AP Practice Exam IV

A 1. $\lim\limits_{x \to b} \frac{b-x}{\sqrt{x}-\sqrt{b}} = \lim\limits_{x \to b} \frac{-1}{\frac{1}{2\sqrt{x}}} = \lim\limits_{x \to b} -2\sqrt{x} = -2\sqrt{b}$

$\frac{"0"}{0}$

A 2. $y' = -\sin x + \sec^2(2x)2$

$y'|_{x=0} = -\sin 0 + 2\sec^2(0) = 0 + 2(1)$
$= 2$

C 3. $g'(x) = F(x)$

$g''(x) = f'(x)$ changes sign at $x=2$ & $x=6$

$y - 1 = 2(x-0) \to y = 2x+1$

D 4. corner at $x=2 \to f'(2)$ undefined

D 5. $u = 3x+5$
$\frac{du}{dx} = 3$
$dx = \frac{du}{3}$

$\int u^2 \frac{du}{3} = \frac{1}{3}\left[\frac{1}{3}u^3\right] + c$

$= \frac{1}{9}(3x+5)^3 + c$

B 6. $avg \; V = \frac{\ln(e) - \ln(1)}{e-1} = \frac{1-0}{e-1} = \frac{1}{e-1}$

A 7. $e^x + e^y \frac{dy}{dx} = 2 + 2\frac{dy}{dx}$

$\frac{dy}{dx}(e^y - 2) = 2 - e^x$

$\frac{dy}{dx} = \frac{2-e^x}{e^y - 2}$

$\frac{dy}{dx}\Big|_{(0,0)} = \frac{2-e^0}{e^0-2} = \frac{2-1}{1-2} = -1$

A 8. $f'(x) = 3\ln x + 3x\left(\frac{1}{x}\right)$

$= 3\ln x + 3$

$= \ln x^3 + 3$

C 9. $\int_{-3}^{-2} -x-2\,dx + \int_{-2}^{3} x+2\,dx =$

$-\frac{1}{2}x^2 - 2x\Big]_{-3}^{-2} + \left[\frac{1}{2}x^2 + 2x\right]_{-2}^{3} =$

$-\frac{1}{2}(4) + 4 + \frac{1}{2}(9) - 6 + \frac{9}{2} + 6 - \frac{1}{2}(4) + 4 =$

$-2 + 4 + \frac{9}{2} - 6 + \frac{9}{2} + 6 - 2 + 4 =$

$4 + \frac{18}{2} = 4 + 9 = 13$

B 10. $x^2 - 2hx = 0$
$x(x-2h) = 0$
$x = 0 \quad x = 2h$

$\int_0^{2h} 0 - x^2 + 2hx\,dx = 36$

$-\frac{1}{3}x^3 + hx^2\Big]_0^{2h} = 36$

$-\frac{8h^3}{3} + 4h^3 = 36$

$-8h^3 + 12h^3 = 72$
$4h^3 = 108$
$h^3 = 27$

$h = 3$

D 11. $f'(x) = e^x - 2 = 0$
$e^x = 2$
$\ln 2 = x$
$f(\ln 2) = e^{\ln 2} - 2\ln 2$

