I. Find $\frac{dy}{dx}$.

lower bound must be a constant

1. $y=\int_{-2}^{x}\sqrt{1+e^{5t}}\,dt$

$\boxed{y'=\sqrt{1+e^{5x}}}$

2. $y=\int_{x^2}^{3}\cot(3t)\,dt = -\int_{3}^{x}\cot(3t)\,dt$

$y' = -\cot(3x^2)\cdot 2x$

$\boxed{= -2x\cot(3x^2)}$

3. $y=\int_{\sin x}^{\cos x} t^2\,dt = \int_{\sin x}^{0} t^2\,dt + \int_{0}^{\cos x} t^2\,dt$

$= -\int_{0}^{\sin x} t^2\,dt + \int_{0}^{\cos x} t^2\,dt$

$y' = -\sin^2 x\cos x + \cos^2 x(-\sin x)$

$\boxed{= -\sin^2 x\cos x - \sin x\cos^2 x}$

II. Use the function $h(x)=\int_3^x \sqrt{t+1}\,dt$ to find the indicated value.

4. Find $h'(8)$.

$h'(x)=\sqrt{x+1}$

$h'(8)=\sqrt{8+1}=3$

5. Find the tangent line to $h(x)$ at $x=4$.

$h(4)=\int_3^4 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2}\Big]_3^4$

$= \frac{2}{3}\left(5^{3/2}-4^{3/2}\right) = \frac{2}{3}\left(5\sqrt{5}-8\right)$

$= \frac{10\sqrt{5}-16}{3}$

$h'(4)=\sqrt{4+1}=\sqrt{5}$

$\boxed{y-\frac{10\sqrt{5}-16}{3}=\sqrt{5}(x-4)}$

III. Solve the following differential equations with the given initial conditions.

6. $\frac{dy}{dx}=\frac{x^3-2x^2+1}{x^2}$ with initial condition $(1,3)$

$\frac{dy}{dx} = x-2+x^{-2}$

$y=\frac{1}{2}x^2-2x-x^{-1}+C = \frac{1}{2}x^2-2x-\frac{1}{x}+C$

$3=\frac{1}{2}-2-1+C$

$3=\frac{1}{2}-3+C$

$3=-2.5+C$

$5.5=C$

$\boxed{y=\frac{1}{2}x^2-2x-\frac{1}{x}+\frac{11}{2}}$

7. $\frac{dy}{dx}=\sqrt{x}+\cos x-\sin x$ with initial condition $(0,0)$

$y=\frac{2}{3}x^{3/2}+\sin x+\cos x+C$

$0=0+\sin(0)+\cos(0)+C$

$0=1+C$

$-1=C$

$\boxed{y=\frac{2}{3}x^{3/2}+\sin x+\cos x-1}$

IV. Multiple Choice: Circle the letter of the correct answer.

8. Evaluate $\int_1^{5e}\frac{1}{x}dx = \ln|x|\Big]_1^{5e} = \ln 5e - \ln 1 = \ln 5 + \ln e = \ln 5 + 1$

a) $\frac{1}{5e}-1$

b) 0

c) $5e$

d) $1+\ln 5$

e) None of These

$=dx$

9. Evaluate $\int \frac{x+2}{x+1}dx = \int \frac{u-1+2}{u}du = \int \frac{u+1}{u}du = \int 1+\frac{1}{u}du = u+\ln|u| = x+1+\ln|x+1|+C$

a) $\frac{x^2+4x}{x^2+2x}+C$ b) $2x+C$ c) $x+C$

$\boxed{\text{d)}}$ $(x+1)+\ln|x+1|+C$ e) None of These

$u=\frac{1}{x+1}=(x+1)^{-1}$

$\frac{du}{dx}=\frac{-1}{(x+1)^2}$

$dx=-(x+1)^2du$

10. Evaluate $\int \frac{e^{1/(x+1)}}{(x+1)^2}dx = \int \frac{e^u}{(x+1)^2}\cdot -(x+1)^2du = -\int e^u du$
$= -e^u+C$

a) $\frac{e^{1/(x+1)}}{2(x+1)}+C$ b) $\frac{e^{-x/(x+1)}}{(x+1)^2}$ $\boxed{\text{c)}}$ $-e^{1/(x+1)}+C$

d) $\frac{e^{x/(x+1)}}{(x+1)^2}$ e) None of These

11. Evaluate $\int \frac{x+3}{x^2+9}dx = \int \frac{x}{x^2+9}dx + 3\int \frac{1}{x^2+9}dx = \frac{1}{2}\ln(x^2+9)+\frac{3}{3}\arctan\left(\frac{x}{3}\right)+C$

a) $\ln|x-3|+C$ b) $\frac{1}{3}\arctan\frac{x}{3}+C$ $\boxed{\text{c)}}$ $\frac{1}{2}\ln(x^2+9)+\arctan\frac{x}{3}+C$

