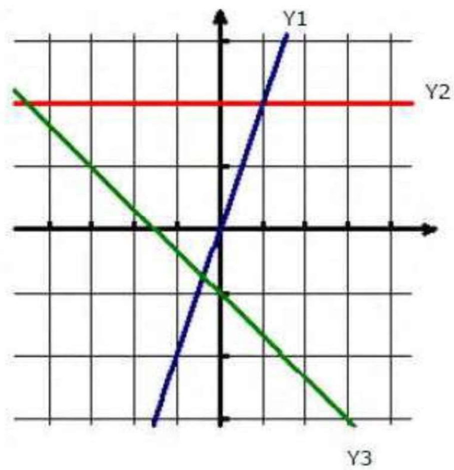

Do Now:

1. Write the simplest rule possible for each of the functions based solely on the graph.

$y_1 =$ $2x$

$y_2 =$ 2

$y_3 =$ $-\frac{2}{3}x-1$

2. The above graphs are actually functions displayed on a zoomed in window. Match each graph with the correct function. Justify your answer.

$y = x^3 + 0.002$ $y_2 = 2$

$y = -\dfrac{2}{3}x - 0.001$ $y_3 = -\frac{2}{3}x-1$

$y = \sin(2x)$ $y_1 = 2x$

3. What is the scale of the above graph? $.001$

4. Algebraically find the equation of the tangent line at $x = 0$ for each function in question 2.

$y' = 3x^2$
$y'|_0 = 0$ $(0, .002)$
$y = .002$

$y' = -\frac{2}{3}$
$y'|_0 = -\frac{2}{3}$ $(0, -.001)$
$y = -\frac{2}{3}x - .001$

$y' = 2\cos(2x)$
$y'|_0 = 2$ $(0, 0)$
$y = 2x$

All differentiable curves are **locally linear**, since we can make the curve appear linear if we get close enough to a specific point. The *tangent line* provides a useful representation of the curve itself if we stay close enough to the point of tangency and creates the **linear approximation** at that point.

Class Work and Homework:

1. Graph the equation $y = \tan\left(\frac{x}{2}\right)$ on a zoom 4 decimal window. Zoom into a very small window.

Write the equation of the line that can be used as the linear approximation (i.e. the tangent line) for this function at $x = 0$. Graph both in the same window.

$y' = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

$y'\big|_{x=0} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x-0)$

$\boxed{y = \frac{1}{2}x}$

2. Find the linearization of $f(x) = \sqrt{x}$ (i.e. the tangent line) at $x = 1$ and use it to approximate $\sqrt{1.02}$ without a calculator. Then use a calculator to determine the accuracy of the approximation. Is your approximation an overestimation or an underestimation?

$f'(x) = \frac{1}{2\sqrt{x}}$ $(1,1)$ $y - 1 = \frac{1}{2}(x-1)$

$y = \frac{1}{2}(x-1) + 1 \Rightarrow L(x) = \frac{1}{2}(x-1)+1$

$f'(1) = \frac{1}{2}$

$L(1.02) \approx \frac{1}{2}(1.02-1)+1 = \frac{1}{2}(.02)+1 = 1.01$

$f''(x) = \frac{-1}{4x^{3/2}}$

$\sqrt{1.02} = 1.00995$

$f''(1) = -\frac{1}{4} < 0$, Since the function is concave down, the approximation is an overestimation.

3. The slope of a function at any point (x, y) is $-\frac{x+1}{y}$. The point (3, 2) is on the graph of *f*.

(a) Write an equation of the line tangent to the graph of *f* at $x = 3$. $y - 2 = -2(x-3)$

$\boxed{y = -2x+8}$

(b) Use the tangent line in part (a) to approximate $f(3.1)$.

