AP Calculus AB – Midterm class notes from Smartboard files.

If f and f^{-1} are both differentiable for all x, with $f(3)=5$ and $f'(3)=7$, then which of the following must be a line tangent to the graph of f^{-1}?

(a) $y=5+7(x-3)$

(b) $y=\frac{1}{5}+\frac{1}{7}(x-3)$

(c) $y=3+7(x-5)$

(d) $y=\frac{1}{3}+\frac{1}{7}(x-5)$

(e) $y=3+\frac{1}{7}(x-5)$

Strategies for evaluating limits:

1. Pull me

2.
3.
4.
5.
6.
7.
8.

1. Direct Substitution
2. Tables
3. Graphical
4. Rationalizing
5. Finding equivalent functions -
   a. Clearing complex fractions
   b. Factoring and simplifying
6. Using Trig Identities
7. Using known Trig limits

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{1-\cos x}{x}=0$$

8. For limits approaching $\pm\infty$, divide by highest degree in denominator

For f(x) to be continuous at a point x = a...

Pull me

1. $f(a)$ *defined*
2. $\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)$

   $\therefore \lim_{x\to a} f(x)$ *exists*
3. $f(a)=\lim_{x\to a} f(x)$

Limit definition of the derivative...

Pull me

1. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
2. $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Differentiability and Continuity...

Pull me

1. Continuity does NOT guarantee differentiability
2. Differentiability guarantees continuity

A function will be non-differentiable when...

Pull me

1. Function is not continuous
2. Sharp turn or cusp
3. Vertical tangent line
4. Endpoints

Multiple Choice Which of the following statements is false for the function

$$f(x) = \begin{cases} \frac{3}{4}x, & 0 \le x < 4 \\ 2, & x = 4 \\ -x + 7, & 4 < x \le 6 \\ 1, & 6 < x < 8? \end{cases}$$

E

(A) $\lim_{x \to 4} f(x)$ exists　　**(B)** $f(4)$ exists

(C) $\lim_{x \to 6} f(x)$ exists　　**(D)** $\lim_{x \to 8^-} f(x)$ exists

(E) f is continuous at $x = 4$

Multiple Choice If the line normal to the graph of f at the point $(1, 2)$ passes through the point $(-1, 1)$, then which of the following gives the value of $f'(1) = ?$ A

(A) -2　　**(B)** 2　　**(C)** $-1/2$　　**(D)** $1/2$　　**(E)** 3

Multiple Choice Which of the following is the slope of the tangent line to $y = \tan^{-1}(2x)$ at $x = 1$? C

(A) $-2/5$ **(B)** $1/5$ **(C)** $2/5$ **(D)** $5/2$ **(E)** 5

Multiple Choice Which of the following is the domain of $f'(x)$ if $f(x) = \log_2(x + 3)$? D

(A) $x < -3$ **(B)** $x \leq 3$ **(C)** $x \neq -3$ **(D)** $x > -3$

(E) $x \geq -3$

Multiple Choice If f is a continuous, decreasing function on $[0, 10]$ with a critical point at $(4, 2)$, which of the following statements *must be false*? E

(A) $f(10)$ is an absolute minimum of f on $[0, 10]$.

(B) $f(4)$ is neither a relative maximum nor a relative minimum.

(C) $f'(4)$ does not exist.

(D) $f'(4) = 0$

(E) $f'(4) < 0$

Multiple Choice If $f(x) = \cos x$, then the Mean Value Theorem guarantees that somewhere between 0 and $\pi/3$, $f'(x) =$ A

(A) $-\frac{3}{2\pi}$ **(B)** $-\frac{\sqrt{3}}{2}$ **(C)** $-\frac{1}{2}$ **(D)** 0 **(E)** $\frac{1}{2}$

Multiple Choice If $a < 0$, the graph of $y = ax^3 + 3x^2 + 4x + 5$ is concave up on A

(A) $\left(-\infty, -\frac{1}{a}\right)$ **(B)** $\left(-\infty, \frac{1}{a}\right)$ **(C)** $\left(-\frac{1}{a}, \infty\right)$

(D) $\left(\frac{1}{a}, \infty\right)$ **(E)** $(-\infty, -1)$

Multiple Choice Which of the following conditions would enable you to conclude that the graph of f has a point of inflection at $x = c$? A 

(A) There is a local maximum of f' at $x = c$.
(B) $f''(c) = 0$.
(C) $f''(c)$ does not exist.
(D) The sign of f' changes at $x = c$.
(E) f is a cubic polynomial and $c = 0$.