \dfrac{\sqrt{3}}{3\cos^2\sqrt{3}}$

(c)



90. $y = 2\csc^3\left(\sqrt{x}\right) = \dfrac{2}{\sin^3\left(\sqrt{x}\right)}, \quad (1, 2\csc^3(1))$

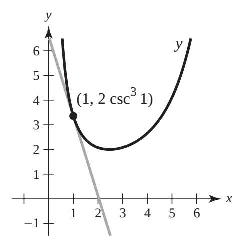
(a) $y' = \dfrac{-3\cos\left(\sqrt{x}\right)}{\sqrt{x}\sin^4\left(\sqrt{x}\right)}$

$y'(1) = \dfrac{-3\cos(1)}{\sin^4(1)}$

(b) $y - 2\csc^3(1) = \dfrac{-3\cos(1)}{\sin^4(1)}(x-1)$

$y = \dfrac{-3\cos(1)}{\sin^4(1)}x + \dfrac{2}{\sin^3(1)} + \dfrac{3\cos(1)}{\sin^4(1)}$

(c)



91. $y = 2x^2 + \sin 2x$

$y' = 4x + 2\cos 2x$

$y'' = 4 - 4\sin 2x$

92. $y = x^{-1} + \tan x$

$y' = -x^{-2} + \sec^2 x$

$y'' = 2x^{-3} + 2\sec x(\sec x \tan x)$

$= \dfrac{2}{x^3} + 2\sec^2 x \tan x$

93. $f(x) = \cot x$

$f'(x) = -\csc^2 x$

$f''(x) = -2\csc x(-\csc x \cdot \cot x)$

$= 2\csc^2 x \cot x$

94. $y = \sin^2 x$

$y' = 2\sin x\cos x = \sin 2x$

$y'' = 2\cos 2x$

95. $f(t) = \dfrac{t}{(1-t)^2}$

$f'(t) = \dfrac{t+1}{(1-t)^3}$

$f''(t) = \dfrac{2(t+2)}{(1-t)^4}$

96. $g(x) = \dfrac{6x-5}{x^2+1}$

$g'(x) = \dfrac{2(-3x^2+5x+3)}{(x^2+1)^2}$

$g''(x) = \dfrac{2(6x^3-15x^2-18x+5)}{(x^2+1)^3}$

97. $g(\theta) = \tan 3\theta - \sin(\theta - 1)$

$g'(\theta) = 3\sec^2 3\theta - \cos(\theta - 1)$

$g''(\theta) = 18\sec^2 3\theta \tan 3\theta + \sin(\theta - 1)$

98. $h(x) = x\sqrt{x^2-1}$

$h'(x) = \dfrac{2x^2-1}{\sqrt{x^2-1}}$

$h''(x) = \dfrac{x(2x^2-3)}{(x^2-1)^{3/2}}$

99. $T = \dfrac{700}{t^2 + 4t + 10}$

$T = 700(t^2 + 4t + 10)^{-1}$

$T' = \dfrac{-1400(t + 2)}{(t^2 + 4t + 10)^2}$

(a) When $t = 1$,

$$T' = \frac{-1400(1 + 2)}{(1 + 4 + 10)^2} \approx -18.667 \text{ deg/hr.}$$

(b) When $t = 3$,

$$T' = \frac{-1400(3 + 2)}{(9 + 12 + 10)^2} \approx -7.284 \text{ deg/hr.}$$

(c) When $t = 5$,

$$T' = \frac{-1400(5 + 2)}{(25 + 20 + 10)^2} \approx -3.240 \text{ deg/hr.}$$

(d) When $t = 10$,

$$T' = \frac{-1400(10 + 2)}{(100 + 40 + 10)^2} \approx -0.747 \text{ deg/hr.}$$

100. $v = \sqrt{2gh} = \sqrt{2(32)h} = 8\sqrt{h}$

$\dfrac{dv}{dh} = \dfrac{4}{\sqrt{h}}$

(a) When $h = 9$, $\dfrac{dv}{dh} = \dfrac{4}{3}$ ft/sec.

