AP Practice Exam 2

MC

A. 1. $\int_0^2 2x^3+3\,dx = \frac{1}{2}x^4+3x\Big]_0^2$

$= \frac{1}{2}(2)^4+3(2)$

$= 14$

A 2. $\frac{1}{2-(-2)}\int_{-2}^{2} f'(x)dx = \frac{1}{4}\left[f(x)\right]_{-2}^{2}$

$= \frac{1}{4}(f(2)-f(-2))$

$= \frac{1}{4}(e-a)$

$= \frac{e-a}{4}$

B 3. $\lim_{x\to\infty} \frac{x^3+2x^2-6x+4}{3x^3-4x^2+5x+1} = \frac{1}{3}$

B 4. $\int_0^1 e^{-2x}dx = -\frac{1}{2}e^{-2x}\Big]_0^1 = -\frac{1}{2}e^{-2}+\frac{1}{2}e^0$

$= \frac{-1}{2e^2}+\frac{1}{2}$

$= \frac{1}{2}-\frac{1}{2e^2}$

C 5. $f'(x)=4x^3-16x=0$

$4x(x-2)(x+2)=0$

$f'(x)$: $-$ | $+$ | $-$ | $+$

$f(x)$: $\searrow$ -2 $\nearrow$ 0 $\searrow$ 2 $\nearrow$

$x=\pm 2$

B 6. $\int \sqrt{x}(x+2)dx =$

$\int x^{3/2}+2x^{1/2}dx =$

$\frac{2}{5}x^{5/2}+\frac{4}{3}x^{3/2}+C$

D 7. $\int_0^8 a(t)dt = v(8)-v(0)$

$v(8)=v(0)+\int_0^8 a(t)dt$

$=4+2(2)+2(3)+2(4)$

$+2(3)= 28$

C 8. $y'=4x^3+2bx+8=0$

$y''=12x^2+2b=0$

$b=-6x^2$

$y'=4x^3-12x^3+8=0$

$-8x^3+8=0$

$x^3=1 \rightarrow x=1$

D 9. $y'=\frac{3(4x-3)-4(3x+4)}{(4x-3)^2}$

$y'=\frac{12x-9-12x-16}{(4x-3)^2}$

$y'=\frac{-25}{(4x-3)^2}$

$y'\big|_{(1,7)}=\frac{-25}{1}=-25$

$y-7=-25(x-1)$

$y=-25x+32$

$y+25x=32$

B 10. $\int 2\tan(x)dx=$

$2\ln|\sec x|=$

$\ln(\sec^2 x)$

A 11. $f'(x)=\frac{f(2)-f(-1)}{2-(-1)}$ on $(-1,2)$

$3x^2=\frac{8+1}{3}$

$3x^2=3$

$x^2=1 \rightarrow x=\pm 1$

solution guaranteed on open interval

C 12. $h'(x)=f'(x)g(x)+g'(x)f(x)$

$h'(3)=f'(3)g(3)+g'(3)f(3)$

$=-\frac{1}{3}(3)+1(1)=0$

B 13. $x+y=xy$

$1+\frac{dy}{dx}=1y+x\frac{dy}{dx}$

$\frac{dy}{dx}-x\frac{dy}{dx}=y-1$

$\frac{dy}{dx}(1-x)=y-1$

$\frac{dy}{dx}=\frac{y-1}{1-x}=\frac{1-y}{x-1}$

C 14. $f(g(x))\Big]_2^4=\int_2^4 f'(g(x))g'(x)dx$

A 15. $\int 8-2t\,dt=-t^2+8t+C=s(t)$

$v(t)=0=8-2t$

$t=4$

changes direction at origin

$s(4)=-4^2+8(4)+C=0$

$16+C=0$

$C=-16$

$s(t)=-t^2+8t-16$

B 16. $u^2+1=x$

$u=\sqrt{x-1}$

$\frac{du}{dx}=\frac{1}{2\sqrt{x-1}}$

$dx=2\sqrt{x-1}du$

$\int_2^5 \frac{\sqrt{x-1}}{x}dx=\int^2 \frac{u}{x}\cdot 2\sqrt{x-1}du$

