Name Answer Key **Date**______________

Calculus I Honors – Precalculus Review **Period**______

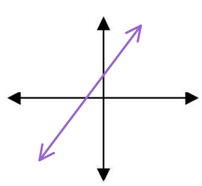
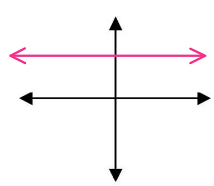
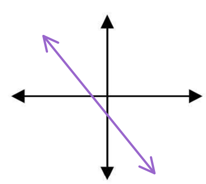
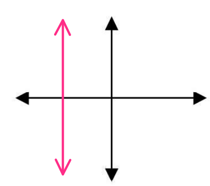
Section I: Linear Functions

Slope-Intercept Form: $y = mx + b$ Point-Slope Form: $y - y_1 = m(x - x_1)$

1. Write the equation of each line in ***point-slope form***:

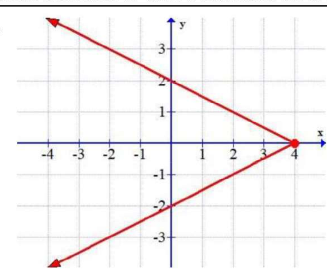
a. given a point and the slope: $(1,-2)$ $m=3$ $\boxed{y+2=3(x-1)}$	b. given two points: $(1,-3)$ $(-7,1)$ $m = \frac{1-(-3)}{-7-1} = \frac{4}{-8} = -\frac{1}{2}$ $\boxed{y+3=-\frac{1}{2}(x-1) \text{ OR } y-1=-\frac{1}{2}(x+7)}$
c. given the point $(-1,-2)$ and is perpendicular to the line $y-2=3(x+1)$. $m_\perp = -\frac{1}{3}$ $\boxed{y+2=-\frac{1}{3}(x+1)}$	d. given the point $(-1,-2)$ and is parallel to the line $3x+2y=1$. $\Rightarrow$ $2y=-3x+1$ $y=-\frac{3}{2}x+\frac{1}{2}$ $m_{\parallel} = -\frac{3}{2}$ $\boxed{y+2=-\frac{3}{2}(x+1)}$

2. Sketch a line with the given slope. Sample answers ONLY!

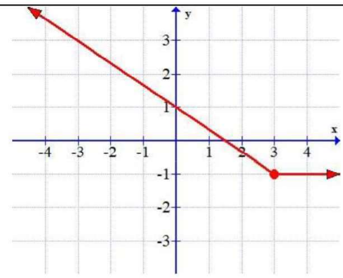
a. $m > 0$	b. $m = 0$	c. $m < 0$	d. *m undefined*
			

Section II: Functions and Relations

3. Determine the domain and range of each relation. Then decide if the relation is a function.

a. $\{(3,1),(2,-3),(-1,5),(-2,-2),(0,2)\}$ Domain $\{-2,-1,0,2,3\}$ Range $\{-3,-2,1,2,5\}$ Function	b.  D: $(-\infty, 4]$ R: $(-\infty,\infty)$ NOT a function (fails Vert Line Test)

c.

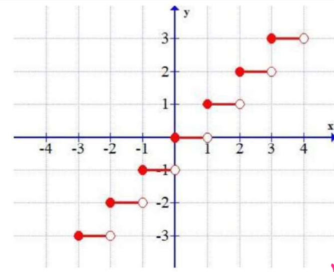


D: $(-\infty,\infty)$

R: $[-1,\infty)$

Function

d.



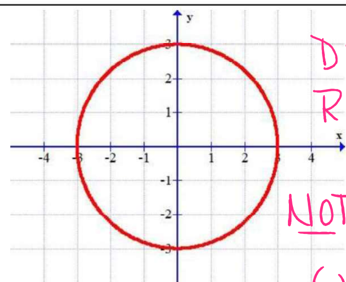
D: $(-\infty,\infty)$

R: $\{\mathbb{Z}\}$

Set of all integers

Function

e.



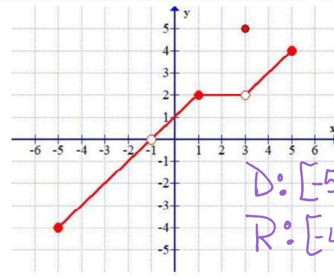
D: $[-3,3]$

R: $[-3,3]$

NOT a function

(VLT)

f.



