

Practice: Evaluate the following.

1) $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x} =$
 $\lim_{x \rightarrow 0} \frac{2\cos(2x)}{3\cos(3x)} =$
 $\frac{2}{3}$

2) $\lim_{x \rightarrow \infty} \frac{x^3}{e^{x/2}} =$
 $\lim_{x \rightarrow \infty} \frac{3x^2}{\frac{1}{2}e^{x/2}} =$
 $\lim_{x \rightarrow \infty} \frac{6x}{\frac{1}{4}e^{x/2}} =$
 $\lim_{x \rightarrow \infty} \frac{6}{\frac{1}{8}e^{x/2}} =$
 0

3) $\lim_{x \rightarrow \infty} \left(x \sin \frac{1}{x}\right) =$
 $\lim_{x \rightarrow \infty} \frac{\sin(\frac{1}{x})}{\frac{1}{x}} =$
 $\lim_{x \rightarrow \infty} \frac{\cos(\frac{1}{x}) \cdot -\frac{1}{x^2}}{-\frac{1}{x^2}} =$
 $\lim_{x \rightarrow \infty} \cos(\frac{1}{x}) =$
 1

4) $\lim_{x \rightarrow \infty} x^{\frac{1}{x}} = y$
 $\ln \lim_{x \rightarrow \infty} x^{\frac{1}{x}} = \ln y$
 $\lim_{x \rightarrow \infty} \frac{1}{x} \ln x = \ln y$
 $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \ln y$
 $\lim_{x \rightarrow \infty} \frac{1}{x} = \ln y$
 $0 = \ln y$
 $1 = y$

5) $\lim_{x \rightarrow 0^+} x^{1+\ln x} = y$
 $\lim_{x \rightarrow 0^+} \frac{\ln 2}{1+\ln x} \ln x = \ln y$
 $\lim_{x \rightarrow 0^+} \frac{\ln 2 \cdot \ln x}{1+\ln x} = \ln y$
 $\lim_{x \rightarrow 0^+} \frac{\ln 2 \cdot \frac{1}{x}}{\frac{1}{x}} = \ln y$
 $\lim_{x \rightarrow 0^+} \frac{\ln 2}{x} \cdot x = \ln y$
 $\lim_{x \rightarrow 0^+} \ln 2 = \ln y$
 $\ln 2 = \ln y$
 $2 = y$

6) $\lim_{x \rightarrow 1^+} (\ln x)^{x-1} = y$
 $\lim_{x \rightarrow 1^+} (x-1) \ln(\ln x) = \ln y$
 $\lim_{x \rightarrow 1^+} \frac{\ln(\ln x)}{\frac{1}{x-1}} = \ln y$
 $\lim_{x \rightarrow 1^+} \frac{\frac{1}{x \ln x}}{\frac{-1}{(x-1)^2}} = \ln y$
 $\lim_{x \rightarrow 1^+} \frac{-(x-1)^2}{x \ln x} = \ln y$
 $\lim_{x \rightarrow 1^+} \frac{-2(x-1)}{x(\frac{1}{x}) + \ln x} = \ln y$
 $\lim_{x \rightarrow 1^+} \frac{-2(x-1)}{1+\ln x} = \ln y$
 $0 = \ln y$
 $y = 1$

7) $\lim_{x \rightarrow 0^+} \frac{4(e^x - 1 - x)}{-7x^3} =$
 $\lim_{x \rightarrow 0^+} \frac{4(e^x - 1)}{-21x^2} =$
 $\lim_{x \rightarrow 0^+} \frac{4e^x}{-42x} =$
 $-\infty$

8) $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt[3]{x}} =$
 $\lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{3x^{2/3}}} =$
 $\lim_{x \rightarrow \infty} \frac{1}{x} \cdot 3x^{2/3} =$
 $\lim_{x \rightarrow \infty} \frac{3}{x^{1/3}} = 0$

9) $\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi} =$
 $\lim_{x \rightarrow \pi} \frac{\cos x}{1} =$
 $\cos \pi =$
 -1

