Name Answer Key ____________ Date______________

Calc I H – Quarterly 3 Review Period ____________

Topic: Analyzing a function

1. Determine the intervals where the following function is increasing, decreasing, concave up, and concave down. Then find all relative extrema and points of inflection.

$f(x) = 4x^3 - x^4$

$f'(x) = 12x^2 - 4x^3$

$f'(x) = 0$

$4x^2(3-x) = 0$

$x = 0, 3$

f': $+$ | $+$ | $-$; f: 0, 3

$f''(x) = 24x - 12x^2$

$f''(x) = 0$

$12x(2-x) = 0$

$x = 0, 2$

f'': $-$ | $+$ | $-$; f: $\cap$ 0 $\cup$ 2 $\cap$

Incr: $(-\infty, 0) \cup (0, 3)$, $f' > 0$

Conc Up: $(0, 2)$, $f'' > 0$

Decr: $(3, \infty)$, $f' < 0$

Conc Down: $(-\infty, 0) \cup (2, \infty)$, $f'' < 0$

Extrema: Rel max $(3, 27)$

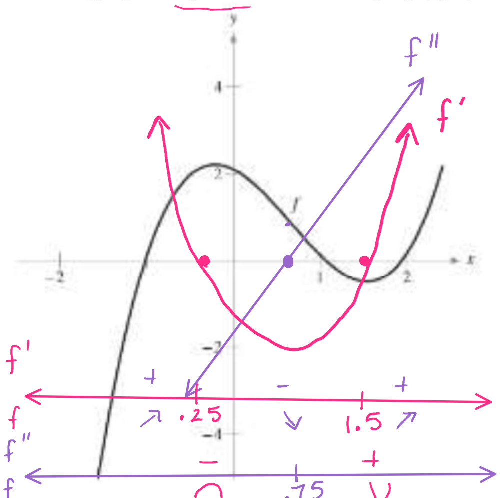
$f' > 0 \rightarrow f' < 0$

POI: $(0, 0)$, $(2, 16)$

f'' changes sign

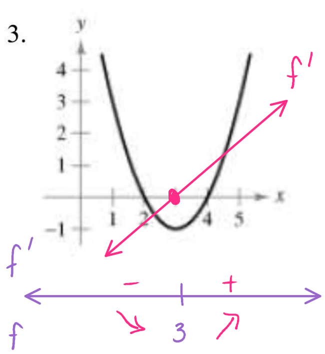
Topic: Graphing Derivatives and 2nd Derivatives

2. The graph of f is shown. Graph f, f', and f'' on the same set of coordinate axes.



For #3-5, sketch the derivative graph for each of the given functions.

3.



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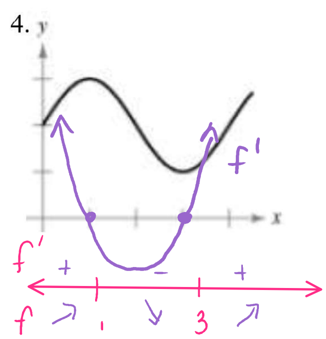

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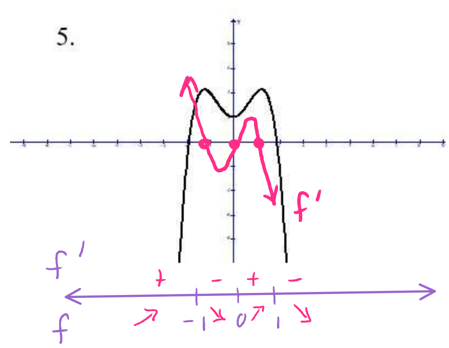


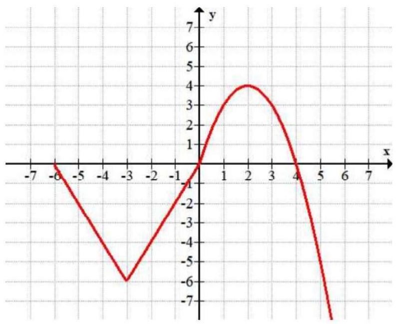
Topic: Extrema

6. a) What is the value (if it exists) of the derivative of the function below at the point (2, 4)?

$f'(2)=0$

b) What is the value (if it exists) of the derivative of the function below at the point (-3, -6)?

$f'(-3)$ und



c) Identify and classify any relative extrema as an ordered pair.

$(-3,-6)$ Relative min

$(2,4)$ Relative max

d) Identify and classify any absolute extrema as an ordered pair.

