Practice Exam 1

Saturday, February 25, 2017 11:01 AM

MC – Part I

B 1. $\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x)$

$25 + 5b = 5\sin\left(\frac{5\pi}{2}\right)$

$25 + 5b = 5$

$b = -4$

C 2. $y' = 6x - 3x^2 = 0 \quad x = 0, 2$

$3x(2 - x) = 0$

y' rel max: $-$ | $+$ | $-$

y: $\searrow$ 0 $\nearrow$ 2 $\searrow$ at $x = 2$

B 3. 

$\rho(x) = \frac{\text{trees}}{mi^2}$

$5\int_0^3 \rho(x)\,dx$

$mi \cdot \frac{\text{trees}}{mi^2} \cdot mi$

C 4. $f(x) = e^{\sin x}$

$f'(x) = \cos x \, e^{\sin x} = 0$

$e^{\sin x} \neq 0 \quad \cos x = 0$

$x = \pi/2, 3\pi/2$

2 zeros

D 5. $\lim_{x \to 0} \frac{3x^2 - \sin x}{2x^2 + x}$ "$\frac{0}{0}$"

$= \lim_{x \to 0} \frac{6x - \cos x}{4x + 1}$

$= \frac{0 - 1}{0 + 1} = -1$

D 6. $y = \ln(3x + 5)$

$\frac{dy}{dx} = \frac{3}{3x + 5} = 3(3x+5)^{-1}$

$\frac{d^2y}{dx^2} = \frac{-3}{(3x+5)^2} \cdot 3 = \frac{-9}{(3x+5)^2}$

D 7. $f(x) = \sqrt{2 - 4\sin x}$

$f'(x) = \frac{1}{2\sqrt{2 - 4\sin x}} \cdot -4\cos x$

$f'(x) = \frac{-2\cos x}{\sqrt{2 - 4\sin x}}$

$f'(\pi) = \frac{-2(-1)}{\sqrt{2 - 0}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

B 8. $\pi(R(x))^2$ is cross section of pipe. Integrating from 10,000 → 30,000 gives Volume in that section.

C 9. $x^2 + y^2 = 169 \quad (5, -12)$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{y}$