(b) When $h = 4$, $\dfrac{dv}{dh} = 2$ ft/sec.

101.

$$\begin{aligned} x^2 + 3xy + y^3 &= 10 \\ 2x + 3xy' + 3y + 3y^2y' &= 0 \\ 3(x + y^2)y' &= -(2x + 3y) \\ y' &= \frac{-(2x + 3y)}{3(x + y^2)} \end{aligned}$$

102.

$$\begin{aligned} x^2 + 9y^2 - 4x + 3y &= 0 \\ 2x + 18yy' - 4 + 3y' &= 0 \\ 3(6y + 1)y' &= 4 - 2x \\ y' &= \frac{4 - 2x}{3(6y + 1)} \end{aligned}$$

103.

$$\begin{aligned} y\sqrt{x} - x\sqrt{y} &= 16 \\ y\left(\frac{1}{2}x^{-1/2}\right) + x^{1/2}y' - x\left(\frac{1}{2}y^{-1/2}y'\right) - y^{1/2} &= 0 \\ \left(\sqrt{x} - \frac{x}{2\sqrt{y}}\right)y' &= \sqrt{y} - \frac{y}{2\sqrt{x}} \\ \frac{2\sqrt{xy} - x}{2\sqrt{y}}y' &= \frac{2\sqrt{xy} - y}{2\sqrt{x}} \\ y' &= \frac{2\sqrt{xy} - y}{2\sqrt{x}} \cdot \frac{2\sqrt{y}}{2\sqrt{xy} - x} = \frac{2y\sqrt{x} - y\sqrt{y}}{2x\sqrt{y} - x\sqrt{x}} \end{aligned}$$

104.

$$\begin{aligned} y^2 &= x^3 - x^2y + xy - y^2 \\ 0 &= x^3 - x^2y + xy - 2y^2 \\ 0 &= 3x^2 - x^2y' - 2xy + xy' + y - 4yy' \\ (x^2 - x + 4y)y' &= 3x^2 - 2xy + y \\ y' &= \frac{3x^2 - 2xy + y}{x^2 - x + 4y} \end{aligned}$$

105.

$$\begin{aligned} x \sin y &= y \cos x \\ (x \cos y)y' + \sin y &= -y \sin x + y' \cos x \\ y'(x \cos y - \cos x) &= -y \sin x - \sin y \\ y' &= \frac{y \sin x + \sin y}{\cos x - x \cos y} \end{aligned}$$

106. $\cos(x + y) = x$

$$-(1 + y')\sin(x + y) = 1$$

$$-y'\sin(x + y) = 1 + \sin(x + y)$$

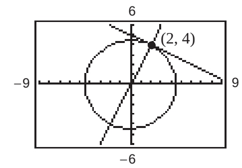
$$y' = -\frac{1 + \sin(x + y)}{\sin(x + y)}$$

$$= -\csc(x + 1) - 1$$

107. $x^2 + y^2 = 20$

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y}$$



At $(2, 4)$: $y' = -\frac{1}{2}$

Tangent line: $y - 4 = -\frac{1}{2}(x - 2)$

$$x + 2y - 10 = 0$$

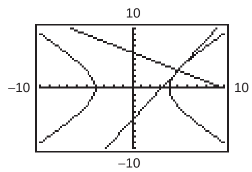
Normal line: $y - 4 = 2(x - 2)$

$$2x - y = 0$$

108. $x^2 - y^2 = 16$

$$2x - 2yy' = 0$$

$$y' = \frac{x}{y}$$



At $(5, 3)$: $y' = \frac{5}{3}$

Tangent line: $y - 3 = \frac{5}{3}(x - 5)$

$$5x - 3y - 16 = 0$$

Normal line: $y - 3 = -\frac{3}{5}(x - 5)$

$$3x + 5y - 30 = 0$$

109. $y = \sqrt{x}$

$$\frac{dy}{dt} = 2 \text{ units/sec}$$

$$\frac{dy}{dt} = \frac{1}{2\sqrt{x}}\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 2\sqrt{x}\,\frac{dy}{dt} = 4\sqrt{x}$$

(a) When $x = \frac{1}{2}$, $\frac{dx}{dt} = 2\sqrt{2}$ units/sec.