D: $[-5,-1)\cup(-1,5]$

R: $[-4,0)\cup(0,4]\cup\{5\}$

Function

4. For $f(x)=x^2+7$, find

a. $f(-1) = (-1)^2+7 = \boxed{8}$

b. $f(-4c) = (-4c)^2+7$

$= \boxed{16c^2+7}$

c. $f(b-1) = (b-1)^2+7$

$= b^2-2b+1+7$

$= \boxed{b^2-2b+8}$

d. $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$; $\Delta x \neq 0$

$f(x+\Delta x) = (x+\Delta x)^2+7$

$= \dfrac{\cancel{x^2}+2x\Delta x+\Delta x^2\cancel{+7}-(\cancel{x^2}\cancel{+7})}{\Delta x}$

$= \dfrac{2x\Delta x+\Delta x^2}{\Delta x} = \dfrac{\cancel{\Delta x}(2x+\Delta x)}{\cancel{\Delta x}}$

$= \boxed{2x+\Delta x}$

5. Find domain for each of the following. Write your answer in interval notation.

a. $f(x)=\sqrt{x-1}$

$x-1\geq 0$

$x\geq 1$

$\boxed{D: [1,\infty)}$

b. $h(x)=\dfrac{1}{2x-1}$

$2x-1\neq 0$

$x\neq \frac{1}{2}$

$\boxed{D: (-\infty,\tfrac{1}{2})\cup(\tfrac{1}{2},\infty)}$

c. $g(x)=\sqrt{25-x^2}$

$25-x^2\geq 0$

$(5+x)(5-x)\geq 0$

Crit # $x=\pm 5$

$-$ | $+$ | $-$

-5 $\quad$ 5

$\boxed{D: [-5,5]}$

d. $j(x)=\dfrac{1}{\sqrt{1-2x}}$

$1-2x>0$

$1>2x$

$x<\frac{1}{2}$

$\boxed{D: (-\infty,\tfrac{1}{2})}$

e. $f(x)=\sqrt[5]{3x+5}$

odd index!

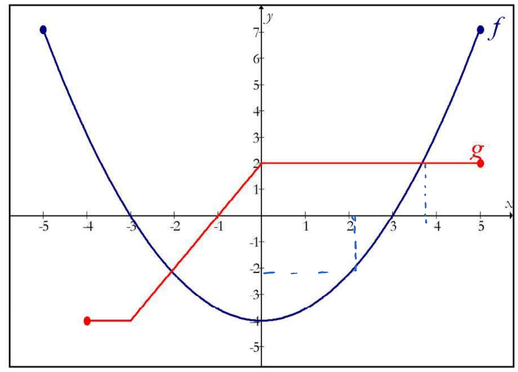
$\boxed{D: (-\infty,\infty)}$

f. $f(x)=\dfrac{4}{x^2+1}$

$x^2+1\geq 0$ for all x!

$\boxed{D: (-\infty,\infty)}$

6. Use the graph given to the right to answer the following questions:



a. Identify the domain and range of *f*. D: [-5, 5] R: [-4, 7]

b. Identify the domain and range of *g*. D: [-4, 5] R: [-4, 2]

c. Identify the value of $f(-3)$.

$f(-3)=0$

d. Identify the value of $g(0)$.

$g(0)=2$

e. Estimate the solution(s) of $f(x)=2$.

$f(x)=2$ when $x \approx \pm 3.75$

f. Estimate the solution(s) of $g(x)=0$.

$g(x)=0$ when $x=-1$

g. Estimate the x-coordinates for which $f(x)=g(x)$. $x \approx -2, 3.75$

h. Estimate the value of $f(g(1))$. $g(1)=2$ $f(2) \approx -2$

i. Estimate the value of $g(f(3))$. $f(3)=0$ $g(0)=2$

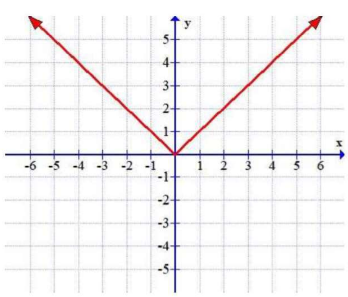
Section III: Parent Functions

7. Match each parent function with the correct graph. Identify all key features of each parent graph.

$f(x)=\frac{1}{x}$ $g(x)=\frac{1}{x^2}$ $h(x)=\sqrt{x}$ $i(x)=x^2$ $j(x)=[\![x]\!]$

$k(x)=\sqrt[3]{x}$ $l(x)=x^3$ $m(x)=\log_a x$ $n(x)=a^x$ $p(x)=|x|$

a.