$\left.\frac{dy}{dx}\right|_{(5,-12)} = \frac{5}{12}$

$y + 12 = \frac{5}{12}(x - 5)$

$12y + 144 = 5x - 25$

$169 = 5x - 12y$

D 10. Speed = |Velocity|

Since pt $|V(D)|$ is greatest, Speed is greatest at pt. D

B 11. $5 = x^2 + 1$

$x^2 = 4$

$x = \pm 2$

$A = \int_0^2 5 - (x^2 + 1)\,dx$

$A = \int_0^2 4 - x^2\,dx$

$A = 4x - \frac{x^3}{3}\Big|_0^2$

$A = 8 - \frac{8}{3} = \frac{16}{3}$

A 12. $\int \frac{1}{\sqrt{4 - x^2}}\,dx \quad u = x, \ a = 2$

$= \arcsin\frac{x}{2} + C$

D 13. $f(x) = 2x^2 + \frac{k}{x}$

$f'(x) = 4x - \frac{k}{x^2}$

$f''(x) = 4 + \frac{2k}{x^3} = 0$

at $x=-1$, $4 - 2k = 0$

$k = 2$

A 14. $\int \sin(3x+4)\,dx$

$u = 3x+4$

$\frac{du}{dx} = 3 \rightarrow dx = \frac{du}{3}$

$\frac{1}{3}\int \sin u\,du$

$= -\frac{1}{3}\cos u + C$

$= -\frac{1}{3}\cos(3x+4) + C$

B 15. $y = \frac{2}{4-x}$

$y' = \frac{2}{(4-x)^2}$

$y'' = \frac{4}{(4-x)^3}$

$x = 4$ only PPOI

y'' + | −

y ∪ 4 ∩

Concave down on $(4, \infty)$

A 16. $f(x) = x^3 + 3x^2 - 2$

$f(-1) = 0$

$f'(x) = 3x^2 + 6x$

$f'(-1) = 3 - 6 = -3$ $\therefore g'(0) = -\frac{1}{3}$

$g(x) = f^{-1}(x)$

$g'(0) = \frac{1}{f'(-1)}$

B 17. $f(x) = \frac{e^x}{1+e^x}$, $x = 1$

$f'(x) = \frac{e^x(1+e^x) - e^x(e^x)}{(1+e^x)^2}$

$f'(x) = \frac{e^x}{(1+e^x)^2}$

$f'(1) = \frac{e}{(1+e)^2}$

C 18. $\frac{ds}{dt} = k\sqrt{\frac{1}{6}s}$

A 19. $\frac{d}{dx}\int_4^{x^2} \frac{dt}{1-\sqrt{t}}$

$= \frac{1}{1-\sqrt{x^2}} \cdot 2x$

$= \frac{2x}{1-x}$

D 20. Dependant on y only (eliminates A & B)

Cannot be C or slope > 0 for all x

C 21. $f'(x) < 0$ decr

$f''(x) > 0$ concave up

B 22. $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$f'(a) = 0$

D 23. $\int_2^8 \frac{f(x)g'(x) - g(x)f'(x)}{(f(x))^2}\,dx$

$= \frac{g(x)}{f(x)}\Big|_2^8$

$= \frac{g(8)}{f(8)} - \frac{g(2)}{f(2)}$

$= \frac{-6}{3} - \frac{2}{1}$

$= -4$

A 24. $g(x) = \int_1^x f(t)\,dt$

$g(-4) = -\int_{-4}^1 f(t)\,dt$

$= -\left(2 - \frac{3}{2}\right) = -\frac{1}{2}$

$g(-2) = -\int_{-2}^1 f(t)\,dt$

$= -\left(-\frac{3}{2}\right) = \frac{3}{2}$

$g(1) = 0$

$g(5) = \int_1^5 f(t)\,dt = 2$

$g(-4) < g(1) < g(-2) < g(5)$

A 25. $\frac{dc}{dt} = \frac{1}{2}\frac{m}{min}$ $\quad \frac{dA}{dt} = ?$

$r = 4m$

$C = 2\pi r$

$\frac{dc}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dc/dt}{2\pi}$

$\frac{dr}{dt} = \frac{1}{4\pi}\frac{m}{min}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(4)\cdot\frac{1}{4\pi}$

$\frac{dA}{dt} = 2\frac{m^2}{min}$

C 26. $\int_{-2}^{1} x f(x)\, dx$

$= \int_{-2}^{0} x^2 dx + \int_{0}^{1}(x^2+x)dx$

$= \frac{x^3}{3}\Big|_{-2}^{0} + \left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Big|_0^1$

$= 0 - \left(\frac{-8}{3}\right) + \frac{1}{3} + \frac{1}{2} - 0$

$= 3\frac{1}{2} = \frac{7}{2}$

D27. $Avf = \frac{1}{0+4}\int_{-4}^{0}\cos\frac{1}{2}x\,dx$

$u = \frac{1}{2}x$ $\quad x=-4,\ u=-2$

$du = \frac{1}{2}dx$ $\quad x=0,\ u=0$

$dx = 2du$

$Avf = \frac{1}{4}\cdot 2\int_{-2}^{0}\cos u\,du$

$= \frac{1}{2}(\sin u)\Big|_{-2}^{0}$

$= \frac{1}{2}(\sin 0) - \frac{1}{2}\sin(-2)$

$= -\frac{1}{2}\sin(-2)$

since $\sin x$ is odd

$\sin(-2) = -\sin(2)$

$Avf = \frac{1}{2}\sin(2)$

D28. $\int_{2}^{8}\frac{dx}{\sqrt{2x}+1}$

$u = \sqrt{2x}$

$\frac{du}{dx} = \frac{1}{\sqrt{2x}}$

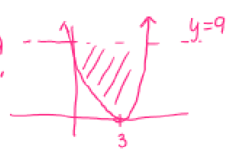
$dx = \sqrt{2x}\,du$

$x=2,\ u=2$

$x=8,\ u=4$

$\int_{2}^{4}\frac{u\,du}{u+1}$

B 29.