$=\int_1^2 \frac{2u^2}{u^2+1}du$

D 17. $f'(x)=3x^2+12x+9=0$

$3(x^2+4x+3)=0$

$3(x+1)(x+3)=0$

$f'(x)$: $+$ | $-$ | $+$

$f(x)$: $\nearrow$ -3 $\searrow$ -1 $\nearrow$

$(-\infty,-3]\cup[-1,\infty)$

D 18. $\int_0^2 2x^3-kx^2+2k\,dx=12$

$\frac{1}{2}x^4-\frac{k}{3}x+2kx\Big]_0^2=12$

$8-\frac{8k}{3}+4k=12$

$-\frac{8k}{3}+4k=4$

$-8k+12k=12$

$4k=12$

$k=3$

D 19. $\int\frac{1}{y}dy=\int x^{-2}dx$

$\ln|y|=-\frac{1}{x}+C$

$|y|=e^{-\frac{1}{x}+C}$

$y=\pm Ce^{-\frac{1}{x}}$

$y=Ce^{-\frac{1}{x}}$

$1=Ce^{-1}$

$1=\frac{C}{e}$

$e=C$

$y=e\cdot e^{-\frac{1}{1}}=e^2$

B 20. $y = \frac{1}{2}\ln(1-x^2)$

$y' = \frac{1}{2} \cdot \frac{1}{1-x^2} \cdot -2x$

$y' = \frac{-x}{1-x^2} = \frac{x}{x^2-1}$

C 21. $s(t)$

$s'(t) = v(t)$

$\underbrace{s''(t) = a(t) < 0}$

concave down

B 22. $y' = 3x^2 - 12x = 0$

$y'' = 6x - 12 = 0$

$x = 2$

y'' — | + (at 2)

$\frown$ 2 $\smile$

C 23. $\frac{ds}{dt} = .05$ cm/sec

$V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$= 3s^2(.05)$

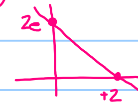
$= .15s^2$

A 24. $y' = e^{2-x}(-1) = -e^{2-x}$

$y'|_{(1,e)} = -e^1 = -e$

$y - e = -e(x-1)$

$y = -ex + 2e$



$A = \frac{1}{2}(2e)(2) = 2e$

C 25. $g'(x) = f(x)$

$g''(x) = f'(x)$

P.O.I. when $g''(x) = f'(x)$

changes sign or

$\underbrace{f(x) \text{ changes direction}}$

extrema

D 26. $y = \frac{1}{3}(x^2-1)^{-2/3} \cdot 2x = \frac{2x}{3(x^2-1)^{2/3}}$

$y'|_{x=3} = \frac{6}{3(8)^{2/3}} = \frac{6}{3 \cdot 4} = \frac{1}{2}$

$(3, 2)$

$m_T = \frac{1}{2}$

$m_N = -2$

$y - 2 = -2(x-3)$

$y = -2x + 8$

$y + 2x = 8$

C 27. $v(t) > 0$ Inc.

$a(t) > 0$ up.

C 28. $g(-3) = \int_0^{-3} f(t)dt = -\int_{-3}^{0} f(t)dt = -\frac{1}{2}(3)(1+3) = -6$

$g(1) = \int_0^1 f(t)dt = \frac{1}{2}(1)(1+2) = \frac{3}{2}$

$g(3) = \int_0^3 f(t)dt = \frac{1}{2}(1)(1+2) + \frac{1}{2}(2)(2) = \frac{3}{2} + 2 = \frac{7}{2}$

$g(5) = \int_0^3 f(t)dt + \int_3^5 f(t)dt = \frac{7}{2} - \frac{1}{2}(2)(2) = \frac{7}{2} - 2 = \frac{3}{2}$