$= 2 - \ln 4$

$f''(x) = e^x$

$f'(\ln 2) = 0 \Big\}$ $x = \ln 2$ yields a minimum
$f''(\ln 2) > 0 \Big\}$

C 12. $MVT \to f'(c) = \frac{f(7) - f(-3)}{7 - (-3)}$

$f'(c) = \frac{2-4}{10}$

$f'(c) = \frac{-2}{10}$

$f'(c) = -\frac{1}{5}$

A 13. $\int_2^4 6t + 2\,dt = 3t^2 + 2t\Big]_2^4 =$

$3(4)^2 + 2(4) - 3(2)^2 - 2(2) =$

$48 + 8 - 12 - 4 =$
$56 - 16 =$
40

B 14. $g'(x) = -(f(x))^{-2} f'(x)$

$= \frac{-f'(x)}{(f(x))^2}$

$g'(2) = \frac{-f'(2)}{(f(2))^2} = \frac{4}{64} = \frac{1}{16}$

D 15. $\int_1^{15} f(t)\,dt = g(15) - g(1) =$

$2(2) + 3(3) + 4(4) + 5(2) =$

$4 + 9 + 16 + 10 = 39$

A 16. $\frac{dV}{dt} = 12 \; ft^3/sec$ Find $\frac{dA}{dt}$ when $V = 36\pi \; ft^3 = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$12 = 4\pi(3)^2 = \frac{dr}{dt}$

$\frac{1}{3\pi} = \frac{dr}{dt}$

$27 = r^3$
$r = 3$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi(3)\left(\frac{1}{3\pi}\right)$

$= 8 \; ft^2/sec$

D 17. $\int_{-2}^{2} 15 - g(x) - g(x) dx =$

$\int_{-2}^{2} 15 - 2g(x) dx =$

$15(4) - 2 \int_{-2}^{2} g(x) dx =$
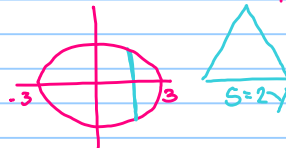
$60 - 2 \int_{-2}^{2} g(x) dx$

C 18. $h'(x) = f'(g(x)) g'(x)$

$h'(2) = f'(g(2)) g'(2)$

$= f'(1) g'(2)$

$= 5 (3) = 15$

$f'(x) = 2x + 3$

$f'(1) = 5$

C 19. $\frac{d}{dx}(C) = $ dollars/item   T

$\frac{d}{dx}(C)$ T
← change cost

$\frac{d}{dx}(C)$ F
← cost not amount

A 20. f is differentiable → $f'(c) = 0$

$f''(c)$ may be undefined (ex. $y = x^{1/3}$)

$f''(c)$ may be 0 (ex. $y = x^4$)