Function: $p(x)=|x|$

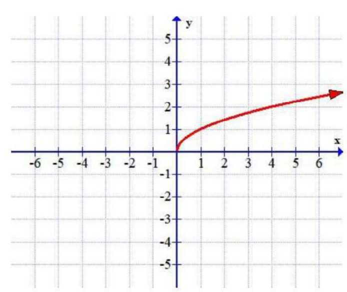
Domain: $(-\infty,\infty)$ Range: $[0,\infty)$

Extrema: Abs min at $(0,0)$

Increasing: $(0,\infty)$ Decreasing: $(-\infty,0)$

Asymptotes: None

b.



Function: $h(x)=\sqrt{x}$

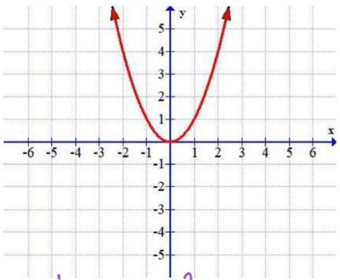
Domain: $[0,\infty)$ Range: $[0,\infty)$

Extrema: Abs min $(0,0)$

Increasing: $(0,\infty)$ Decreasing: None

Asymptotes: None

c.



Function: $i(x) = x^2$

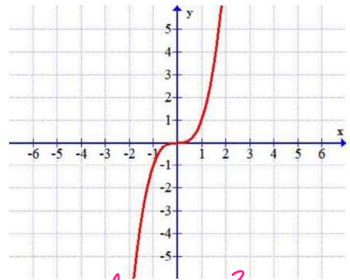
Domain: $(-\infty, \infty)$ Range: $[0, \infty)$

Extrema: Abs min $(0, 0)$

Increasing: $(0, \infty)$ Decreasing: $(-\infty, 0)$

Asymptotes: None

d.



Function: $l(x) = x^3$

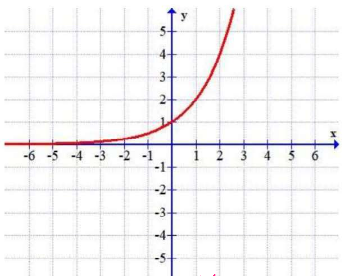
Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Extrema: None

Increasing: $(-\infty, \infty)$ Decreasing: None

Asymptotes: None

e.



Function: $n(x) = a^x$

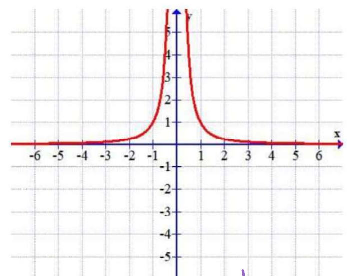
Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

Extrema: None

Increasing: $(-\infty, \infty)$ Decreasing: None

Asymptotes: HA: $y = 0$

f.



Function: $g(x) = \frac{1}{x^2}$

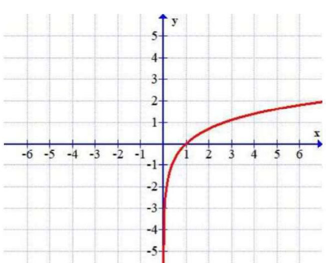
Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$

Extrema: None

Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$

Asymptotes: VA: $x = 0$, HA: $y = 0$

g.



Function: $m(x) = \log_a(x)$

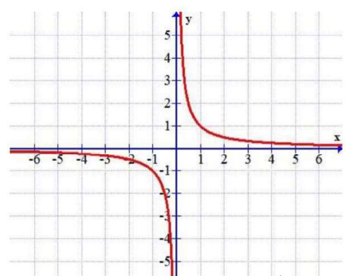
Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Extrema: None

Increasing: $(0, \infty)$ Decreasing: None

Asymptotes: VA: $x = 0$

h.