OR

$g'(x) = f(x)$ changes from positive to negative

$x = 3$

A 29. $\frac{dy}{dx} = x^2y^2$

$\frac{d^2y}{dx^2} = 2xy^2 + x^2 \cdot 2y\frac{dy}{dx}$

$= 2xy^2 + 2x^2y(x^2y^2)$

$= 2xy^2 + 2x^4y^3$

A 30. $\frac{d}{dx}\int_x^{x^3} \sin(t^2)dt =$

$\frac{d}{dx}\left(-\int_0^x \sin(t^2)dt + \int_0^{x^3} \sin(t^2)dt\right) =$

$-\sin(x^2) + \sin(x^6) \cdot 3x^2 =$

$3x^2\sin(x^6) - \sin(x^2)$

B 31. $f''(2) > 0$ $f'(x)$ increasing

$f'(2) = 0$ & $f'(x) < 0$ to $f'(x) > 0$ $x = 2$ min

$f(2) < f(1) = 0$ $f(x)$ dec $(0, 2)$

$f(2) < f'(2) < f''(2)$

D 32. $f'(x)$ is increasing $\rightarrow$ $f(x)$ concave up only

& no P.O.I

$f'(0) = 0$ & $f'(x) < 0$ to $f'(x) > 0 \Rightarrow x = 0$ min.

A 33. $\int_0^3 m(t)dt = 10.667$

A 34. True $f'(x) < 0 \rightarrow f(x)$ decreasing

True $\lim_{x\to\infty} f'(x) = 0 \rightarrow f(x)$ approaches $y = -3$

& flattens out

False $\lim_{x\to-\infty} f'(x) = 3 \rightarrow f(x)$ approaches $y = 3$

& flattens out $\rightarrow f'(x) = 0 \neq 3$

B 35. $g'(x) = f(x)$

$g''(x) = f'(x)$

concavity of $g(x) \rightarrow$ down, up, down

$f'(x) \rightarrow$ neg, pos, neg

C 36. $\int_{-8}^{8} 4\sqrt{64-x^2}\,dx \approx 402.124$

D 37. $f'(x)=0 \Rightarrow x=.618$ & $f'(x)<0$ to $f'(x)>0$

$\int_{.618}^{5} f'(x)\,dx = f(5)-f(.618) < 0$

$f(5) < f(.618)$

D 38. $f'(4) = \dfrac{1.16016 - 1.15548}{4.001 - 3.999}$

$= 2.34$

B 39. $\Delta x = \frac{1}{n} = \frac{2-1}{n}$

$\lim_{n\to\infty} \underbrace{\frac{1}{n}}_{\text{width}} \left[\frac{1}{\overbrace{1+\frac{1}{n}}^{\text{height}}} + \frac{1}{1+2(\frac{1}{n})} + \dots \right]$

start point, Δx

$\int_{1}^{2} \frac{1}{x}\,dx$

D 40. largest area enclosed by graphs

C 41. $f(x) = \tan(3^x) = 0$

$x = 1.042$

$f'(1.042) = 3.451$

C 42. $2(.25)+2(.68)+2(.95) = 3.76$

D 44. $2\sin x = x$

$x = 1.89549$

$\pi \int_{0}^{1.89549} (2\sin x)^2 - x^2\,dx = 6.678$

B 45. $y' = e^x + xe^x$

$y'' = e^x + e^x + xe^x = 2e^x + xe^x$

$y''' = e^x + e^x + e^x + xe^x = 3e^x + xe^x$

$y^{(n)} = ne^x + xe^x = (n+x)e^x$

C 43. I. $f'(x)$ changes sign → $f(x)$ changes direction → false

II. $f'(x)$ changes from neg to positive → minimum → true

III. $f'(x)$ changes direction 3 times → $f(x)$ has 3 P.O.I → true

OE

1a) $T(x) = 70$

$x = 8.575 \approx 8.5$ 8:30am

b) $T(x) = 77$

$x = 10.507 \approx 10.5$ 10:30am

c) $C(8.5) = .16\int_{8.5}^{18} T(x) - 70\,dx = \$18.26$

d) $C(10.5) = .16\int_{10.5}^{18} T(x) - 77\,dx = \$8.79$

$\$18.26 - \$8.79 = \$9.47$

2a) $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\int_{\pi/6}^{5\pi/6} \sin(x) - \frac{1}{2}\,dx = .684$ or $.685$

b) $\pi \int_{\pi/6}^{5\pi/6} \sin^2 x - (\frac{1}{2})^2\,dx = 3.005$

c) $\frac{\pi}{2} \int_{\pi/6}^{5\pi/6} \left(\frac{\sin x - \frac{1}{2}}{2}\right)^2 dx = .107$ or $.106$