(b) When $x = 1$, $\frac{dx}{dt} = 4$ units/sec.

(c) When $x = 4$, $\frac{dx}{dt} = 8$ units/sec.

110. Surface area $= A = 6x^2$, $x =$ length of edge

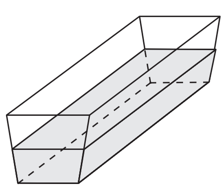
$$\frac{dx}{dt} = 5$$

$$\frac{dA}{dt} = 12x\frac{dx}{dt} = 12(4.5)(5) = 270 \text{ cm}^2\text{/sec}$$

111. $\frac{s}{h} = \frac{1/2}{2}$

$$s = \frac{1}{4}h$$

$$\frac{dV}{dt} = 1$$



Width of water at depth h:

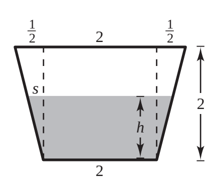
$$w = 2 + 2s = 2 + 2\left(\frac{1}{4}h\right) = \frac{4 + h}{2}$$

$$V = \frac{5}{2}\left(2 + \frac{4 + h}{2}\right)h = \frac{5}{4}(8 + h)h$$

$$\frac{dV}{dt} = \frac{5}{2}(4 + h)\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2(dV/dt)}{5(4 + h)}$$

When $h = 1$, $\frac{dh}{dt} = \frac{2}{25}$ m/min.

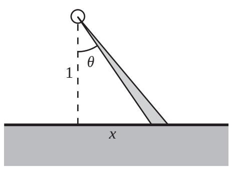


112.

$$\tan\theta = x$$
$$\frac{d\theta}{dt} = 3(2\pi) \text{ rad/min}$$
$$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$$
$$\frac{dx}{dt} = (\tan^2\theta + 1)(6\pi) = 6\pi(x^2 + 1)$$

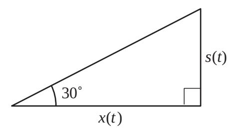
When $x = \frac{1}{2}$,

$$\frac{dx}{dt} = 6\pi\left(\frac{1}{4} + 1\right) = \frac{15\pi}{2} \text{ km/min} = 450\pi \text{ km/hr.}$$



113.

$$s(t) = 60 - 4.9t^2$$
$$s'(t) = -9.8t$$
$$s = 35 = 60 - 4.9t^2$$
$$4.9t^2 = 25$$
$$t = \frac{5}{\sqrt{4.9}}$$
$$\tan 30 = \frac{1}{\sqrt{3}} = \frac{s(t)}{x(t)}$$
$$x(t) = \sqrt{3}\,s(t)$$
$$\frac{dx}{dt} = \sqrt{3}\frac{ds}{dt} = \sqrt{3}(-9.8)\frac{5}{\sqrt{4.9}} \approx -38.34 \text{ m/sec}$$



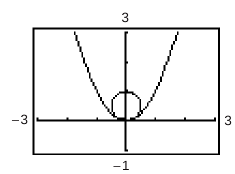
Problem Solving for Chapter 2

1. (a) $x^2 + (y - r)^2 = r^2$, Circle

$x^2 = y$, Parabola

Substituting:

$$(y - r)^2 = r^2 - y$$
$$y^2 - 2ry + r^2 = r^2 - y$$
$$y^2 - 2ry + y = 0$$
$$y(y - 2r + 1) = 0$$



Since you want only one solution, let $1 - 2r = 0 \Longrightarrow r = \frac{1}{2}$. Graph $y = x^2$ and $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$.