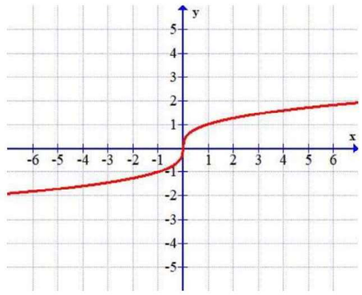
Function: $f(x) = \frac{1}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$

Extrema: None

Increasing: None Decreasing: $(-\infty, 0) \cup (0, \infty)$

Asymptotes: VA: $x = 0$, HA: $y = 0$

i. 

Function: $k(x) = \sqrt[3]{x}$

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Extrema: None

Increasing: $(-\infty, \infty)$ Decreasing: None

Asymptotes: None

Section IV: Simplifying Expressions

8. Perform the operation indicated and simplify each expression. Identify any domain restrictions.

a. $\frac{x^2+x}{x+2}\cdot\frac{4-x^2}{x^2+3x} = \frac{\cancel{x}(x+1)}{\cancel{x+2}}\cdot\frac{\cancel{(2+x)}(2-x)}{\cancel{x}(x+3)}$

$= \boxed{\frac{(x+1)(2-x)}{x+3}, \quad x \neq -3, -2, 0}$

b. $\frac{x^2-6x+9}{x^2-9} \div \frac{x-9}{x+3} = \frac{x^2-6x+9}{x^2-9}\cdot\frac{x+3}{x-9}$

$= \frac{(x-3)^{\cancel{2}}}{(\cancel{x+3})(\cancel{x-3})}\cdot\frac{\cancel{x+3}}{x-9}$

$= \boxed{\frac{x-3}{x-9} \quad x \neq \pm 3, 9}$

c. $\frac{4}{x^2-9} - \frac{2}{x^2-3x} = \frac{4}{(x+3)(x-3)}\cdot\frac{x}{x} - \frac{2}{x(x-3)}\cdot\frac{(x+3)}{(x+3)}$

LCD: $x(x+3)(x-3)$

$= \frac{4x-2x-6}{x(x+3)(x-3)} = \frac{2x-6}{x(x+3)(x-3)}$

$= \frac{2(\cancel{x-3})}{x(x+3)(\cancel{x-3})} = \boxed{\frac{2}{x(x+3)}, \quad x \neq 0, \pm 3}$

d. $x+2-\frac{4x}{x+2} = \frac{(x+2)(x+2)}{(x+2)} - \frac{4x}{x+2}$

$= \frac{x^2+\cancel{4x}+4-\cancel{4x}}{x+2}$

$= \boxed{\frac{x^2+4}{x+2}, \quad x \neq -2}$

e. $\frac{\frac{1}{x+1}-1}{x}\cdot\frac{(x+1)}{(x+1)}$ Hint: Multiply by LCD in num & denom

LCD: $x+1$

$= \frac{1-(x+1)}{x(x+1)} = \frac{\cancel{1}-x-\cancel{1}}{x(x+1)} = \frac{-\cancel{x}}{\cancel{x}(x+1)}$

$= \boxed{\frac{-1}{x+1}, \quad x \neq 0, -1}$

f. $\frac{\frac{1}{x-2}+\frac{1}{2}}{x}\cdot\frac{2(x-2)}{2(x-2)}$ LCD: $2(x-2)$

$= \frac{\cancel{2}+x-\cancel{2}}{2x(x-2)} = \frac{\cancel{x}}{2\cancel{x}(x-2)}$

$= \boxed{\frac{1}{2(x-2)}, \quad x \neq 0, 2}$

g. $\dfrac{\left(x-\frac{1}{x}\right)}{\left(\frac{1-x^2}{x}\right)} \cdot \dfrac{x}{x} = \dfrac{x^2-1}{1-x^2} = \boxed{-1,\ x\neq 0, \pm 1}$	

9. Given: $f(x)=x+3$ and $g(x)=\dfrac{x-5}{x^2-2x-15}$, a) perform the given operation and write the answer in simplified **form**, <u>and</u> b) find the **domain** of the combination of functions.