$y' = -\frac{x+1}{y}$

$L(3.1) = -2(3.1)+8 = 1.8$

$y'\big|_{(3,2)} = -2$

$\boxed{f(3.1) \approx L(3.1) = 1.8}$

4. Let f be the function that is differentiable for all real numbers. The table below gives the values of f and its derivative for selected values in the interval $-0.9 \le x \le 0.9$. The second derivative is always positve, which means the function is concave up, on the same closed interval. Write an equation of the line tangent to the graph of f where $x = -0.6$. Use this line to approximate the value of $f(-0.5)$. Is this approximation greater or less than the actual value of $f(-0.5)$? Give a reason to support your answer.

x	-0.9	-0.6	-0.3	0	0.3	0.6	0.9
$f(x)$	-34	-87	-99	-100	-84	-51	21
$f'(x)$	-69	-30	-9	0	1	9	90

$y+87=-30(x+.6)$

$L(x)=-30x-105$

$\boxed{f(-\tfrac{1}{2})\approx L(-\tfrac{1}{2})=-30(-\tfrac{1}{2})-105=-90}$

$f''(x)>0 \rightarrow$ concave up (tangent line below curve)

$\boxed{\therefore \text{ under approximation}}$

5. Find the linearization of $f(x)=\cos(x)$ at $x=\frac{\pi}{2}$ and use it to approximate $\cos(1.75)$ without a calculator. Then use a calculator to determine the accuracy of the approximation.

$f'(x)=-\sin x$

$f'(\tfrac{\pi}{2})=-1$

$y-0=-1(x-\tfrac{\pi}{2})$

$L(x)=-x+\tfrac{\pi}{2}$

$f(1.75)\approx \boxed{L(1.75)=-1.75+\tfrac{\pi}{2}\approx-.179}$

$f(1.75)=\cos(1.75)=-.178$

6. Use linearization to approximate **(a)** $\sqrt{123}$ and **(b)** $\sqrt[3]{123}$ without a calculator. Then compare the result to the actual value.

a) $f'(x)=\frac{1}{2\sqrt{x}}$

$f'(121)=\frac{1}{22}$

$y-11=\frac{1}{22}(x-121)$

$L(x)=\frac{1}{22}(x-121)+11$

$\boxed{L(123)=11+\tfrac{1}{22}(123-121)=11+\tfrac{2}{22}\approx 11.\overline{09}}$

$f(123)=\sqrt{123}=11.0905$

b) $f'(x)=\frac{1}{3x^{2/3}}$

$f'(125)=\frac{1}{75}$

$y-5=\frac{1}{75}(x-125)$

$L(x)=\frac{1}{75}(x-125)+5$

$\boxed{L(123)=\tfrac{1}{75}(123-125)+5=4.9\overline{73}}$

$f(123)=\sqrt[3]{123}=4.9732$

7. Estimate $(16.5)^{\frac{1}{4}} - 16^{\frac{1}{4}}$ using linear approximation.

$f(x) = x^{\frac{1}{4}}$

$f'(x) = \frac{1}{4x^{3/4}}$

$f'(16) = \frac{1}{32}$

$L(x) = \frac{1}{32}(x-16)+2$

$L(16.5) = \frac{1}{64}+2$

$(16.5)^{\frac{1}{4}} - 16^{\frac{1}{4}} \approx \frac{1}{64}+2-2 = \boxed{\frac{1}{64}}$

8. Approximate the value of sin 31° using radians.

$f'(x) = \cos(x)$

$f'(\frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$L(x) = \frac{\sqrt{3}}{2}(x-\frac{\pi}{6}) + \frac{1}{2}$

$L(\frac{\pi}{6}+\frac{\pi}{180}) = \frac{1}{2} + \frac{\sqrt{3}}{2}(\frac{\pi}{6}+\frac{\pi}{180}-\frac{\pi}{6}) = \frac{1}{2} + \frac{\sqrt{3}\pi}{360} = \boxed{0.515115}$

$\sin(31) = .515038$

9. If f is a differentiable function, $f(2) = 6$ and $f'(2) = -\frac{1}{2}$, find the approximate value of $f(2.1)$.

$L(x) = -\frac{1}{2}(x-2)+6$

$\boxed{f(2.1) \approx L(2.1) = 5.95}$

10. Write an equation of the tangent line to $f(x) = x^3$ at (2, 8). Use the tangent line to find the approximate values of $f(1.9)$ and $f(2.01)$. Are these estimations greater or less than the actual value of $f(1.9)$ and $f(2.01)$?

$f'(x) = 3x^2$

$f'(2) = 12$

$L(x) = 12(x-2)+8$

$\boxed{f(1.9) \approx L(1.9) = 6.8}$

$\boxed{f(2.1) \approx L(2.1) = 9.2}$

$f''(x) = 6x$

$f''(2) = 12 > 0$ concave up (tangent line below)

$\boxed{\text{underapproximation}}$