$(x-3)^2 = 9$

$x-3 = \pm 3$

$x = 0, 6$

$V = \pi\int_{0}^{6}(9-(x-3)^2)dx$

Due to symmetry in line $x=3$

$V = 2\pi\int_{0}^{3}(9-(x-3)^2)dx$

B 30. Since $f(x)$ is even

$\int_{-7}^{-1} f(x)dx = \int_{1}^{7} f(x)dx$

$\int_{1}^{7} f(x)dx = \int_{0}^{7} f(x)dx - \int_{0}^{1} f(x)dx$

$= 1 - 5 = -4$

OR

$\int_{0}^{7} f(x)\,dx = \int_{0}^{1} f(x)dx + \int_{1}^{7} f(x)dx$

$1 = 5 + \int_{1}^{7} f(x)dx$

$\int_{1}^{7} f(x)dx = -4$

MC Part B

B 31. $f(x)=\tan x \quad g(x)=x^2$

$f'(x)=\sec^2 x \quad g'(x)=2x$

$\sec^2 x = 2x$

$\sec^2 x - 2x = 0$

$x = 2.083$

A 32. $f(t)=\frac{1}{t},\ t>0$

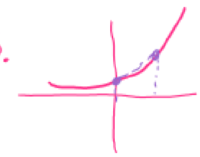
$\frac{f(b)-f(a)}{b-a} = \frac{(\frac{1}{b}-\frac{1}{a})ab}{(b-a)ab}$

$= \frac{a-b}{ab(b-a)} = \frac{-1}{ab}$

$f'(t) = \frac{-1}{t^2} = \frac{-1}{ab}$

$t^2 = ab \quad t = \pm\sqrt{ab}$

$t>0 \quad t=\sqrt{ab}$

A 33. 

Since concave up, top of trapezoid is above curve.

$L < A \quad T > A$

$R > A \quad T < R$

$L < A < T < R$

D 34. $R(t)$ $\frac{gal}{hr} \Rightarrow V'$

$V = \int_0^3 R(t)\,dt$

$\frac{gal}{hr} \cdot hr = gallons$

A 35. $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(3) \cdot (-3)$

$= (-5)(-3)$

$= 15$

C 36. $f'(c) = \frac{f(b)-f(0)}{b-0}$

Since f is increasing,

$f(b) > f(0)$

so $f'(c) > 0$

C 37. 

even functions Symmetrical to y axis

$A = 2\int_0^{1.137} \sqrt{4-x^2} - (e^{x^2}-2)\,dx$

$A = 5.0496$

C 38. $\lim_{x\to 1^-} \frac{x^3-1}{|x^3-1|}$

when $x<1$, $x^3-1=-c$

$\therefore |x^3-1| = c$

$\lim_{x\to 1^-} \frac{x^3-1}{|x^3-1|} = \frac{-c}{c} = -1$

D 39. $f(x)$ has 4 POI

Since $f'(x)$ changes direction 4 times

So $f''(x)$ changes sign 4 times.

A 40. $Avf = \frac{1}{12-0}\int_0^{12} f(x)\,dx$

$\int_0^{12} f(x)\,dx = 8(4) + \frac{1}{4}\pi(4)^2$

$= 32 + 4\pi$

$Avf = \frac{1}{12}(32+4\pi)$

≈ 3.714

A 41. $\int_1^5 f(x)\,dx = g(x)\Big|_1^5$

$10.882 = g(5) - g(1)$

$10.882 = 7 - g(1)$

$g(1) = -3.882$

B 42. $A(t) = 4000 + 48(t-3) - 4(t-3)^3$

$A'(t) = 48 - 12(t-3)^2 = 0$

$A''(t) = -24(t-3) = 0$

$t = 3$

		3	
$A''(t)$	+		−
$A'(t)$	↗		↘

Rate of Production is greatest at $t = 3$ hrs

$8am + 3 = 11am$

A43. $g(x)=\int_3^x (5+4t-t^2)(2^{-t})\,dt$

$g'(x)=(5+4x-x^2)(2^{-x})$

$$\begin{array}{c|ccccc} g'(x) & & + & & - & \\ g(x) & 3 & \nearrow & 5 & \searrow & 7 \end{array}$$