a. $f(x)-g(x)$ $x+3-\dfrac{\cancel{x-5}}{(x+3)\cancel{(x-5)}} = \dfrac{(x+3)^2}{(x+3)} - \dfrac{1}{(x+3)}$ $= \dfrac{x^2+6x+9-1}{x+3} = \dfrac{x^2+6x+8}{x+3}$ $\boxed{\text{or } \dfrac{(x+4)(x+2)}{x+3} \quad D: (-\infty,-3)\cup(-3,5)\cup(5,\infty)}$	b. $g(x)\cdot f(x)$ $\dfrac{\cancel{x-5}}{\cancel{(x+3)}\cancel{(x-5)}} \cdot \cancel{x+3} = \boxed{1}$ $\boxed{D: (-\infty,-3)\cup(-3,5)\cup(5,\infty)}$
c. $g(f(x)) = g(x+3)$ $= \dfrac{x+3-5}{(x+3)^2-2(x+3)-15}$ $= \dfrac{x-2}{x^2+6x+9-2x-6-15} = \dfrac{x-2}{x^2+4x-12}$ $= \dfrac{\cancel{x-2}}{(x+6)\cancel{(x-2)}} = \boxed{\dfrac{1}{x+6}} \quad x\neq -6, 2$ $\boxed{D: (-\infty,-6)\cup(-6,2)\cup(2,\infty)}$	d. $\dfrac{f(x)}{g(x)} = \dfrac{x+3}{\frac{\cancel{x-5}}{(x+3)\cancel{(x-5)}}}$ $= (x+3)^2$ $\boxed{D: (-\infty,-3)\cup(-3,5)\cup(5,\infty)}$

10. Rationalize each function and simplify.

a. $\dfrac{x}{\sqrt{x+4}} \cdot \dfrac{\sqrt{x+4}}{\sqrt{x+4}}$ $= \boxed{\dfrac{x\sqrt{x+4}}{x+4}},\ x\neq -4$	b. $\dfrac{x}{\sqrt{x+4}-2} \cdot \dfrac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$ $= \dfrac{x(\sqrt{x+4}+2)}{x+\cancel{4}-\cancel{4}} = \dfrac{\cancel{x}(\sqrt{x+4}+2)}{\cancel{x}}$ $= \boxed{\sqrt{x+4}+2},\ x\neq 0$	c. $\dfrac{4-x}{\sqrt{x}-2} \cdot \dfrac{\sqrt{x}+2}{\sqrt{x}+2}$ $= \dfrac{\overset{-1}{\cancel{4-x}}(\sqrt{x}+2)}{\underset{1}{\cancel{x-4}}}$ $= \boxed{-\sqrt{x}-2},\ x\neq 4$

Section V: Exponential and Log Functions

11. Rewrite each function in rational form. Simplify if necessary.

a. $x^{2/3}$ $= \boxed{\sqrt[3]{x^2} = (\sqrt[3]{x})^2}$	b. $x^{3/2}$ $= \boxed{\sqrt{x^3} = (\sqrt{x})^3}$	c. $x^{-2/3} = \left(\frac{1}{x}\right)^{2/3}$ $= \boxed{\frac{1}{\sqrt[3]{x^2}} = \frac{1}{(\sqrt[3]{x})^2}}$	d. $\frac{3x^2}{x^{4/3}} = 3x^{2-4/3}$ $= 3x^{2/3} = \boxed{3\sqrt[3]{x^2} = 3(\sqrt[3]{x})^2}$

12. Rewrite each function in exponential form. Simplify if necessary.

a. $\sqrt[4]{x^5}$ $= \boxed{x^{5/4}}$	b. $\frac{2}{\sqrt{x^5}} = \frac{2}{x^{5/2}}$ $= \boxed{2x^{-5/2}}$	c. $\sqrt[3]{x^6} = x^{6/3}$ $= \boxed{x^2}$	d. $4x^3 \cdot \sqrt[2]{x^5}$ $= 4x^3 \cdot x^{5/2} = 4x^{3+\frac{5}{2}}$ $= \boxed{4x^{11/2}}$

13. Condense and write as a single logarithm.

a. $\log_4 3 + 5\log_4 x$ $= \log_4 3 + \log_4 x^5$ $= \boxed{\log_4(3x^5)}$	b. $\log 3 - 5\log x$ $= \log 3 - \log x^5$ $= \boxed{\log \frac{3}{x^5}}$
c. $\ln 2 + 4\ln x - 3\ln y - \ln 8$ $= \ln 2 + \ln x^4 - \ln y^3 - \ln 8$ $= \ln \frac{2x^4}{8y^3} = \boxed{\ln \frac{x^4}{4y^3}}$	d. $2\ln 4 - \frac{1}{2}\ln x + \ln y - 3\ln 2$ $= \ln 4^2 - \ln x^{1/2} + \ln y - \ln 2^3$ $= \ln \frac{16y}{8\sqrt{x}} = \boxed{\ln \frac{2y}{\sqrt{x}}}$

