Name Answer Key　　　　　　　　Date________

Calc I H – Final Exam Review 2　　　　　　　　Per ________

7.) Evaluate the following integrals:

a) $\int_0^{\pi} \frac{\sin 2x}{\cos^2 2x} dx$

$u = \cos 2x$ 　 $x=0, u=1$

$\frac{du}{dx} = -2\sin 2x$ 　 $x=\pi, u=1$

$dx = \frac{du}{-2\sin 2x}$

$= \int_1^1 \frac{\cancel{\sin 2x}}{u^2} \cdot \frac{du}{-2\cancel{\sin 2x}}$

$= -\frac{1}{2}\int_1^1 u^{-2} du = \boxed{0}$

b) $\int_0^1 \frac{x^3}{\sqrt{x^4+1}} dx$

$u = x^4+1$

$\frac{du}{dx} = 4x^3 \Rightarrow dx = \frac{du}{4x^3}$

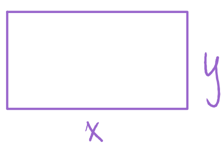
$x=0, u=1, \; x=1, u=2$

$\int_1^2 \frac{\cancel{x^3}}{\sqrt{u}} \cdot \frac{du}{4\cancel{x^3}} = \frac{1}{4}\int_1^2 u^{-1/2} du$

$= \frac{1}{4}\left[2u^{1/2}\right]_1^2 = \frac{1}{2}(\sqrt{2}-\sqrt{1})$

$= \boxed{\frac{\sqrt{2}-1}{2} \approx .207}$

8.) Find the length and width of a rectangle that has a perimeter of 48 meters and a maximum area. What is the maximum area?



$P = 2x + 2y = 48 \Rightarrow 2y = 48 - 2x$

$y = 24 - x$

$A = xy$

$A = x(24-x)$

$\frac{d}{dx}\left(A = 24x - x^2\right)$

$A'(x) = 24 - 2x = 0$

$2x = 24$

$x = 12$

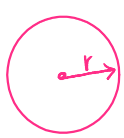
$A''(x) = -2 < 0$

Since $A'(12) = 0$ & $A''(12) < 0$, max area occurs at $x = 12m$

$y = 24 - 12 = 12m$

$A = (12)(12) = \boxed{144m^2}$

9.) A chemical substance is spreading in a nearly circular shape on the surface of the water in a large holding tank pool. At the same time the radius of the chemical is increasing at a rate of 1ft/min, the diameter is 100 ft. At what rate is the area of the chemical spreading?



$\frac{dr}{dt} = \frac{1 ft}{min}$ when $d = 100ft$

$r = 50ft$

$\frac{d}{dt}\left(A = \pi r^2\right)$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(50)(1)$

$= \boxed{100\pi \frac{ft^2}{min}}$

10.) Find the particular solution of $f(x)$, given $f''(x)=3x^2+1$, $f'(0)=1$, and $f(0)=2$.

$\int f''(x)\,dx = \int 3x^2+1\,dx$

$f'(x) = x^3 + x + C$

$f'(0) = 0 + 0 + C = 1$

$C = 1$

$\int f'(x)\,dx = \int x^3 + x + 1\,dx$

$f(x) = \frac{x^4}{4} + \frac{x^2}{2} + x + C$

$f(0) = 0 + 0 + 0 + C = 2$

$C = 2$

$\boxed{f(x) = \frac{x^4}{4} + \frac{x^2}{2} + x + 2}$

11.) Evaluate algebraically: $\lim_{x\to 0}\frac{\sqrt{9-x}-3}{x}$ "$\frac{0}{0}$"

$\lim_{x\to 0}\frac{\sqrt{9-x}-3}{x}\cdot\frac{\sqrt{9-x}+3}{\sqrt{9-x}+3} = \lim_{x\to 0}\frac{\cancel{9}-x-\cancel{9}}{x(\sqrt{9-x}+3)} = \lim_{x\to 0}\frac{-\cancel{x}\,1}{\cancel{x}\,1(\sqrt{9-x}+3)}$

$= \lim_{x\to 0}\frac{-1}{\sqrt{9-x}+3} = \frac{-1}{\sqrt{9-0}+3} = \boxed{-\frac{1}{6}}$

12.) Find $f''(x)$ if $f(x)=3x^4-6x^3+3x-2$.

$f'(x) = 12x^3 - 18x^2 + 3$

$\boxed{f''(x) = 36x^2 - 36x}$

13.) Given: $\int_0^3 f(x)dx = a$, $\int_3^8 f(x)dx = b$, and $\int_0^8 g(x)dx = c$. Find:

a) $\int_0^8 [f(x)-g(x)]dx =$ $a+b-c$

$\int_0^3 f(x)dx + \int_3^8 f(x)dx - \int_0^8 g(x)dx$

$a + b - c$

b) $\int_0^8 [f(x)+3g(x)]dx =$

$a+b+3(c) = a+b+3c$

c) $\int_8^0 f(x)dx = -\left(\int_0^3 f(x)dx + \int_3^8 f(x)dx\right)$

$= -(a+b)$