Name Answer Key　　　　　　　　　　Date________

Calc I H – Integration Bonus Review　　　　　　　Period ________

1. Evaluate the indefinite or definite integral. Check when applicable on the calculator.

a. $\int_0^{\frac{\pi}{2}} 2\sin x\sqrt{\cos x}\,dx$

$u=\cos x$, $\frac{du}{dx}=-\sin x$, $dx=\frac{du}{-\sin x}$; $x=0, u=1$; $x=\pi/2, u=0$

$\int_1^0 2\sin x\sqrt{u}\cdot\frac{du}{-\sin x}$

$=-2\int_1^0 u^{1/2}du = 2\int_0^1 u^{1/2}du =$

$2\cdot\frac{2}{3}u^{3/2}\Big|_0^1 = \frac{4}{3}(1-0) = \boxed{\frac{4}{3}}$

b. $\int \frac{x+3}{x^2+3x}dx = \int \frac{x+3}{x(x+3)}dx$

$=\int \frac{1}{x}dx$

$=\boxed{\ln|x|+C}$

c. $\int \frac{t^2}{(16-t^3)^2}dt$

$u=16-t^3$, $\frac{du}{dx}=-3t^2$, $dx=\frac{du}{-3t^2}$

$\int \frac{t^2}{u^2}\cdot\frac{du}{-3t^2}$

$=-\frac{1}{3}\int u^{-2}du = -\frac{1}{3}\cdot\frac{u^{-1}}{-1}+C$

$=\boxed{\frac{1}{3(16-t^3)}+C}$

d. $\int_0^{\frac{\pi}{3}} \sec\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)dx$

$u=\frac{x}{2}$, $\frac{du}{dx}=\frac{1}{2}$, $dx=2du$; $x=0, u=0$; $x=\frac{\pi}{3}, u=\frac{\pi}{6}$

$\int_0^{\pi/6} \sec u\tan u\cdot 2du = 2\sec u\Big|_0^{\pi/6}$

$=2(\sec(\pi/6)-\sec(0))$

$=\boxed{2\left(\frac{2}{\sqrt{3}}-1\right)}$

e. $\int_1^4 \frac{4x^4-2x^3-3x}{x^2}dx$

$=\int_1^4\left(4x^2-2x-\frac{3}{x}\right)dx$

$=\left[\frac{4}{3}x^3-x^2-3\ln x\right]_1^4$

$=\frac{4}{3}(64)-16-3\ln 4-\left(\frac{4}{3}-1-3\ln 1\right)$ ($3\ln 1 = 0$)

$=\boxed{69-3\ln 4}$

f. $\int(\cos x)e^{\sin x}dx$

$u=\sin x$, $\frac{du}{dx}=\cos x$, $dx=\frac{du}{\cos x}$

$=\int(\cos x)e^u\cdot\frac{du}{\cos x}$

$=\int e^u du$

$=e^u+C$

$=\boxed{e^{\sin x}+C}$

2. Solve the following differential equations with the given initial conditions.

a. $\frac{dy}{dx}=\frac{x^3-2x^2+1}{x^2}$ with initial condition $(1,3)$

$\int dy = \int\frac{x^3-2x^2+1}{x^2}dx = \int(x-2+x^{-2})dx$

$y=\frac{x^2}{2}-2x+\frac{x^{-1}}{-1}+C$

$y=\frac{x^2}{2}-2x-\frac{1}{x}+C$

$(1,3)$: $3=\frac{1}{2}-2-1+C$

$3=-\frac{5}{2}+C$

$C=\frac{11}{2}$

$\boxed{y=\frac{x^2}{2}-2x-\frac{1}{x}+\frac{11}{2}}$

b. $\frac{dy}{dx}=\sqrt{x}+\cos x-\sin x$ with initial condition $(0,0)$

$\int dy=\int(x^{1/2}+\cos x-\sin x)dx$

$y=\frac{2}{3}x^{3/2}-\sin x-\cos x+C$

$0=0-\sin(0)-\cos(0)+C$

$1=C$

$\boxed{y=\frac{2}{3}x^{3/2}-\sin x-\cos x+1}$

3. $\int \frac{3x^2}{\sqrt{x^3+1}}dx =$

$u = x^3+1$, $\frac{du}{dx} = 3x^2$, $dx = \frac{du}{3x^2}$

$\int \frac{3x^2}{\sqrt{u}} \cdot \frac{du}{3x^2} = \int u^{-1/2}du = 2u^{1/2}+C$
$= 2\sqrt{x^3+1}+C$