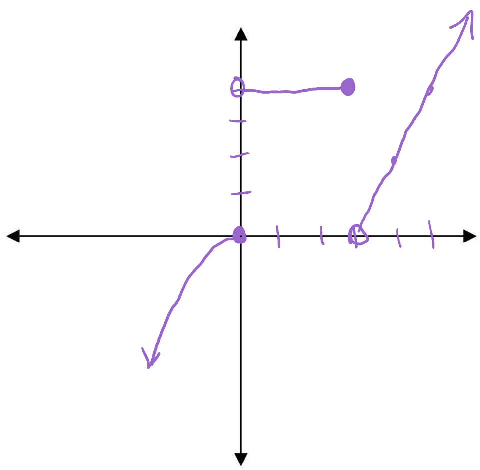
14. Expand each logarithmic expression.

a. $\log_7 \frac{5x}{y^4}$ $= \log_7 5 + \log_7 x - \log_7 y^4$ $= \boxed{\log_7 5 + \log_7 x - 4\log_7 y}$	b. $\log \frac{x^3}{9y^2}$ $= \log x^3 - \log 9 - \log y^2$ $= \boxed{3\log x - \log 9 - 2\log y}$
c. $\ln 27\sqrt[4]{a}$ $= \ln 27 + \ln a^{1/4}$ $= \boxed{\ln 27 + \frac{1}{4}\ln a}$	d. $\ln \frac{3x^2+1}{2x^3-3x^2}$ $= \ln(3x^2+1) - \ln(\underbrace{2x^3-3x^2}_{x^2(2x-3)})$ OR $\boxed{\ln(3x^2+1) - 2\ln x - \ln(2x-3)}$

Section VI: Piecewise Functions

15. Graph each function, then evaluate each at the given value.

a. $f(x)=\begin{cases} -x^2, & x \le 0 \\ 4, & 0 < x \le 3 \\ 2x-6, & x > 3 \end{cases}$



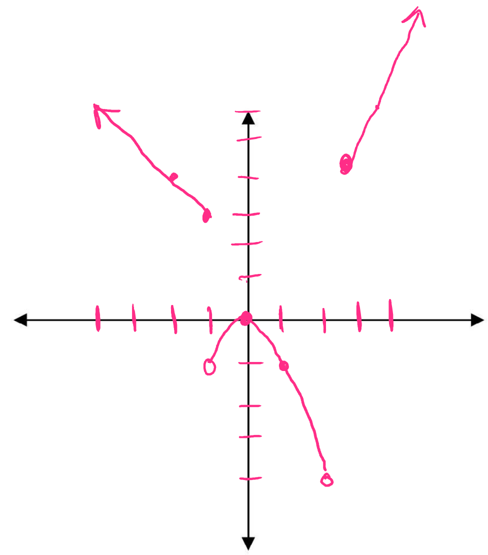
$f(-5) = -(-5)^2 = -25$

$f(3) = 4$

$f(1) = 4$

$f(10) = 2(10)-6 = 14$

b. $g(x)=\begin{cases} |x|+2, & x \le -1 \\ -x^2, & -1 < x < 2 \\ 2x, & x \ge 2 \end{cases}$



$g(-5) = |-5|+2 = 7$

$g(-1) = |-1|+2 = 3$

$g(0) = -(0)^2 = 0$

$g(5) = 2(5) = 10$

16. Evaluate each function at the given value.

a. $h(x)=\begin{cases} 2x+3, & x < 0 \\ x^2, & 0 < x < 2 \\ 1, & x \ge 2 \end{cases}$

$h(-1) = 2(-1)+3 = 1$

$h(0) =$ undefined

$h(1.5) = \left(\frac{3}{2}\right)^2 = \frac{9}{4} = 2.25$

$h(2) = 1$

b. $j(x)=\begin{cases} 3x+2, & x \le -2 \\ -1, & -2 < x \le 0 \\ \sqrt{x}, & x \ge 1 \end{cases}$