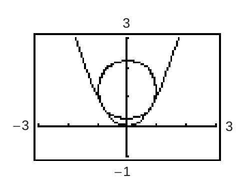
(b) Let (x, y) be a point of tangency:

$x^2 + (y - b)^2 = 1 \Longrightarrow 2x + 2(y - b)y' = 0 \Longrightarrow y' = \dfrac{x}{b - y}$, Circle

$y = x^2 \Longrightarrow y' = 2x$, Parabola

Equating:

$$2x = \frac{x}{b - y}$$
$$2(b - y) = 1$$
$$b - y = \frac{1}{2} \Longrightarrow b = y + \frac{1}{2}$$



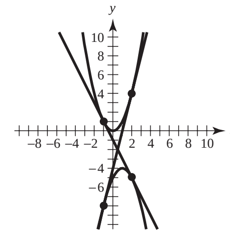
Also, $x^2 + (y - b)^2 = 1$ and $y = x^2$ imply:

$$y + (y - b)^2 = 1 \Longrightarrow y + \left[y - \left(y + \frac{1}{2}\right)\right]^2 = 1 \Longrightarrow y + \frac{1}{4} = 1 \Longrightarrow y = \frac{3}{4} \text{ and } b = \frac{5}{4}$$

Center: $\left(0, \dfrac{5}{4}\right)$

Graph $y = x^2$ and $x^2 + \left(y - \frac{5}{4}\right)^2 = 1$.

2. Let (a, a^2) and $(b, -b^2 + 2b - 5)$ be the points of tangency. For $y = x^2, y' = 2x$ and for $y = -x^2 + 2x - 5, y' = -2x + 2$. Thus, $2a = -2b + 2 \Rightarrow a + b = 1$, or $a = 1 - b$. Furthermore, the slope of the common tangent line is



$$\frac{a^2 - (-b^2 + 2b - 5)}{a - b} = \frac{(1 - b)^2 + b^2 - 2b + 5}{(1 - b) - b} = -2b + 2$$

$$\Rightarrow \frac{1 - 2b + b^2 + b^2 - 2b + 5}{1 - 2b} = -2b + 2$$

$$\Rightarrow 2b^2 - 4b + 6 = 4b^2 - 6b + 2$$

$$\Rightarrow 2b^2 - 2b - 4 = 0$$

$$\Rightarrow b^2 - b - 2 = 0$$

$$\Rightarrow (b - 2)(b + 1) = 0$$

$$b = 2, -1$$

For $b = 2, a = 1 - b = -1$ and the points of tangency are $(-1, 1)$ and $(2, -5)$. The tangent line has slope -2: $y - 1 = -2(x = 1) \Rightarrow y = -2x - 1$

For $b = -1, a = 1 - b = 2$ and the points of tangency are $(2, 4)$ and $(-1, -8)$. The tangent line has slope 4: $y - 4 = 4(x - 2) \Rightarrow y = 4x - 4$

3. (a)
$f(x) = \cos x$ $P_1(x) = a_0 + a_1x$
$f(0) = 1$ $P_1(0) = a_0 \Rightarrow a_0 = 1$
$f'(0) = 0$ $P'_1(0) = a_1 \Rightarrow a_1 = 0$
$P_1(x) = 1$

(b)
$f(x) = \cos x$ $P_2(x) = a_0 + a_1x + a_2x^2$
$f(0) = 1$ $P_2(0) = a_0 \Rightarrow a_0 = 1$
$f'(0) = 0$ $P'_2(0) = a_1 \Rightarrow a_1 = 0$
$f''(0) = -1$ $P''_2(0) = 2a_2 \Rightarrow a_2 = -\frac{1}{2}$
$P_2(x) = 1 - \frac{1}{2}x^2$

(c)

x	-1.0	-0.1	-0.001	0	0.001	0.1	1.0
$\cos x$	0.5403	0.9950	≈ 1	1	≈ 1	0.9950	0.5403
$P_2(x)$	0.5	0.9950	≈ 1	1	≈ 1	0.9950	0.5

$P_2(x)$ is a good approximation of $f(x) = \cos x$ when x is near 0.