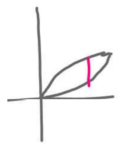
d) $\ln(x^2+9)+\frac{1}{3}\arctan\frac{x}{3}+C$ e) None of These

12. GC Calculator Active: The rate of change of the altitude of a hot-air balloon is given by $r(t)=t^3-4t^2+6$ for $0\le t\le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

$\boxed{\text{a)}}$ $\int_{1.572}^{3.514} r(t)dt$ b) $\int_{0}^{8} r(t)dt$ c) $\int_{0}^{2.667} r(t)dt$

d) $\int_{1.572}^{3.514} r'(t)dt$ e) $\int_{0}^{2.667} r'(t)dt$

13. Using Let R be the region in the first quadrant bounded below by the graph of $y = x^2$ and above by the graph $y = \sqrt{x}$. R is the base of a solid whose cross sections perpendicular to the x-axis are squares. What is the volume of the solid?



$x^2 = \sqrt{x}$
$x^4 = x$
$x^4 - x = 0$
$x(x^3 - 1) = 0$
$x = 0 \quad x = 1$

$\boxed{\text{a)}}$ 0.129 b) 0.300 c) 0.333

d) 0.700 e) 1.271

$s^2 = (\sqrt{x} - x^2)^2$

$\int_0^1 (\sqrt{x} - x^2)^2 \, dx = .129$

14. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\, dx$ is equivalent to $\int_1^5 \sqrt{u} \frac{du}{2}$

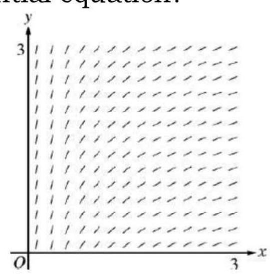
$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

a) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\, du$ b) $\frac{1}{2}\int_0^2 \sqrt{u}\, du$ $\boxed{\text{c)}}$ $\frac{1}{2}\int_1^5 \sqrt{u}\, du$

d) $\int_0^2 \sqrt{u}\, du$ e) $\int_1^5 \sqrt{u}\, du$

15. The slope field from a certain differential equation is shown below. Which of the following could b specific solution to that differential equation?



vertical pattern → dependent on x

a) $y = x^2$ b) $y = e^x$ c) $y = e^{-x}$

d) $y = \cos x$ $\boxed{\text{e)}}$ $y = \ln x$

16. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

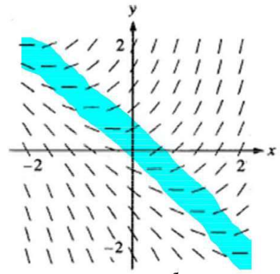
$y = -2\cos x + c$
$1 = -2\cos(\pi) + c$

a) $y = 2\cos x + 3$ b) $y = 2\cos x - 1$ c) $y = -2\cos x + 3$

$1 = -2(-1) + c$
$1 = 2 + c$
$-1 = c$

d) $y = -2\cos x + 1$ $\boxed{\text{e)}}$ $y = -2\cos x - 1$

17. Shown below is the slope field for which of the following differential equations?



no horizontal or vertical pattern → dependent on x & y

$\frac{dy}{dx} = 0$ along $y = -x$

~~a)~~ $\frac{dy}{dx} = 1 + x$ ~~b)~~ $\frac{dy}{dx} = x^2$ $\boxed{\text{c)}}$ $\frac{dy}{dx} = y + x$

d) $\frac{dy}{dx} = \frac{x}{y}$ ~~e)~~ $\frac{dy}{dx} = \ln y$

18. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not head the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant? p → heard $\underbrace{n}_{\text{Total}}-p$ → did not hear

a) $\frac{dp}{dt} = kp$ $\boxed{\text{b)}}$ $\frac{dp}{dt} = kp(N - p)$ c) $\frac{dp}{dt} = kp(p - N)$

d) $\frac{dp}{dt} = kt(N - t)$ e) $\frac{dp}{dt} = kt(t - N)$

$\frac{dP}{dt} = k(p(N-P)$

19. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

$\boxed{\text{a)}}$ $\frac{dP}{dt} = 200$ b) $\frac{dP}{dt} = 200t$ c) $\frac{dP}{dt} = 100t^2$

d) $\frac{dP}{dt} = 200P$ e) $\frac{dP}{dt} = 100P^2$

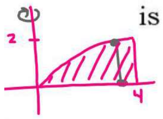
20. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$

a) 10.667 $\boxed{\text{b)}}$ 11.833 c) 14.583

d) 21.333 e) 32

$\int_1^2 x^3 - 8x^2 + 18x - 5 - x - 5\,dx + \int_2^5 x + 5 - x^3 + 8x^2 - 18x + 5\,dx =$

21. Consider the region bounded below by the x-axis, above by $y = \sqrt{x}$ and on the right by $x = 4$. Which of the following integrals represent the volume generated when this region is revolved about the y-axis?