g increasing on $(3,5)$

$g(3)=0$

$g(7)>0$ since

area $(3,5) >$ area $(5,7)$

$+ \quad > \quad -$

more area above than below

C44. $P(t)=6000-\frac{5500}{e^{.159t}}$

$\lim_{t\to\infty} 6000-\cancel{\frac{5500}{e^{.159t}}}^{\,0}=6000$

$\frac{1}{2}$ Limiting value is 3000

$6000-\frac{5500}{e^{.159t}}=3000$

$3000-\frac{5500}{e^{.159t}}=0$

$e^{.159t}=\frac{30}{55}$

$t=\frac{\ln(30/55)}{.159}\approx 3.8$

4th year

C45. $\lim_{h\to 0}\frac{g(x+h)-g(x-h)}{h}$

$\frac{g(x+h)-g(x-h)}{x+h-(x-h)}$

$g'(x)=\lim_{h\to 0}\frac{g(x+h)-g(x-h)}{2h}$

$2g'(x)=\lim_{h\to 0}\frac{g(x+h)-g(x-h)}{h}$

$2g'(x)=6-4x$

$g'(x)=3-2x$

$g''(x)=-2$

$g'''(x)=0$

$g'(0)=3 \quad g''(0)=-2$

$g'''(0)=0$

OE Part A

1. $L(t)=167.5\sin\left(\frac{2\pi}{366}(t-80)\right)+731$

$L'(t)=\frac{2\pi}{366}\left(167.5\cos\left(\frac{2\pi}{366}(t-80)\right)\right)=0$

$$\begin{array}{c|ccccccc} L'(t) & & + & & - & & + & \\ L(t) & 0 & \nearrow & 171.5 & \searrow & 354.5 & \nearrow & 365 \end{array}$$

Use GC!

a) Rel max occurs at $t=171.5$
On the 172 day or June 21.

b) $\int_0^{365} L(t)\,dt = 266{,}980$ min.
or 266,979 min

c) $\frac{1}{365}\int_0^{365} L(t)\,dt=\frac{266{,}980}{365}=731.451$ min
731 min.

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2. $f(x)=50\cos\left(\frac{x}{100}\right)$ pt A $\left(a, 50\cos\left(\frac{a}{100}\right)\right)$

$f'(x)=\frac{-50}{100}\sin\left(\frac{x}{100}\right)$

$f'(x)=-\frac{1}{2}\sin\left(\frac{x}{100}\right)$

a) $f'(a)=-\frac{1}{2}\sin\left(\frac{a}{100}\right)$

$y-50\cos\left(\frac{a}{100}\right)=-\frac{1}{2}\sin\left(\frac{a}{100}\right)(x-a)$

b) Person's eyes are at $(0,55)$

$55-50\cos\left(\frac{a}{100}\right)=\frac{a}{2}\sin\left(\frac{a}{100}\right)$

$a=45.935$

flag is 25 ft tall
height is at -25ft.

c) Period of f: $\frac{2\pi}{b}=200\pi$

Halfway will be low point at $x=100\pi$

$f(100\pi)=50\cos\pi=-50$ ft

$y(100\pi)=-\frac{1}{2}\sin\left(\frac{45.935}{100}\right)(100\pi-45.935)+50\cos\left(\frac{45.935}{100}\right)$

$y(100\pi)=-14.643$ ft.

Since the lowest the person can see is -14.643ft and the flag is at -25 ft, the person cannot see the flag.

OE - Part II

3. $x(t) = 7t - 4t^2 + \int_0^t s^2\,ds$

$x'(t) = 7 - 8t + t^2$

$x''(t) = -8 + 2t$

a) $x(3) = 7(3) - 4(3)^2 + \int_0^3 s^2\,ds$

$= 21 - 36 + \frac{s^3}{3}\Big|_0^3 = -15 + 9 = -6$

$x'(3) = 7 - 8(3) + 3^2 = 7 - 24 + 9 = -8$

b) Speed = |velocity|

Speed (3) = 8

$x''(3) = -8 + 6 = -2$

The speed is increasing since velocity & acceleration have the same sign.

c) $7 - 8t + t^2 = 0$

$t^2 - 8t + 7 = 0$

$(t-7)(t-1) = 0$

$t = 1$

On the interval $(0,4)$, the particle will change direction at $t=1$, since $x'(t) > 0 \rightarrow x'(t) < 0$ at $t=1$.