(d)
$f(x) = \sin x$ $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$
$f(0) = 0$ $P_3(0) = a_0 \Rightarrow a_0 = 0$
$f'(0) = 1$ $P'_3(0) = a_1 \Rightarrow a_1 = 1$
$f''(0) = 0$ $P''_3(0) = 2a_2 \Rightarrow a_2 = 0$
$f'''(0) = -1$ $P'''_3(0) = 6a_3 \Rightarrow a_3 = -\frac{1}{6}$
$P_3(x) = x - \frac{1}{6}x^3$

4. (a) $y = x^2, y' = 2x$, Slope $= 4$ at $(2, 4)$

Tangent line: $y - 4 = 4(x - 2)$

$y = 4x - 4$

(b) Slope of normal line: $-\frac{1}{4}$

Normal line: $y - 4 = -\frac{1}{4}(x - 2)$

$y = -\frac{1}{4}x + \frac{9}{2}$

$y = -\frac{1}{4}x + \frac{9}{2} = x^2$

$\Rightarrow 4x^2 + x - 18 = 0$

$\Rightarrow (4x + 9)(x - 2) = 0$

$x = 2, -\frac{9}{4}$

Second intersection point: $\left(-\frac{9}{4}, \frac{81}{16}\right)$

(c) Tangent line: $y = 0$

Normal line: $x = 0$

—CONTINUED—

4. —CONTINUED—

(d) Let (a, a^2), $a \neq 0$, be a point on the parabola $y = x^2$. Tangent line at (a, a^2) is $y = 2a(x - a) + a^2$. Normal line at (a, a^2) is $y = -(1/2a)(x - a) + a^2$. To find points of intersection, solve:

$$x^2 = -\frac{1}{2a}(x - a) + a^2$$

$$x^2 + \frac{1}{2a}x = a^2 + \frac{1}{2}$$

$$x^2 + \frac{1}{2a}x + \frac{1}{16a^2} = a^2 + \frac{1}{2} + \frac{1}{16a^2}$$

$$\left(x + \frac{1}{4a}\right)^2 = \left(a + \frac{1}{4a}\right)^2$$

$$x + \frac{1}{4a} = \pm\left(a + \frac{1}{4a}\right)$$

$$x + \frac{1}{4a} = a + \frac{1}{4a} \Longrightarrow x = a \quad \text{(Point of tangency)}$$

$$x + \frac{1}{4a} = -\left(a + \frac{1}{4a}\right) \Longrightarrow x = -a - \frac{1}{2a} = -\frac{2a^2 + 1}{2a}$$

The normal line intersects a second time at $x = -\dfrac{2a^2 + 1}{2a}$.

5. Let $p(x) = Ax^3 + Bx^2 + Cx + D$

$p'(x) = 3Ax^2 + 2Bx + C.$

At $(1, 1)$:	$A + B + C + D = 1$	Equation 1
	$3A + 2B + C = 14$	Equation 2
At $(-1, -3)$:	$-A + B - C + D = -3$	Equation 3
	$3A - 2B + C = -2$	Equation 4

Adding Equations 1 and 3: $2B + 2D = -2$

Subtracting Equations 1 and 3: $2A + 2C = 4$

Adding Equations 2 and 4: $6A + 2C = 12$

Subtracting Equations 2 and 4: $4B = 16$

Hence, $B = 4$ and $D = \frac{1}{2}(-2 - 2B) = -5$. Subtracting $2A + 2C = 4$ and $6A + 2C = 12$, you obtain $4A = 8 \Longrightarrow A = 2$. Finally, $C = \frac{1}{2}(4 - 2A) = 0$. Thus, $p(x) = 2x^3 + 4x^2 - 5$.

