

Name Answer Key

Date \_\_\_\_\_

Calc I H - Final Exam Review 3

Per \_\_\_\_\_

14.) Find  $\frac{dy}{dx}$ , given  $y = -x^3 \cos 2x - \csc(3x+1)$ .

$$\begin{aligned}\frac{dy}{dx} &= -3x^2 \cos 2x - x^3(-2 \sin 2x) - (-3 \csc(3x+1) \cot(3x+1)) \\ &= -3x^2 \cos(2x) + 2x^3 \sin(2x) + 3 \csc(3x+1) \cot(3x+1)\end{aligned}$$

15.) Find  $y'$ , given  $y = x^2 \sin(4x-2) + \tan(2x-1)$ .

$$y' = 2x \sin(4x-2) + 4x^2 \cos(4x-2) + 2 \sec^2(2x-1)$$

16.) Find an equation of the tangent line to the curve at the given  $x$ -value:

$$y = -2x^3 + x^2 + 11 \text{ at } x = 1.$$

$$(1, 10) \quad m = -4$$

$$y(1) = -2 + 1 + 11 = 10$$

$$y' = -6x^2 + 2x$$

$$y'(1) = -6 + 2 = -4 = m$$

$$y - 10 = -4(x - 1)$$

17.) Evaluate the following integrals:

a)  $\int x^3 (x^4 + 10)^{10} dx$

$$u = x^4 + 10 \\ \frac{du}{dx} = 4x^3 \Rightarrow dx = \frac{du}{4x^3}$$

$$\int x^3 u^{10} \cdot \frac{du}{4x^3} = \frac{1}{4} \int u^{10} du$$

$$= \frac{1}{4} \cdot \frac{u^{11}}{11} + C = \frac{(x^4 + 10)^{11}}{44} + C$$

b)  $\int x e^{x^2} dx$

$$u = x^2 \\ \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$$

$$\int x e^u \cdot \frac{du}{2x} = \frac{1}{2} \int e^u du$$

$$= \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

18.) Differentiate  $y = \frac{4x+1}{x^2-1}$

$$y' = \frac{4(x^2-1) - 2x(4x+1)}{(x^2-1)^2}$$

$$y' = \frac{4x^2 - 4 - 8x^2 - 2x}{(x^2-1)^2} = \frac{-4x^2 - 2x - 4}{(x^2-1)^2}$$

19.) Evaluate:  $\int 5 \csc 4x \cot 4x dx$

$$u = 4x \\ \frac{du}{dx} = 4 \Rightarrow dx = \frac{du}{4}$$

$$5 \int \csc u \cot u \cdot \frac{du}{4}$$

$$= -\frac{5}{4} \csc u + C = \frac{-5}{4} \csc(4x) + C$$

20.) Given the function  $f(x) = x^3 - 12x$ , use calculus to analyze and graph of the function.

$$f'(x) = 3x^2 - 12 = 0 \\ x^2 - 4 = 0 \\ (x+2)(x-2) = 0 \\ x = \pm 2$$

$$f''(x) = 6x = 0 \\ x = 0$$

$$f(-2) = -8 + 24 = 16 \\ f(2) = 8 - 24 = -16$$

$$f(x) = 0 = x^3 - 12x \\ 0 = x(x^2 - 12) \\ x = 0, \pm\sqrt{12} \\ = 0, \pm 2\sqrt{3}$$

Domain:  $(-\infty, \infty)$

x-intercept(s):  $(0,0), (-2\sqrt{3},0), (2\sqrt{3},0)$

y-intercept:  $(0,0)$

Vertical asymptote(s): none

Horizontal asymptote(s): none

Slant asymptote(s): none

Incr:  $(-\infty, -2) \cup (2, \infty) f'(x) > 0$

Decr:  $(-2, 2) f'(x) < 0$

Extrema:  $(-2, 16) \text{ max } f' > 0 \rightarrow f' < 0$

$(2, -16) \text{ min } f' < 0 \rightarrow f' > 0$

Conc Up:  $(0, \infty) f'' > 0$

Conc Down:  $(-\infty, 0) f'' < 0$

POI:  $(0,0)$