D 21. $x^2 + y^2 = 9$

$A = \frac{\sqrt{3}}{4} s^2 = \frac{\sqrt{3}}{4}(4y^2)$
$= \sqrt{3}(9 - x^2)$

-3 ... 3   s = 2y

$\int_{-3}^{3} \sqrt{3}(9 - x^2) dx =$

$\sqrt{3}[9x - \frac{1}{3}x^3]_{-3}^{3} =$

$\sqrt{3}(27 - 9 + 27 - 9) =$

$36\sqrt{3}$

B 22. $\int_{2}^{x} f(x) dx = y(x) - y(2)$

$\int_{2}^{x} f(x) dx = y(x) - 4$

$4 + \int_{2}^{x} f(x) dx = y(x)$

A 23. $y' = \frac{1}{\sqrt{1 - (\frac{3x}{4})^2}} \cdot \frac{3}{4}$

$= \frac{1}{\sqrt{\frac{16 - 9x^2}{16}}} \cdot \frac{3}{4}$

$= \frac{1}{\frac{\sqrt{16 - 9x^2}}{4}} \cdot \frac{3}{4}$

$= \frac{3}{\sqrt{16 - 9x^2}}$

B 24. $\lim_{x \to \infty} \frac{ae^x + b}{e^x + 1} = 3$

$\frac{\infty}{\infty}$ $\lim_{x \to \infty} \frac{ae^x}{e^x} = 3$

a = 3

$\lim_{x \to -\infty} \frac{ae^x + b}{e^x + 1} = -5$

b = -5

a + b = 3 - 5 = -2

A 25. $\frac{dy}{dx} = \frac{2y}{x}$

$\int \frac{1}{y} dy = \int \frac{2}{x} dx$

$\ln|y| = 2\ln|x| + C$

$\ln|y| = \ln x^2 + C$

$y = \pm e^{\ln x^2 + C}$

$y = \pm Ce^{\ln x^2}$

$y = Cx^2$

C 26. $f'(x) = \lim_{h \to 0} \frac{\tan(2x + 2h) - \tan(2x)}{h}$

$f(x) = \tan(2x)$

$f'(x) = 2\sec^2(2x)$

D 29. $g'(x) = 4 + \frac{f'(x)x - f(x)}{x^2}$

$g'(3) = 4 + \frac{f'(3) \cdot 3 - f(3)}{9}$

$= 4 + \frac{4(3) + 6}{9}$

$= 6$

$g(3) = 4(3) + \frac{f(3)}{3} = 12 + \frac{-6}{3}$
$= 10$

$y - 10 = 6(x - 3)$

B 27. $\sin(\frac{x}{2}) = 0$

$\frac{x}{2} = 0$ $\frac{x}{2} = \pi$
$x = 0$ $x = 2\pi$

$\int_{0}^{2\pi} \sin(\frac{x}{2}) dx = -2\cos(\frac{x}{2})]_{0}^{2\pi} =$

$= -2\cos(\pi) + 2\cos 0$
$= -2(-1) + 2\cos(1)$
$= 4$

C 28. $f'(x) = \frac{1}{2}x + 1$

$f(x) = \frac{1}{4}x^2 + x + C$

$f(2) = \frac{1}{4}(2)^2 + 2 + C = -3$

$3 + C = -3$
$C = -6$

$f(x) = \frac{1}{4}x^2 + x - 6$

$f(4) = \frac{1}{4}(4)^2 + 4 - 6$
$= 4 + 4 - 6$
$= 2$

C 30 $y = -x + 1$

$\pi \int_{0}^{1} (-x + 1 + 1)^2 - (1)^2 dx =$

$\pi \int_{0}^{1} (-x + 2)^2 - 1 dx =$

$\pi \int_{0}^{1} x^2 - 4x + 4 - 1 dx =$

$\pi \int_{0}^{1} x^2 - 4x + 3 dx = \pi [\frac{1}{3}x^3 - 2x^2 + 3x]_{0}^{1} =$

$\pi(\frac{1}{3} - 2 + 3) = \pi(\frac{1}{3} + \frac{3}{3}) = \frac{4\pi}{3}$

$$\int_0^1 F(x)\,dx < \int_0^2 F(x)\,dx$$

B
31. $\int_0^2 f(x)\,dx > 0$

$\int_0^4 f(x)\,dx < 0$

$\int_0^5 f(x)\,dx < 0$

* compare signed areas

C 32. $x'(t) = v(t) = 4t^3 - 30t^2 + 58t - 36$

$|v(2)| = 8$

$|v(3)| = 24$

$|v(4)| = 28$

$|v(5)| = 4$

$speed = |v(t)|$

D 33.
$f'(x) > 0 \rightarrow F(x)$ Inc

$f'(x)$ Inc on $(-\infty, 0) \rightarrow$
$\quad F(x)$ c. up

$f'(x)$ Dec on $(0, \infty) \rightarrow$
$\quad F(x)$ c. down

As $x \rightarrow \pm\infty$, $f'(x) \rightarrow 0 \rightarrow$
$\quad F(x)$ approaches
$\quad$ a H.A.

C 34. $f'(t) = e^{t\cos t}(\cos(t) - t\sin(t))$
A maximum occurs when
$f'(t)$ changes from positive
to negative or at an endpoint.

