Name Answer Key Date

Calc I H – Final Exam Review 4 Per

21.) Given $f(x)=\frac{2x^2+x-15}{x^2-9}$, $= \frac{(2x-5)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \frac{2x-5}{x-3}$, $x \neq -3$

(hole at $x=-3$)

a) Find all asymptotes for the graph of *f*.

VA: $x=3$ $\lim_{x\to 3^+} f(x)=\infty$, $\lim_{x\to 3^-} f(x)=-\infty$ $\Big\}$ $\lim_{x\to 3} f(x)$ DNE

HA: $y=2$ $\lim_{x\to\infty} \frac{2x-5}{x-3} = \lim_{x\to\infty} \frac{2-\cancel{5/x}^{\,0}}{1-\cancel{3/x}^{\,0}}$

$=2$

b) Find $\lim_{x\to -3} f(x)$.

$\lim_{x\to -3} f(x) = \frac{2(-3)-5}{-3-3} = \frac{-11}{-6} = \boxed{\frac{11}{6}}$

c) Find $\lim_{x\to 0} f(x)$.

$\lim_{x\to 0} f(x) = \frac{2(0)-5}{0-3} = \boxed{\frac{5}{3}}$

22.) Find $\lim_{x\to\infty} f(x)$: Remember: Divide by highest degree in denominator!

a) $f(x)=\frac{(-2x+3)\frac{1}{x^2}}{(3x^2+1)\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{-\cancel{\frac{2}{x}}^{\,0}+\cancel{\frac{3}{x^2}}^{\,0}}{3+\cancel{\frac{1}{x^2}}^{\,0}} = 0$

b) $f(x)=\frac{(-2x^2+3)\frac{1}{x^2}}{(3x^2+1)\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{-2+\cancel{\frac{3}{x^2}}^{\,0}}{3+\cancel{\frac{1}{x^2}}^{\,0}} = \frac{-2}{3}$

c) $f(x)=\frac{(-2x^3+3)\frac{1}{x^2}}{(3x^2+1)\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{-2x+\cancel{\frac{3}{x^2}}^{\,0}}{3+\cancel{\frac{1}{x^2}}} = -\infty$

23.) Evaluate the following integrals:

a) $\int \sin(4\theta-7)\,d\theta$

$u=4\theta-7$

$\frac{du}{d\theta}=4 \Rightarrow d\theta=\frac{du}{4}$

$\int \sin(u)\cdot\frac{du}{4}$

$= -\frac{1}{4}\cos u + C = \boxed{-\frac{1}{4}\cos(4\theta-7)+C}$

b) $\int \sqrt{9-3x}\,dx$

$u=9-3x$

$\frac{du}{dx}=-3 \Rightarrow dx=\frac{du}{-3}$

$\int \sqrt{u}\cdot\frac{du}{-3} = -\frac{1}{3}\int u^{1/2}\,du$

$= -\frac{1}{3}\cdot\frac{2}{3}u^{3/2}+C = \boxed{-\frac{2}{9}(9-3x)^{3/2}+C}$

24.) Determine what value of a will make each function continuous:

a) $g(x)=\begin{cases} 2x-5, & x \le a \\ 3x+5, & x > a \end{cases}$

$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

$2a-5 = 3a+5$

$-10 = a$

$\boxed{a=-10}$

b) $h(x)=\begin{cases} ax+5, & x < 2 \\ 2x^2-7, & x \ge 2 \end{cases}$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$2a+5 = 2(2)^2-7$

$2a+5 = 1$

$2a = -4$

$a = -2$

$\boxed{a=-2}$

25.) Find y' if $y=e^{5x}(5x-3)^4$.

$y' = 5e^{5x}(5x-3)^4 + e^{5x}\cdot 4(5x-3)^3 \cdot 5$

$y' = 5e^{5x}(5x-3)^4 + 20e^{5x}(5x-3)^3$

GCF: $5e^{5x}(5x-3)^3$

$y' = 5e^{5x}(5x-3)^3(5x-3+4)$

$\boxed{y' = 5e^{5x}(5x-3)^3(5x+1)}$

26.) Differentiate: $\frac{d}{dx}\left(x^2-3y^2=5xy\right)$

$2x - 6y\frac{dy}{dx} = 5y + 5x\frac{dy}{dx}$

$2x - 5y = (5x+6y)\frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{2x-5y}{5x+6y}}$