a. $2\sqrt{x^3+1}+C$ b. $\frac{3}{2}\sqrt{x^3+1}+C$ c. $\sqrt{x^3+1}+C$ d. $\ln\sqrt{x^3+1}+C$

4. $\int (x^2+1)^2 dx =$ $\int (x^4+2x^2+1)dx$

a. $\frac{(x^2+1)^3}{3}+C$ b. $\frac{(x^2+1)^3}{6x}+C$ c. $\left(\frac{x^3}{3}+x\right)^2+C$ d. $\frac{x^5}{5}+\frac{2x^3}{3}+x+C$

5. $\int_1^2 (4x^3-6x)dx =$ $\left(x^4-3x^2\right)\Big|_1^2 = 16-3(4)-(1-3) = 4+2=6$

a. 2   b. 4   c. 6   d. 36

6. $\int_{-3}^{0}(2x-1)e^{x^2-x}dx$

$u = x^2-x$, $\frac{du}{dx} = 2x-1$, $dx = \frac{du}{2x-1}$
$x=-3,\ u=12$; $x=0,\ u=0$

$\int_{12}^{0}(2x-1)e^u \cdot \frac{du}{2x-1} = \int_{12}^{0} e^u du = e^u\Big|_{12}^{0} = e^0-e^{12}$

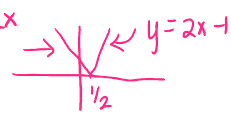
a. $1-\frac{1}{e^6}$ b. $\frac{1}{e^6}-1$ c. $1-e^{12}$ d. $e^{12}-1$

7. $\int_1^e \left(\frac{x^2-1}{x}\right)dx =$ $\int_1^e \left(x-\frac{1}{x}\right)dx = \left(\frac{x^2}{2}-\ln x\right)\Big|_1^e = \frac{e^2}{2}-\ln e-\left(\frac{1}{2}-\ln 1\right) = \frac{e^2}{2}-1-\frac{1}{2}$
$= \frac{e^2}{2}-\frac{3}{2}$

a. $e-\frac{1}{e}$ b. $\frac{e^2-3}{2}$ c. $\frac{e^2}{2}-e+\frac{1}{2}$ d. e^2-2

8. $\int \frac{2}{x^3}dx$ $= \int 2x^{-3}dx = \frac{2x^{-2}}{-2}+C = -\frac{1}{x^2}+C$

a. $-6\ln(x^3)+c$ b. $2\ln(x^3)+c$ c. $-\frac{1}{2x^4}+c$ d. $-\frac{1}{x^2}+c$



9. What is the value of $\int_{-1}^{5}|2x-1|\,dx$?

$\int_{-1}^{1/2}(1-2x)dx + \int_{1/2}^{5}(2x-1)dx$
$= (x-x^2)\Big|_{-1}^{1/2} + (x^2-x)\Big|_{1/2}^{5}$
$= \frac{1}{2}-\frac{1}{4}-(-1-1)+25-5-\left(\frac{1}{4}-\frac{1}{2}\right) =$
22.5

a. 18   b. 22.5   c. -20   d. -18

10. Using the substitution $u=\sqrt{x}$ ($x^{1/2}$), $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ is equal to which of the following?

a. $2\int_1^{16} e^u\,du$ b. $2\int_1^{4} e^u\,du$ c. $2\int_1^{2} e^u\,du$ d. $\frac{1}{2}\int_1^{2} e^u\,du$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ $dx = 2\sqrt{x}\,dx$ $x=1,\ u=1,\ x=4,\ u=2$ $\int_1^2 \frac{e^u}{\sqrt{x}} \cdot 2\sqrt{x}\,dx$