Name Answer Key **Date**_____

Calc I H – Final Exam Review 1 **Per** _____

1.) Find the area of the region bounded by the curves:

a) $y = 4 - x^2$ and $y = x - 8$.

dx persp.

$4 - x^2 = x - 8$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$x = -4, 3$

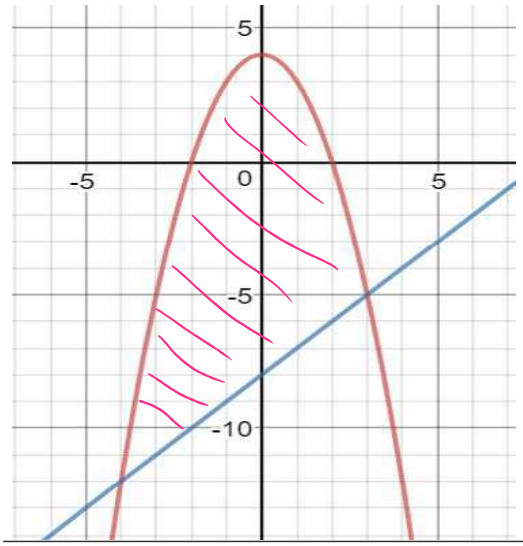
$A = \int_{-4}^{3} 4 - x^2 - (x-8)\,dx$

$= \int_{-4}^{3} (-x^2 - x + 12)\,dx$

$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 12x \right]_{-4}^{3}$

$= -9 - \frac{9}{2} + 36 - \left(\frac{64}{3} - 8 - 48 \right)$

$= \boxed{\frac{343}{6}\,u^2 \approx 57.167\,u^2}$



b) $x = y^2$ and $x = 9$.

dy persp

$y^2 = 9$

$y = \pm 3$

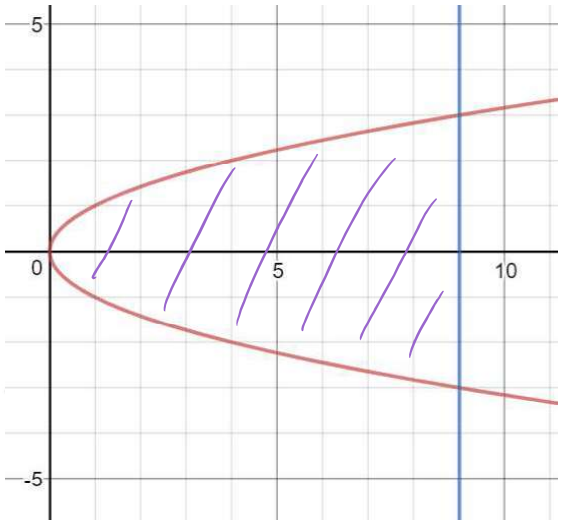
$A = \int_{-3}^{3} (9 - y^2)\,dy$

$= 2\int_{0}^{3} (9 - y^2)\,dy$

$= 2\left[9y - \frac{y^3}{3} \right]_0^3$

$= 2(27 - 9 - 0)$

$= \boxed{36\,u^2}$



2.) Find the volume of the solid generated by revolving the region bounded by the following curves:

a) $y=\sqrt{25-x^2}$, $y=0$, $x=0$, in the **first** quadrant, about the *y*-axis.

disk - dy perspective

$y=\sqrt{25-x^2}$

$y^2=25-x^2$

$x^2=25-y^2$

$x=\pm\sqrt{25-y^2}$

$x=\sqrt{25-y^2}$ (1st quad!)

$R=\sqrt{25-y^2}$

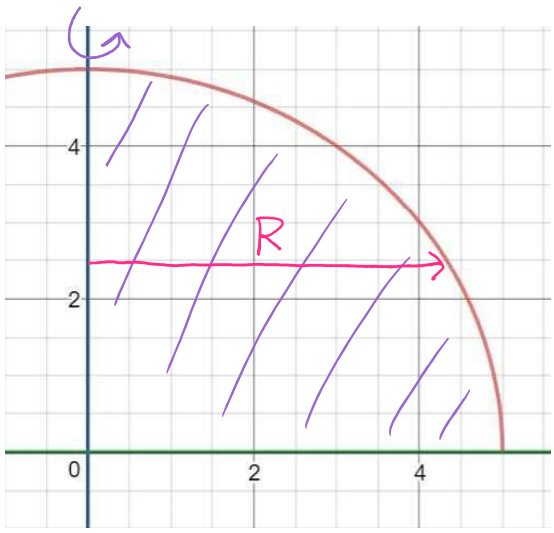
$V=\pi\int_0^5\left(\sqrt{25-y^2}\right)^2dy$

$=\pi\int_0^5(25-y^2)dy$

$=\pi\left[25y-y^3/3\right]_0^5$

$=\pi\left[125-\frac{125}{3}-0\right]$

$=\boxed{\frac{250\pi}{3}\approx 261.799u^3}$



b) $y=9-x^2$, $y=5$ about the line $y=3$.