6. $f(x) = a + b\cos cx$

$f'(x) = -bc\sin cx$

At $(0, 1)$:	$a + b = 1$	Equation 1
At $\left(\frac{\pi}{4}, \frac{3}{2}\right)$:	$a + b\cos\left(\frac{c\pi}{4}\right) = \frac{3}{2}$	Equation 2
	$-bc\sin\left(\frac{c\pi}{4}\right) = 1$	Equation 3

From Equation 1, $a = 1 - b$. Equation 2 becomes

$$(1 - b) + b\cos\left(\frac{c\pi}{4}\right) = \frac{3}{2} \Longrightarrow -b + b\cos\frac{c\pi}{4} = \frac{1}{2}.$$

From Equation 3, $b = \dfrac{-1}{c\sin(c\pi/4)}$. Thus:

$$\frac{1}{c\sin(c\pi/4)} + \frac{-1}{c\sin(c\pi/4)}\cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}$$

$$1 - \cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}c\sin\left(\frac{c\pi}{4}\right)$$

Graphing the equation

$$g(c) = \frac{1}{2}c\sin\left(\frac{c\pi}{4}\right) + \cos\left(\frac{c\pi}{4}\right) - 1,$$

you see that many values of c will work. One answer:

$$c = 2, b = -\frac{1}{2}, a = \frac{3}{2} \Longrightarrow f(x) = \frac{3}{2} - \frac{1}{2}\cos 2x$$

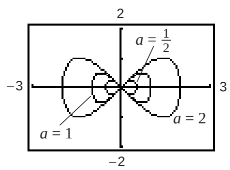
7. (a) $x^4 = a^2x^2 - a^2y^2$

$a^2y^2 = a^2x^2 - x^4$

$y = \dfrac{\pm\sqrt{a^2x^2 - x^4}}{a}$

Graph: $y_1 = \dfrac{\sqrt{a^2x^2 - x^4}}{a}$ and $y_2 = -\dfrac{\sqrt{a^2x^2 - x^4}}{a}$.

(b)



$(\pm a, 0)$ are the x-intercepts, along with $(0, 0)$.

(c) Differentiating implicitly:

$4x^3 = 2a^2x - 2a^2yy'$

$y' = \dfrac{2a^2x - 4x^3}{2a^2y}$

$= \dfrac{x(a^2 - 2x^2)}{a^2y} = 0 \Longrightarrow 2x^2 = a^2 \Longrightarrow x = \dfrac{\pm a}{\sqrt{2}}$

$\left(\dfrac{a^2}{2}\right)^2 = a^2\left(\dfrac{a^2}{2}\right) - a^2y^2$

$\dfrac{a^4}{4} = \dfrac{a^4}{2} - a^2y^2$

$a^2y^2 = \dfrac{a^4}{4}$

$y^2 = \dfrac{a^2}{4}$

$y = \pm\dfrac{a}{2}$

Four points: $\left(\dfrac{a}{\sqrt{2}}, \dfrac{a}{2}\right), \left(\dfrac{a}{\sqrt{2}}, -\dfrac{a}{2}\right), \left(-\dfrac{a}{\sqrt{2}}, \dfrac{a}{2}\right), \left(\dfrac{-a}{\sqrt{2}}, -\dfrac{a}{2}\right)$

8. (a) $b^2y^2 = x^3(a - x); \quad a, b > 0$

$y^2 = \dfrac{x^3(a - x)}{b^2}$

Graph $y_1 = \dfrac{\sqrt{x^3(a - x)}}{b}$ and $y_2 = -\dfrac{\sqrt{x^3(a - x)}}{b}$.

(b) a determines the x-intercept on the right: $(a, 0)$. b affects the height.

(c) Differentiating implicitly:

$2b^2yy' = 3x^2(a - x) - x^3 = 3ax^2 - 4x^3$

$y' = \dfrac{(3ax^2 - 4x^3)}{2b^2y} = 0$

$\Longrightarrow 3ax^2 = 4x^3$

$3a = 4x$

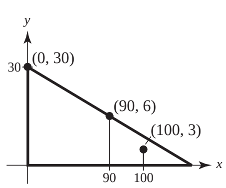
$x = \dfrac{3a}{4}$

$b^2y^2 = \left(\dfrac{3a}{4}\right)^3\left(a - \dfrac{3a}{4}\right) = \dfrac{27a^3}{64}\left(\dfrac{1}{4}a\right)$