$\int_0^4 x(\sqrt{x})dx$

$\int_0^4 x^{3/2}\,dx$

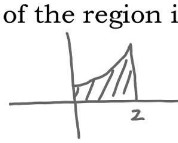
$\int_0^2 (4)^2-(y^2)^2 dy$

I. $\pi\int_0^2 y^4\,dy$ $\quad$ $\boxed{\text{II.}}$ $\pi\int_0^2 (16-y^4)dy$ $\quad$ III. $\pi\int_0^2 (4-y^2)^2\,dy$

a) I only $\quad$ $\boxed{\text{b)}}$ II only $\quad$ c) III only

d) I & II $\quad$ e) I & III

22. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

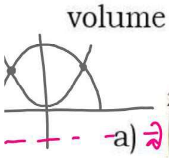


$\int_0^2 e^{\frac{x}{2}}dx = 2e^{\frac{x}{2}}\Big]_0^2$
$= 2e - 2e^0$
$= 2e - 2$

$\boxed{\text{a)}}$ $2e-2$ $\quad$ b) $2e$ $\quad$ c) $\frac{e}{2}-1$

d) $\frac{e-1}{2}$ $\quad$ e) $e-1$

23. Let R be the region bounded above by $y = 8 - x^2$ and below by $y = x^2$. What integral gives the volume of the solid obtained by rotating R about the line $y = -1$?



$8-x^2 = x^2$
$8 = 2x^2$
$x = \pm 2$

a) $\int_{-2}^{2} \pi\left[\left(8-x^2\right)^2 - \left(x^2\right)^2\right]dx$ $\quad$ b) $\int_{-2}^{2} \pi\left(8-2x^2\right)^2 dx$ $\quad$ c) $\int_0^8 \pi\left(8-2y^2\right)^2 dy$

d) $\int_{-2}^{2} \pi\left[\left(7-x^2\right)^2 - \left(x^2-1\right)^2\right]dx$ $\quad$ $\boxed{\text{e)}}$ $\int_{-2}^{2} \pi\left[\left(9-x^2\right)^2 - \left(1+x^2\right)^2\right]dx$

$\pi\int_{-2}^{2} (8-x^2+1)^2 - (x^2+1)^2\,dx$

V. Applications of Integration: Answer all parts of the question.

24. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$, with units.

$$\frac{A(30)-A(0)}{30-0} \approx -.197 \text{ pounds/days}$$

b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

$A'(15) = -.164$ pounds/days

The amount of clippings is decreasing at .164 lbs/day at $t=15$.

c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval.

$$A(t) = \frac{1}{30}\int_0^{30} A(t)\,dt$$

$A(t) = 2.7526$ when $t = 12.415$

d) For t > 30, $L(t)$ the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$$\left.\begin{aligned} A'(30) &= -.0560 \\ A(30) &= .783 \end{aligned}\right\}$$ You must use exact values → store each value!

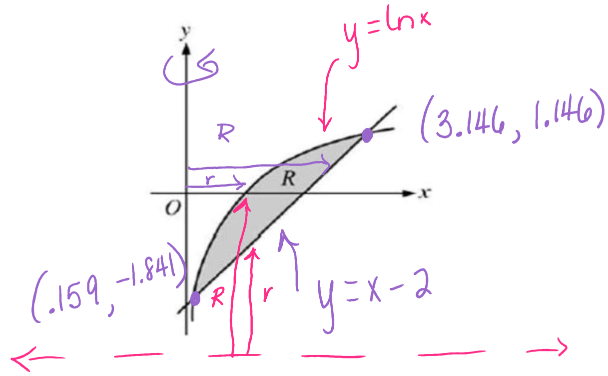
$$L(x) - A(30) = A'(30)(x-30)$$

$$\frac{\frac{1}{2} - A(30)}{A'(30)} + 30 = x$$

$$35.054 = x$$

25. Let R be the shaded region bounded by the graph of $y = \ln x$ and $y = x - 2$, as shown below.

$x = e^y$ $\quad$ $x = y + 2$



a) Find the area of R.

b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

a) $\int_{.159}^{3.146} (\ln x - (x-2))\,dx = \boxed{1.949}$ (3pts – 2pts integrand, 1pt limits)

b) $R = \ln x - (-3) = \ln x + 3$

$r = x - 2 - (-3) = x + 1$

$V = \pi \int_{.159}^{3.146} (\ln x + 3)^2 - (x+1)^2\,dx = \boxed{34.199}$ (3 pts – 2 pts int. – 1 pt limits)

c) $R = y + 2$

$r = e^y$

$\boxed{V = \int_{-.1841}^{1.146} (y+2)^2 - (e^y)^2\,dy}$ (3 pts – 2pts int. – 1pt limits)