washer, dx

$9-x^2=5$

$x^2=4$

$x=\pm 2$

$R=9-x^2-3$

$=6-x^2$

$r=5-3=2$

$V=\pi\int_{-2}^{2}(6-x^2)^2-2^2\,dx$

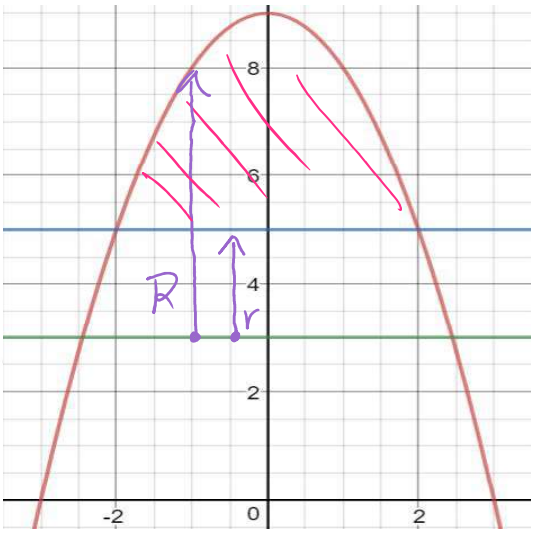
$=2\pi\int_0^2(36-12x^2+x^4-4)\,dx$

$=2\pi\int_0^2(32-12x^2+x^4)\,dx$

$=2\pi\left[32x-4x^3+\frac{x^5}{5}\right]_0^2$

$=2\pi\left[64-32+\frac{32}{5}-0\right]$

$=\boxed{\frac{384\pi}{5}u^3\approx 241.274u^3}$



3.) Evaluate the following integrals:

a) $\int_{-2}^{3}(9x-4)^2\,dx$

$=\int_{-2}^{3}(81x^2-72x+16)\,dx$

$=\left(27x^3-36x^2+16x\right)_{-2}^{3}$

$=729-324+48-(-216-144-32)$

$=\boxed{845}$

b) $\int\cos(2x+1)\,dx$

$u=2x+1$

$\frac{du}{dx}=2\Rightarrow dx=\frac{du}{2}$

$\int\cos(u)\cdot\frac{du}{2}$

$=\frac{1}{2}\sin u+C=\boxed{\frac{1}{2}\sin(2x+1)+C}$

4.) A ball is tossed in the air from a bridge and its height (s, in feet) above the ground, t seconds after it is thrown is given by:

$$s(t) = -16t^2 + 45t + 24$$

(a) What is the initial height of the bridge? $s(0) = 24$ ft

(b) Find the initial velocity of the ball. $V(t) = s'(t) = -32t + 45$

$V(0) = 45$ ft/s

(c) When is its velocity -10 ft/sec?

(d) What is the velocity 2 seconds after it is thrown?

(e) What is the acceleration 1 second after it is thrown?

c) $-10 = -32t + 45$

$-55 = -32t$

$t = 55/32 \approx 1.719$ sec

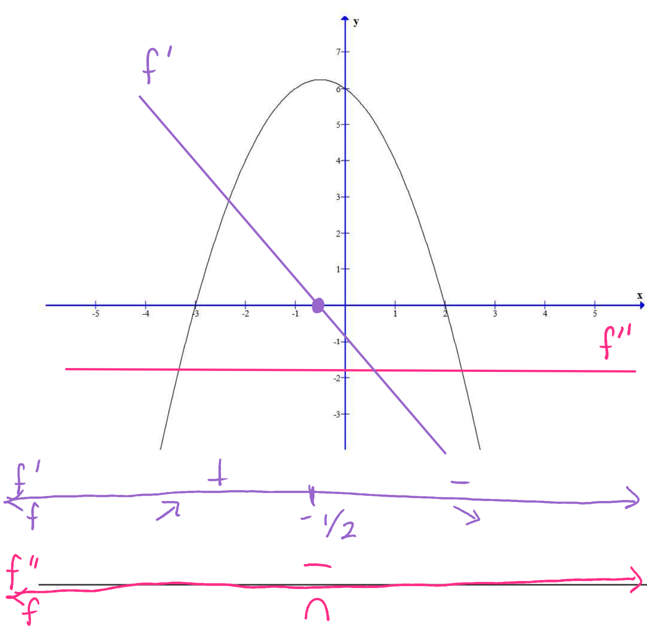
d) $V(2) = -32(2) + 45$

$V(2) = -19$ ft/sec

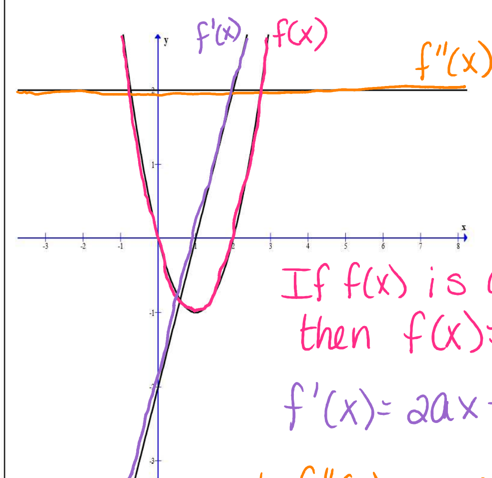
e) $a(t) = v'(t) = -32$

$a(1) = -32$ ft/s²

5.) The graph of f is shown below. Sketch the graphs of f' and f'' on the same set of axes.



6.) The graphs of f, f', and f'' are shown below. Which is which? Explain your reasoning.



If $f(x)$ is quad then $f(x) = ax^2 + bx + C$

$f'(x) = 2ax + b$ (linear)

& $f''(x) = 2a$ (horiz Line)