$y^2 = \dfrac{27a^4}{256b^2} \Longrightarrow y = \pm\dfrac{3\sqrt{3}a^2}{16b}$

Two points: $\left(\dfrac{3a}{4}, \dfrac{3\sqrt{3}a^2}{16b}\right), \left(\dfrac{3a}{4}, \dfrac{-3\sqrt{3}a^2}{16b}\right)$

9. (a)



Not drawn to scale

Line determined by $(0, 30)$ and $(90, 6)$:

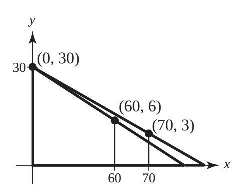
$y - 30 = \dfrac{30 - 6}{0 - 90}(x - 0)$

$= -\dfrac{24}{90}x = -\dfrac{4}{15}x \Longrightarrow y = -\dfrac{4}{15}x + 30$

When $x = 100$:

$y = \dfrac{-4}{15}(100) + 30 = \dfrac{10}{3} > 3 \Longrightarrow$ Shadow determined by man

(b)



Not drawn to scale

Line determined by $(0, 30)$ and $(60, 6)$:

$y - 30 = \dfrac{30 - 6}{0 - 60}(x - 0) = -\dfrac{2}{5}x \Longrightarrow y = -\dfrac{2}{5}x + 30$

When $x = 70$:

$y = \dfrac{-2}{5}(70) + 30$

$= 2 < 3 \Longrightarrow$ Shadow determined by child

—CONTINUED—

9. —CONTINUED—

(c) Need $(0, 30)$, $(d, 6)$, $(d + 10, 3)$ collinear.

$$\frac{30 - 6}{0 - d} = \frac{6 - 3}{d - (d + 10)} \Rightarrow \frac{24}{d} = \frac{3}{10} \Rightarrow d = 80 \text{ feet}$$

(d) Let y be the distance from the base of the street light to the tip of the shadow. We know that $dx/dt = -5$.

For $x > 80$, the shadow is determined by the man.

$$\frac{y}{30} = \frac{y - x}{6} \Rightarrow y = \frac{5}{4}x \text{ and } \frac{dy}{dt} = \frac{5}{4}\frac{dx}{dt} = \frac{-25}{4}$$

For $x < 80$, the shadow is determined by the child.

$$\frac{y}{30} = \frac{y - x - 10}{3} \Rightarrow y = \frac{10}{9}x + \frac{100}{9} \text{ and } \frac{dy}{dt} = \frac{10}{9}\frac{dx}{dt} = \frac{-50}{9}$$

Therefore:

$$\frac{dy}{dt} = \begin{cases} -\frac{25}{4}, & x > 80 \\ -\frac{50}{9}, & 0 < x < 80 \end{cases}$$

dy/dt is not continuous at $x = 80$.

10. (a) $y = x^{1/3} \Rightarrow \frac{dy}{dt} = \frac{1}{3}x^{-2/3}\frac{dx}{dt}$

$$1 = \frac{1}{3}(8)^{-2/3}\frac{dx}{dt}$$

$$\frac{dx}{dt} = 12 \text{ cm/sec}$$

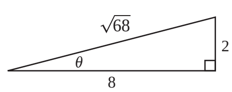
(b) $D = \sqrt{x^2 + y^2} \Rightarrow \frac{dD}{dt} = \frac{1}{2}(x^2 + y^2)\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$

$$= \frac{x(dx/dt) + y(dy/dt)}{\sqrt{x^2 + y^2}}$$

$$= \frac{8(12) + 2(1)}{\sqrt{64 + 4}}$$

$$= \frac{98}{\sqrt{68}} = \frac{49}{\sqrt{17}} \text{ cm/sec}$$

(c) $\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{x(dy/dt) - y(dx/dt)}{x^2}$



From the triangle, $\sec\theta = \sqrt{68}/8$. Hence

$$\frac{d\theta}{dt} = \frac{8(1) - 2(12)}{64(68/64)} = \frac{-16}{68} = \frac{-4}{17} \text{ rad/sec.}$$

11. $L'(x) = \lim_{\Delta x \to 0} \frac{L(x + \Delta x) - L(x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{L(x) + L(\Delta x) - L(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{L(\Delta x)}{\Delta x}$$

Also, $L'(0) = \lim_{\Delta x \to 0} \frac{L(\Delta x) - L(0)}{\Delta x}$.