$f(0) = 0$
$f(.860) = .753$
* $f(6.437) = 577.827$
$f(10) = -1$

A 35.
$2 = 2e^{2t}$
$e^{2t} = 1$
$\ln 1 = 2t$
$t = 0$

$4 = 2e^{2t}$
$2 = e^{2t}$
$\ln 2 = 2t$
$t = \frac{1}{2}\ln 2$

$\int_0^{\frac{1}{2}\ln 2} 2e^{2t}\,dt = 1$

B 36. $\Delta x = \frac{b-a}{n} = \frac{1}{n}$ ∴ $b - a = 1$

$\lim_{n \to \infty} \frac{1}{n}\left(\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \ldots + \left(\frac{n-1}{n}\right)^2\right)$
$\quad\quad\quad\quad$ width $\quad$ height

$\left(\frac{1}{n}\right)^2 = (\Delta x)^2$
$\quad\quad\quad\quad\quad$ input

$\int_0^1 x^2\,dx$

B 37. $y'(t) = \frac{3}{4}\cos(3t) + \frac{3}{6}\sin(3t)$

$y''(t) = \frac{-9}{4}\sin(3t) + \frac{3}{2}\cos(3t)$

$y''(t)$ changes from positive (above x-axis) to

negative (below x-axis) 4 times on $[0,8]$

B 38. Area of Square = 4

Below: $\int_0^2 -x^2 + 2x\,dx = \frac{4}{3}$

Above: $4 - \frac{4}{3} = \frac{8}{3}$

Prob $= \frac{\frac{8}{3}}{4} = \frac{2}{3}$

D
42. g c. down to g. c up
∴ $g'(x)$ dec to $g'(x)$ Inc

$\quad\quad$ minimum of $g'(x)$

C 43. $2y\frac{dy}{dx} - 3x^2 - 30x = 0$

$\frac{dy}{dx} = \frac{3x^2 + 30x}{2y} = 0$

$3x^2 + 30x = 0$
$3x(x + 10) = 0$
$x = 0 \quad x = -10$

C 39. $f'(x) = \sin^{-1}(x)$
$f'(.4) = \sin^{-1}(.4) \approx .412$

D 41. $y = \sin x + \cos x \rightarrow \frac{dy}{dx} = \cos x - \sin x$
* Substitute + check *

I. $\underbrace{\sin x + \cos x}_{y} + \underbrace{\cos x - \sin x}_{y'} \neq 2\sin x$

✓ II. $\underbrace{\sin x + \cos x}_{y} + \underbrace{\cos x - \sin x}_{y'} = 2\cos x$

✓ III. $\cos x - \sin x - \sin x - \cos x = -2\sin x$

D 44. Since $g(x)$ is odd the signed
areas on $[-a, 0]$ and
$[0, a]$ are opposite
in value so a-c
are true.

B 40. $f'(6) > 0$ b/c $F(x)$ Inc on $(-\infty, 8)$

* $f'(4) < 0$ b/c $F(x)$ is concave
$\quad\quad$ down on $(-\infty, 10)$

$f''(10) = 0$ b/c $x = 10$ is a P.O.I

$f''(12) > 0$ b/c $f(x)$ is concave
$\quad\quad$ up on $(10, \infty)$

C 45. $avg = \frac{1}{a-(-a)}\int_{-a}^a F(x)\,dx = 0$

∴ $\int_{-a}^a F(x)\,dx = 0$

∴ $F(x)$ is odd

1. a) $s(0)=0$

$s(t)=\int v(t)\,dt = -\cos(t)-e^t+c$

$s(0)=-\cos(0)-e^0+c=0$

$-1-1+c=0$

$c=2$

$s(t)=-\cos(t)-e^t+2$

b) $v(t)=\sin(t)+e^t=0$

$t\approx 3.183$

c) $\dfrac{1}{5-0}\int_0^5 s(t)\,dt \approx 1.993$

d) $\int_0^5 |v(t)|\,dt \approx 4.206$

2. a) At P, $d=40$.