$r = \frac{\sin x - \frac{1}{2}}{2}$ $\sin x - \frac{1}{2} = d$

3a) $\underbrace{(5,6) \quad h'(5) = .7}$

$y - 6 = .7(x-5)$

$L(x) = .7(x-5)+6$

$L(4) = .7(4-5)+6 = 5.3$

$h''(t) > 0$ → concave up → $L(4)$ underapproximation

b) $V = 10(20)h$

$\frac{dV}{dt} = 200\frac{dh}{dt}$

$\left.\frac{dV}{dt}\right|_{t=2} = 200(.5)$

$= 100\ \text{in}^3/\text{min}$

c) $\int_{0}^{15} h'(t)\,dt = 2(.4)+3(.5)+4(.7)+4(1)+2(1.1) =$

11.3 in

From $t=0$ to $t=15$, the depth changed by 11.3 in.

d) $h''(t) > 0$ → $h'(t)$ increasing → underapproximation

4a) $y' = \frac{1(x+c) - x(1)}{(x+c)^2} = \frac{c}{(x+c)^2}$ ✓

$\frac{dy}{dx} = \frac{y - y^2}{x}$

$= \frac{\frac{x}{x+c} - \left(\frac{x}{x+c}\right)^2}{x}$

$= \frac{x(x+c) - x^2}{x(x+c)^2}$

$= \frac{cx}{x(x+c)^2}$

$\frac{dy}{dx} = \frac{c}{(x+c)^2}$ ✓

$y\big|_{(0,0)} = \frac{0}{0+c} = 0$ ✓

b) $y = \frac{x}{x+c}$ at $(1,2)$ $\quad 2 = \frac{1}{1+c}$

$2+2c = 1$

$2c = -1$

$c = -\frac{1}{2}$

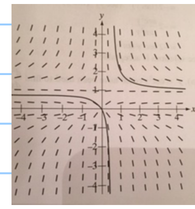
$y = \frac{x}{x-\frac{1}{2}} = \frac{2x}{2x-1}$

$y' = \frac{c}{(x+c)^2}$

$y'\big|_{(0,0)} = \frac{-\frac{1}{2}}{(0-\frac{1}{2})^2} = -2$

c) $y = \frac{2x}{2x-1}$ $\quad$ H.A. $y=1$ $\quad$ V.A. $x = \frac{1}{2}$

d)



5a) $\sec^2 y \frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\frac{dy}{dx}(\sec^2 y - 1) = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y - 1} = \frac{1}{\tan^2 y} = \cot^2 y$

b) $\frac{dy}{dx} = \frac{1}{\tan^2 y} = \text{und} \rightarrow \tan^2 y = 0$

$y = \pm\pi, 0$

$\tan y = x + y$

$\tan 0 = x + 0 \qquad \tan \pi = x + \pi \qquad \tan(-\pi) = x - \pi$

$0 = x \qquad 0 = x + \pi \qquad 0 = x - \pi$

$(0,0) \qquad (\pi, -\pi) \qquad (\pi, -\pi)$

c) $\frac{dy}{dx} = (\cot y)^2$

$\frac{d^2y}{dx^2} = 2\cot y\,(-\csc^2 y)\frac{dy}{dx}$

$= -2\csc^2 y \cot y\,(\cot^2 y) = -2\csc^2 y \cot^3 y$

6a) $f'(3) = 0$ and $f'(x)$ changes from positive to negative $\Rightarrow x = 3$ is a rel. maximum

b) $f'(-3) = 0$ and $f'(x)$ changes from negative to positive $\Rightarrow x = -3$ is a rel. minimum

c) f concave down if $f'(x)$ decreasing $\Rightarrow (-2,0) \cup (2,4)$

d) $f'(x)$ is even $\Rightarrow$ $\underbrace{f(x) \text{ is odd} \quad f(0) = 0}_{\text{symmetric to origin}}$ $\qquad \int_{-a}^{a} f(x)\,dx = 0$