But, $L(0) = 0$ because

$L(0) = L(0 + 0) = L(0) + L(0) \Rightarrow L(0) = 0$.

Thus, $L'(x) = L'(0)$ for all x. The graph of L is a line through the origin of slope $L'(0)$.

12. $E'(x) = \lim_{\Delta x \to 0} \frac{E(x + \Delta x) - E(x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{E(x)E(\Delta x) - E(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} E(x)\left(\frac{E(\Delta x) - 1}{\Delta x}\right)$$

$$= E(x) \lim_{\Delta x \to 0} \frac{E(\Delta x) - 1}{\Delta x}$$

But, $E'(0) = \lim_{\Delta x \to 0} \frac{E(\Delta x) - E(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{E(\Delta x) - 1}{\Delta x} = 1$.

Thus, $E'(x) = E(x)E'(0) = E(x)$ exists for all x.

For example: $E(x) = e^x$.

13. (a)

z (degrees)	0.1	0.01	0.0001
$\dfrac{\sin z}{z}$	0.0174524	0.0174533	0.0174533

(b) $\displaystyle\lim_{z\to 0}\frac{\sin z}{z} \approx 0.0174533$

In fact, $\displaystyle\lim_{z\to 0}\frac{\sin z}{z} = \frac{\pi}{180}$.

(c) $$\frac{d}{dz}(\sin z) = \lim_{\Delta z\to 0}\frac{\sin(z + \Delta z) - \sin z}{\Delta z}$$

$$= \lim_{\Delta z\to 0}\frac{\sin z \cdot \cos \Delta z + \sin \Delta z \cdot \cos z - \sin z}{\Delta z}$$

$$= \lim_{\Delta z\to 0}\left[\sin z\left(\frac{\cos \Delta z - 1}{\Delta z}\right)\right] + \lim_{\Delta z\to 0}\left[\cos z\left(\frac{\sin \Delta z}{\Delta z}\right)\right]$$

$$= (\sin z)(0) + (\cos z)\left(\frac{\pi}{180}\right) = \frac{\pi}{180}\cos z$$

(d) $S(90) = \sin\left(\dfrac{\pi}{180}90\right) = \sin\dfrac{\pi}{2} = 1$

$C(180) = \cos\left(\dfrac{\pi}{180}180\right) = -1$

$\dfrac{d}{dz}S(z) = \dfrac{d}{dz}\sin(cz) = c \cdot \cos(cz) = \dfrac{\pi}{180}C(z)$

(e) The formulas for the derivatives are more complicated in degrees.

14. (a) $v(t) = -\frac{27}{5}t + 27$ ft/sec

$a(t) = -\frac{27}{5}$ ft/sec^2

(b) $v(t) = -\frac{27}{5}t + 27 = 0 \Longrightarrow \frac{27}{5}t = 27 \Longrightarrow t = 5$ seconds

$S(5) = -\frac{27}{10}(5)^2 + 27(5) + 6 = 73.5$ feet

(c) The acceleration due to gravity on Earth is greater in magnitude than that on the moon.

15. $j(t) = a'(t)$

(a) $j(t)$ is the rate of change of acceleration.

(b) $s(t) = -8.25t^2 + 66t$

$v(t) = -16.5t + 66$

$a(t) = -16.5$

$a'(t) = j(t) = 0$

The acceleration is constant, so $j(t) = 0$.

(c) a is position.

b is acceleration.

c is jerk.

d is velocity.