$(2,4)$ Absolute max

7. Use the Second Derivative Test to find all relative extrema of the function $f(x)=x^4-8x^3+5$.

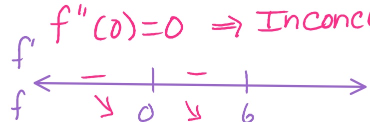
$f'(x)=4x^3-24x^2=0$

$4x^2(x-6)=0$

$x=0,6$

$f''(x)=12x^2-48x$

$f''(0)=0 \Rightarrow$ Inconclusive



NOT extrema

$f''(6)=144>0$

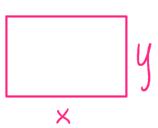
Conc. up

$f'(6)=0$ ∪

$(6,-427)$ Rel min

Topic: Optimization - You **must** show CALCULUS work in order to receive credit.

8 ~~11~~. Find the length and width of a rectangle that has a perimeter of 100 meters and a maximum area.



$100=2x+2y$

$50=x+y$

$x=50-y$

$A=xy$

$A=(50-y)y$

$A=50y-y^2$

$A'=50-2y=0$

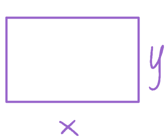
$y=25$

$A''=-2<0,\ A'(25)=0$

Area max when $y=25$

$\boxed{25\text{ m} \times 25\text{ m}}$

9 ~~12~~. Find the length and width of a rectangle that has an area of 64 square feet and a minimum perimeter.



$64=xy$

$x=\frac{64}{y}$

$P=2x+2y$

$P=2\left(\frac{64}{y}\right)+2y$

$P=\frac{128}{y}+2y$

$P'=\frac{-128}{y^2}+2=0$

$y^2=64$

$y=\pm 8$ NOT in domain

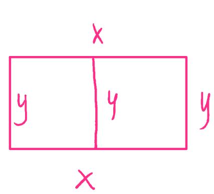
$P''=\frac{256}{y^3},\ P''(8)>0$

& $P'(8)=0$ ∴ min

Perimeter when $y=8$

$\boxed{8\text{ ft} \times 8\text{ ft}}$

10 ~~13~~. A rancher has 400 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions will enclose the maximum area?



$400=2x+3y$

$x=200-\frac{3}{2}y$

$A=xy$

$A=\left(200-\frac{3}{2}y\right)y$

$A=200y-\frac{3}{2}y^2$

$A'=200-3y=0$

$y=\frac{200}{3}$

$A''=-3$ & $A'\left(\frac{200}{3}\right)=0$ ∩

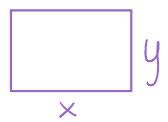
∴ Area max when

$y=200/3$ ft.

$x=100$ ft

$\boxed{\frac{200}{3}\text{ ft} \times 100\text{ ft}}$ outer Corral

11
~~14~~. What are the dimensions that would give the minimum perimeter of a rectangle with an area of $225\,cm^2$?



$225 = xy$

$x = \frac{225}{y}$

$P = 2x + 2y$

$P = \frac{450}{y} + 2y$

$P' = -\frac{450}{y^2} + 2 = 0$

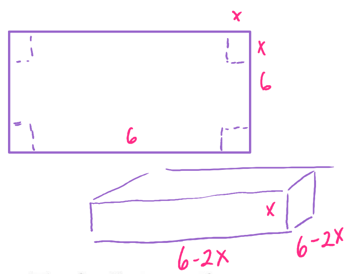
$y^2 = 225$

$y = \pm$ Not in domain

$P'' = \frac{900}{y^3}$, $P''(15) > 0$, $P'(15) = 0$ } $\cup$ min area when $y = 15$ cm

$\boxed{15\text{ cm} \times 15\text{ cm}}$

12
~~15~~. An open-top is made from cutting congruent squares of side length x from the corners of a 6- by 6-in. sheet of tin and bending the sides up. Find how large the cut out squares should be so that the box can hold as much as possible.



$A = x(6-2x)(6-2x)$

$A = 36x - 24x^2 + 4x^3$

$A' = \frac{36 - 48x + 12x^2 = 0}{12}$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3, 1$

A': $+$ | $-$; A: $\nearrow$ $\searrow$; 0, 1, 3

max area when $x = 1$ in since $A' > 0 \to A' < 0$

$\boxed{\text{Squares } 1\text{ in} \times 1\text{ in}}$

Topic: Integration

13 ~~16~~. Evaluate each of the following indefinite integrals.