d)

$x'(t)$	[+	−]	
$x(t)$	0	1	4

$x(0) = 0$

$x(1) = 7 - 4 + \int_0^1 s^2\,ds = 3 + \frac{s^3}{3}\Big|_0^1 = 3\frac{1}{3}$

$x(4) = 7(4) - 4(4)^2 + \int_0^4 s^2\,ds = 28 - 64 + \frac{s^3}{3}\Big|_0^4$

$= -36 + \frac{64}{3} = \frac{-108}{3} + \frac{64}{3} = \frac{-44}{3}$

The particle is furthest right at $t = 1$, $x(1) = 3\frac{1}{3}$.

It is farthest left at $t = 4$, $x(4) = -44/3$.

4. $h(x) = f(g(x)) - g(f(x))$

$h'(x) = f'(g(x))g'(x) - g'(f(x))f'(x)$

a) $h(x)$ is differentiable on $(1,4)$ which implies continuity.

$h(1) = f(g(1)) - g(f(1))$

$= f(2) - g(3) = 5 - 5 = 0$

$h(4) = f(g(4)) - g(f(4))$

$= f(5) - g(2) = 4 - 4 = 0$

Since $\frac{h(4) - h(1)}{4 - 1} = 0$, by the MVT, there must exist a pt c on $(1,4)$ such that $f'(c) = 0$.

b) $h'(3) = f'(g(3))g'(3) - g'(f(3))f'(3) = 0$

$= f'(5)(k) - g'(1)(5) = 0$

$8k - 9(5) = 0$

$8k = 45$ $\boxed{k = 45/8}$

c) $w(x) = 7 + \int_1^{f(x)} g(t)\,dt$

$w(3) = 7 + \int_1^{f(3)} g(t)\,dt = 7 + 0 = 7$

$w'(x) = g(f(x))\,f'(x)$

$w'(3) = g(f(3))\,f'(3) = g(1)\cdot(5) = 2(5) = 10$

$\boxed{y - 7 = 10(x-3)}$

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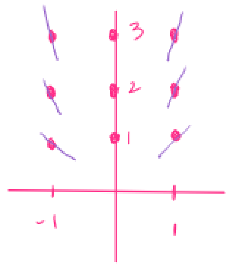
5. a)



b) $\frac{dy}{dx} = xy^2 \quad y(1) = 1$

$\lim_{x\to 1} \frac{y-1}{x^2-1} = \lim_{x\to 1} \frac{dy/dx}{2x}$

$= \frac{xy^2}{2x} = \frac{y^2}{2} = \boxed{\frac{1}{2}}$

c) $\frac{dy}{dx} = xy^2 \quad y(1) = 1$

$\int \frac{dy}{y^2} = \int x\,dx$

$-\frac{1}{y} = \frac{x^2}{2} + C$

$-1 = \frac{1}{2} + C$

$C = -\frac{3}{2}$

$-\frac{1}{y} = \frac{x^2-3}{2}$

$\boxed{y = \frac{2}{3-x^2}}$

d) $y = \frac{2}{3-x^2}$

vertical asymptotes

$x = \pm\sqrt{3}$

$\lim_{x\to\pm\sqrt{3}} \frac{2}{3-x^2} = \infty$

horizontal asymptotes

$y = 0$

$\lim_{x\to\pm\infty} \frac{2}{3-x^2} = 0$

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6) $G(x) = G(-2) + \int_{-2}^{x} f(t)\,dt$

$G'(x) = f(x)$

$G''(x) = f'(x)$

a) G is concave down on $(-1, \frac{3}{2})$ since $G'(x) = f(x)$ is decreasing.

or $G''(x) = f'(x) < 0$

b) $y = mx + 7$

$m = ? \quad G(0) = ?$

$G'(0) = f(0) = 2$

$\boxed{\begin{array}{c} m = 2 \\ G(0) = 7 \end{array}}$ $\quad G(0) = y = 7$

$y = 2x + 7$

c) $\text{Av } f = 0 = \frac{1}{3}\int_0^3 f(x)\,dx = G(x)\Big|_0^3$

$0 = G(3) - G(0)$

$0 = G(3) - 7$

$\boxed{G(3) = 7}$