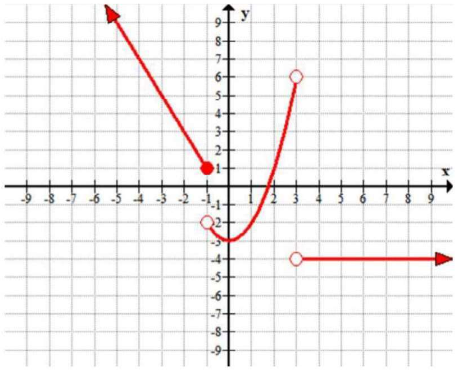
$j(-4) = 3(-4)+2 = -10$

$j(0) = -1$

$j\left(\frac{1}{2}\right) =$ Undefined

$j(9) = \sqrt{9} = 3$

17. Write the equation of the piecewise function graphed below.



$$f(x)=\begin{cases}-2x-1, & x\le -1\\ x^2-3, & -1<x<3\\ -4, & x>3\end{cases}$$

Section VII: Intercepts and intersection of graphs

18. For each function, find all *x*- and *y*-intercepts. Write as ordered pairs.

a. $f(x)=x^2-5x-66$ $=(x-11)(x+6)=0$ $x=11,-6$ X int: $(11,0),(-6,0)$ y int: $f(0)=-66$ $(0,-66)$	b. $g(x)=x^2\sqrt{25-x^2}=0$ $x=0$ $25-x^2=0$ $(5+x)(5-x)=0$ $x=\pm5$ x int: $(0,0),(-5,0),(5,0)$ y int: $g(0)=0$ $(0,0)$	c. $h(x)=\dfrac{9-x^2}{x^2-4x-21}=\dfrac{(3-x)\cancel{(3+x)}}{(x-7)\cancel{(x+3)}}$ Note: hole at $x=-3$ x int: $3-x=0$ $x=3$ $(3,0)$ y int: $h(0)=\frac{3}{-7}$ $(0,-3/7)$

19. Find the points of intersection of the graphs of each set of functions.

a. $x+y=4$ and $x^2+y=6$ $y=4-x$ $x^2+4-x=6$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2,-1$ $2+y=4$ $y=2$ $\boxed{(2,2)}$ $-1+y=4$ $y=5$ $\boxed{(-1,5)}$	b. $x^2+2x-y=8$ and $x+y=-8$ $y=-x-8$ $x^2+2x-(-x-8)=8$ $x^2+2x+x+8=8$ $x^2+3x=0$ $x(x+3)=0$ $x=0,-3$ $0+y=-8$ $y=-8$ $\boxed{(0,-8)}$ $-3+y=-8$ $y=-5$ $\boxed{(-3,-5)}$

Section VIII: Solving Equations

20. Solve each equation by factoring, completing the square or quadratic formula.

a. $49 - x^2 = 0$

$(7+x)(7-x)=0$

$\boxed{x=\pm 7}$

b. $x^2 - 5x - 36 = 0$

$(x-9)(x+4)=0$

$\boxed{x=9,\ -4}$

c. $2x^2 + x - 15 = 0$

$(2x-5)(x+3)=0$

$\boxed{x=5/2,\ -3}$

d. $4x^3 - 25x = 0$

$x(4x^2-25)=0$

$x(2x+5)(2x-5)=0$

$\boxed{x=0,\ \pm 5/2}$

e. $3x^2 - 13x = 10$

-30 ; -15, 2 ; -13

$3x^2 - 13x - 10 = 0$

$3x^2 - 15x + 2x - 10 = 0$

$3x(x-5) + 2(x-5) = 0$

$(3x+2)(x-5)=0$

$\boxed{x=-2/3,\ 5}$

OR

$x = \dfrac{13 \pm \sqrt{(-13)^2 - 4(3)(-10)}}{2(3)} = \dfrac{13 \pm \sqrt{289}}{6}$

$x = \dfrac{13+17}{6}$ or $\dfrac{13-17}{6} = \dfrac{-4}{6}$

$\boxed{x = 5,\ -2/3}$

f. $x^2 + 3x + 6 = 0$

NOT factorable

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(6)}}{2(1)}$

$= \dfrac{-3 \pm \sqrt{-15}}{2}$

No Real Solution!

No complex solutions in calc ☺ !