a) $\int \left(-9z^2 + 10\right) dz$

$= -3z^3 + 10z + C$

b) $\int \left(16x^3 - 10\sqrt[7]{x^3} + \frac{4}{x^3}\right) dx$

$= \int (16x^3 - 10x^{3/7} + 4x^{-3})\, dx$

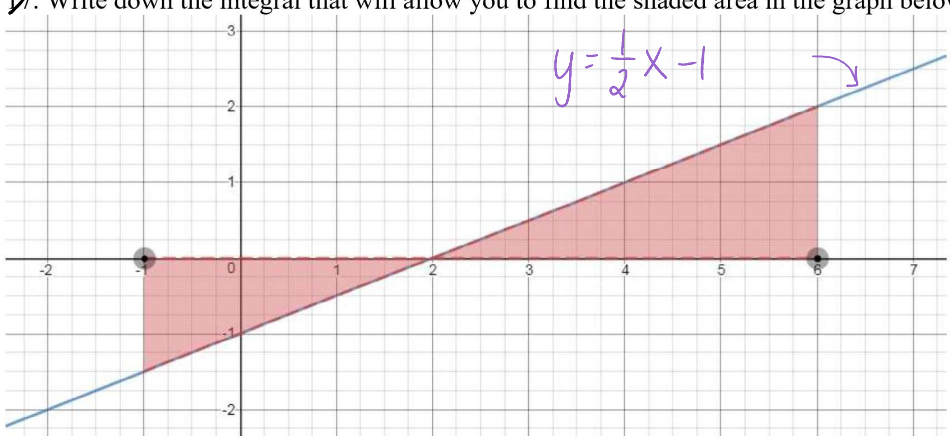
$= 4x^4 - 7x^{10/7} - \frac{2}{x^2} + C$

c) $\int \frac{4u^2 + 12u - 15}{u^4} du$

$= \int (4u^{-2} + 12u^{-3} - 15u^{-4})\, du$

$= -\frac{4}{u} - \frac{6}{u^2} + \frac{5}{u^3} + C$

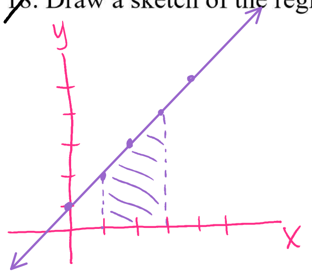
14
~~17~~. Write down the integral that will allow you to find the shaded area in the graph below.



$\int_{-1}^{6} \left(\tfrac{1}{2}x - 1\right) dx$

15

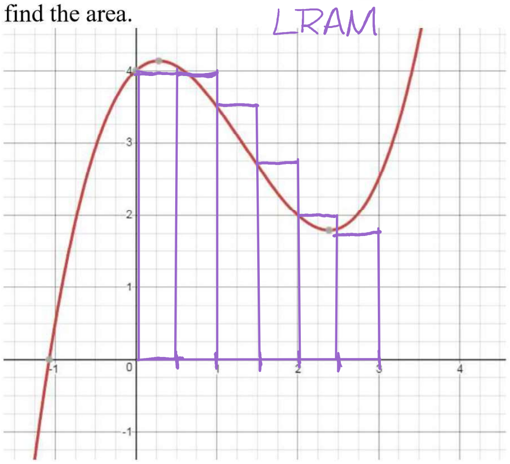
~~18~~. Draw a sketch of the region represented by $\int_1^3 (x+1)dx$



Topic: Rectangular approximation of area under a curve

16\. The region R is enclosed between the function $f(x)=\frac{1}{2}x^3-2x^2+x+4$ and the $x-axis$ for $0\le x\le 3$. On the graph below draw LRAM and RRAM using 6 equal subintervals that would approximate the area of R. Then, set-up the formula and find the area.

LRAM



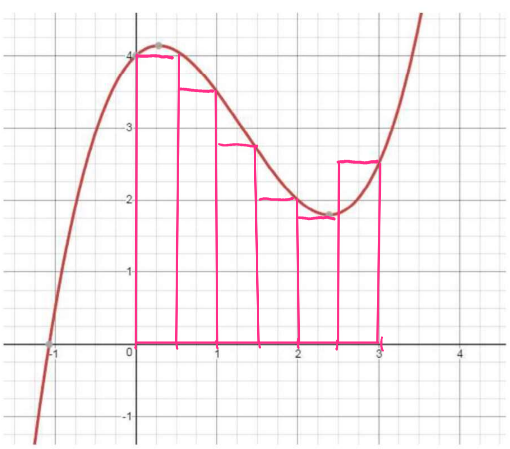
$\Delta x = \frac{3-0}{6} = \frac{1}{2}$

$A = \frac{1}{2}\left(f(0)+f(\frac{1}{2})+f(1)+f(\frac{3}{2})+f(2)+f(\frac{5}{2})\right)$

$A \approx 9.03125\, u^2$

$\boxed{A \approx 9.031\, u^2}$

RRAM



$A = \frac{1}{2}\left(f(\frac{1}{2})+f(1)+f(\frac{3}{2})+f(2)+f(\frac{5}{2})+f(3)\right)$

$A \approx 8.28125\, u^2$

$\boxed{A \approx 8.281\, u^2}$