10) $\lim_{x \rightarrow 1} \frac{\int_1^x (\ln t) dt}{(x-1)^2} =$
 $\lim_{x \rightarrow 1} \frac{\ln x}{2(x-1)} =$
 $\lim_{x \rightarrow 1} \frac{\frac{1}{x}}{2} =$
 $\lim_{x \rightarrow 1} \frac{1}{2x} = \frac{1}{2}$

11) $\lim_{x \rightarrow 0^+} (1+3x)^{\frac{1}{2x}} =$
 $\lim_{x \rightarrow 0^+} \frac{1}{2x} \ln(1+3x) = \ln y$
 $\lim_{x \rightarrow 0^+} \frac{\ln(1+3x)}{2x} = \ln y$
 $\lim_{x \rightarrow 0^+} \frac{\frac{3}{1+3x}}{2} = \ln y$
 $\lim_{x \rightarrow 0^+} \frac{3}{2+6x} = \ln y$
 $\frac{3}{2} = \ln y$
 $e^{3/2} = y$

12) $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right) =$
 $\lim_{x \rightarrow 1^+} \frac{x-1-\ln x}{(x-1)\ln x} =$
 $\lim_{x \rightarrow 1^+} \frac{1-\frac{1}{x}}{\ln x + \frac{x-1}{x}} =$
 $\lim_{x \rightarrow 1^+} \frac{x-1}{x \ln x + x-1} =$
 $\lim_{x \rightarrow 1^+} \frac{1}{\ln x + x(\frac{1}{x}) + 1} =$
 $\lim_{x \rightarrow 1^+} \frac{1}{\ln x + 2} = \frac{1}{2}$

"0/0"

$$13) \lim_{x \rightarrow 0^+} \frac{\arctan x}{x^{2/3}} =$$

$$\lim_{x \rightarrow 0^+} \frac{1}{\frac{2}{3} x^{-1/3}} =$$

$$\lim_{x \rightarrow 0^+} \frac{1}{\frac{2}{3} x^{1/3}} =$$

$$\lim_{x \rightarrow 0^+} \frac{3x^{1/3}}{2(1+x^2)} =$$

$$\frac{0}{2} =$$

$$\boxed{0}$$

"∞/∞"

$$14) \lim_{x \rightarrow \infty} \frac{\int_1^x \sqrt{t} dt}{\sqrt{x}} =$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{2\sqrt{x}} =$$

$$\lim_{x \rightarrow \infty} \frac{1}{2} =$$

$$\boxed{\frac{1}{2}}$$

$$15) \lim_{x \rightarrow \infty} 3x \sin\left(\frac{1}{3x}\right) =$$

$$\lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{3x}\right)}{\frac{1}{3x}} =$$

$$\lim_{x \rightarrow \infty} \frac{\cos\left(\frac{1}{3x}\right) \cdot \frac{-1}{(3x)^2} \cdot 3}{\frac{-1}{(3x)^2} \cdot 3} =$$

$$\lim_{x \rightarrow \infty} \cos\left(\frac{1}{3x}\right) =$$

$$\boxed{1}$$

$$16) \lim_{x \rightarrow 0^+} 3x^{\frac{x}{2}} = y$$

$$3 \lim_{x \rightarrow 0^+} x^{\frac{x}{2}} = y$$

$$\lim_{x \rightarrow 0^+} \frac{x}{2} \ln x = \ln\left(\frac{y}{3}\right)$$

$$\lim_{x \rightarrow 0^+} \frac{\ln x}{2 \cdot \frac{1}{x}} = \ln\left(\frac{y}{3}\right)$$

$$\lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^2}} = \ln\left(\frac{y}{3}\right)$$

$$\lim_{x \rightarrow 0^+} \frac{x}{-2} = \ln\left(\frac{y}{3}\right)$$

$$0 = \ln\left(\frac{y}{3}\right)$$

$$1 = \frac{y}{3}$$

$$\boxed{3 = y}$$