$10,000=\dfrac{k}{40^2}$

$k=16,000,000=1.6\times 10^7$

b) $\cos\theta=\dfrac{40}{d} \rightarrow d=\dfrac{40}{\cos\theta}$

$L=\dfrac{k}{d^2}=\dfrac{k}{\left(\frac{40}{\cos\theta}\right)^2}=\dfrac{k}{\frac{1600}{\cos^2\theta}}=\dfrac{k\cos^2\theta}{1600}$

$L=\dfrac{(1.6)(10^7)\cos^2\theta}{(1.6)(10^3)}$

$L=10^4\cos^2\theta$

c) $\dfrac{dL}{dt}=2(10^4)\cos\theta(\sin\theta)\dfrac{d\theta}{dt}$

$=2(10^4)\left(\frac{1}{\sqrt2}\right)\left(\frac{-1}{\sqrt2}\right)\left(\frac{\pi}{30}\right)$

$=-1,047.198 \quad \text{Lumens/sec}$

The strength is decreasing at 1,047.198 lumens/sec

3. a) $\int_1^5 1-\frac{1}{x}\,dx = x-\ln|x|\big]_1^5$

$=5-\ln 5-1+\ln 1$

$=4-\ln 5$

b) $\pi\int_1^5 \left(2-\frac{1}{x}\right)^2-(2-1)^2\,dx$

c) $\int_1^5 2^x-2\,dx=$

$\left[\dfrac{1}{\ln 2}2^x-2x\right]_1^5 =$

$\dfrac{32}{\ln 2}-10-\dfrac{1}{\ln 2}\cdot 2+2=$

$\dfrac{30}{\ln 2}-8$

4. a) $\dfrac{dh}{dt}=-kh^{1/2}$ $\qquad h(0)=16$

$\int h^{-1/2}\,dh=\int -k\,dt$

$2h^{1/2}=-kt+c \rightarrow 2\sqrt{h}=-kt+8$

$\qquad\qquad\qquad\qquad \sqrt{h}=\frac{-k}{2}t+4$

$2\sqrt{16}=0+c$

$8=c \qquad\qquad h(t)=\left(\frac{-k}{2}t+4\right)^2$

b) $h(8)=12.25 \quad h(8)=\left(\frac{-8k}{2}+4\right)^2=12.25$

$\qquad\qquad\qquad\qquad -4k+4=3.5$

$\qquad\qquad\qquad\qquad -4k=\frac{-1}{2}$

$h(t)=\left(\frac{-1}{16}t+4\right)^2 \qquad k=\frac{1}{8}$

c) $h(t)=\left(\frac{-1}{16}t+4\right)^2=0$

$\frac{-1}{16}t+4=0$

$4=\frac{1}{16}t$

$t=64 \text{ hours}$

5. a) $m_c=\dfrac{15-(-15)}{-3-3}=\dfrac{-5}{}=-5$

$\dfrac{dy}{dx}=4-3x^2=-5$

$\qquad -3x^2=-9$

$\qquad x^2=3$

$\qquad x=\pm\sqrt{3}$

$(\sqrt3,\sqrt3) \quad (-\sqrt3,-\sqrt3)$

b) Chord: $y-15=-5(x+3)$

$\qquad\qquad y=-5x$

$D(x)=(4x-x^3)-(-5x)=-x^3+9x$

c) $D'(x)=-3x^2+9=0$

$3x^2=9$

$x=\pm\sqrt3 \quad$ so $\ x=\sqrt3 \ $ on $[0,3]$

$D''(x)=-6x$

$\left.\begin{array}{l}D''(\sqrt3)=0 \\ D''(\sqrt3)<0\end{array}\right\} \ \begin{array}{l}X=\sqrt3 \text{ yields}\\ \text{a max value}\end{array}$

$D(\sqrt3)=-(\sqrt3)^3+9\sqrt3=-3\sqrt3+9\sqrt3=6\sqrt3$

$y=4x-x^3=x(4-x^2)$

6. a) $f(x)$ is Inc. on $(0,2)\cup(3,4)$ since $f'(x)>0$.

b) $f(x)$ has a rel. minimum value at $x=3$ since $f'(x)$ changes from negative to positive.

c) $f(x)$ has a rel. maximum value at $x=2$ since $f'(x)$ changes from positive to negative.

d.