

Use calculators only when asked to find the exact values or to check your answers.

1. Consider the curve $\frac{d}{dx}(x^2 - xy + y^2 = 19)$.

a. Write an equation of the line tangent to the curve at the point $(-2, 3)$.

$$2x - y - x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0 \quad \left. \frac{dy}{dx} \right|_{(-2,3)} = \frac{3 - 2(-2)}{2(3) - (-2)} = \frac{7}{8} \quad \boxed{y - 3 = \frac{7}{8}(x + 2)}$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

b. Use your answer in part a to approximate q , where $(-2.168, q)$ is on the curve near $(-2, 3)$.

$$L(x) = \frac{7}{8}(x + 2) + 3 \quad q \approx L(-2.168) = \boxed{2.853}$$

c. Compare the actual value, correct to three decimal places, of q to your answer in part b and draw a tentative conclusion about the concavity of the curve at $(-2, 3)$.

$$(-2.168)^2 + 2.168y + y^2 = 19 \quad y \approx 2.850$$

$$y^2 + 2.168y - 14.300 = 0$$

Since the approximation is an overestimation, the curve is concave down.

2. Let f be the function that is differentiable for all real numbers. The table below gives the values of f and its derivative for selected values in the interval $-1.5 \leq x \leq 1.5$. The second derivative is always positive in the same closed interval. Write an equation of the line tangent to the graph of f where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater or less than the actual value of $f(1.2)$? Give a reason to support your answer.

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

$$(1, -4) \quad f'(1) = m = 5$$

$$y + 4 = 5(x - 1)$$

$$L(x) = 5(x - 1) - 4$$

$$f(1.2) \approx L(1.2) = -3$$

$$\boxed{f(1.2) \approx -3}$$

Since $f''(1.2) > 0$, curve is concave up and this is an underapproximation.

3. Box office revenue at a multiplex cinema in Paris is $R(p) = 3600p - 10p^3$ euros per showing when the ticket price is p euros. Calculate $R(p)$ for $p = 9$ and use the Linear Approximation to estimate ΔR if p is raised or lowered by 0.5 euros.

$$R'(p) = 3600 - 30p^2$$

$$R'(9) = 1170$$

$$(9, 25,110)$$

$$y - 25,110 = 1170(x - 9)$$

$$L(x) = 1170(x - 9) + 25,110$$

$$L(9.5) = 25,695$$

$$L(8.5) = 24,525$$

$$\Delta R = \$585$$

$$\Delta R(9.5) = 25,695 - 25,110 = 585$$

$$\Delta R(8.5) = 25,110 - 24,525 = 585$$

4. What is $f'(2)$ if the linearization of $f(x)$ at $a=2$ is $L(x)=2x+4$?

$$f'(2) \approx L'(2) = 2$$

5. Compute $\frac{dy}{dx}$ at the point $P=(2,1)$ on the curve $y^3 + 3xy = 7$ and show that the linearization at

P is $L(x) = -\frac{1}{3}x + \frac{5}{3}$. Use $L(x)$ to estimate the y -coordinate of the point on the curve where $x=2.1$.

$$3y^2 \frac{dy}{dx} + 3y + 3x \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y}{y^2+x}$$

$$\left. \frac{dy}{dx} \right|_{(2,1)} = -\frac{1}{3}$$

$$y-1 = -\frac{1}{3}(x-2)$$

$$L(x) = -\frac{1}{3}(x-2) + 1$$

$$L(x) = -\frac{1}{3}x + \frac{2}{3} + \frac{3}{3}$$

$$L(x) = -\frac{1}{3}x + \frac{5}{3}$$

$$L(2.1) = -\frac{1}{3}\left(\frac{21}{10}\right) + \frac{5}{3}$$

$$y(2.1) \approx \frac{29}{30}$$

6. The cube root of 27 is 3. How much larger is the cube root of 27.2? Estimate using Linear Approximation.

$$y = \sqrt[3]{x}$$

$$y' = \frac{1}{3x^{2/3}}$$

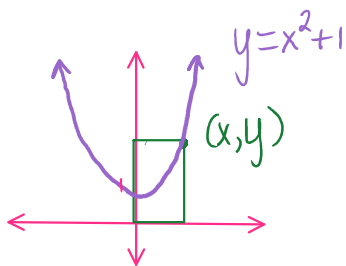
$$y'(27) = \frac{1}{3(27)^{2/3}} = \frac{1}{27}$$

$$y-3 = \frac{1}{27}(x-27)$$

$$L(x) = \frac{1}{27}(x-27) + 3$$

$$y(27.2) \approx L(27.2) = \frac{1}{27}(.2) + 3 \approx 3.007$$

7. A rectangle has one side on the x -axis, one side on the y -axis, and a corner on the graph of $y = x^2 + 1$. Use Linear Approximation of the area formula to estimate the increase in the area of the rectangle if the base grows from 2 inches to 2.3 inches. Then, use a calculator to estimate the exact increase.



$$A = xy$$

$$A(x) = x(x^2 + 1) = x^3 + x$$

$$A'(x) = 3x^2 + 1$$

$$A'(2) = 13$$

$$A(2) = 10$$

$$(2,10) \quad m=13$$

$$y-10 = 13(x-2)$$

$$L(x) = 13(x-2) + 10$$

$$A(2.3) \approx L(2.3) = 13.9 \text{ in}^2$$

$$A(2) = 10 \quad \therefore 3.9 \text{ in}^2 \text{ increase}$$

$$\text{Actual } A(2) = 14.467 \text{ in}^2$$

$$\text{actual increase } 1.467 \text{ in}^2$$