Name Answer Key Date________

Calc I H – 2.3 Review **Period______**

1. Find the derivative. $R(x) = x^4 \cos x$
   a) $R'(x) = 4x^3 \sin x - x^4 \cos x$
   b) $R'(x) = -4x^3 \cos x - x^4 \sin x$
   c) $R'(x) = 4x^3 \cos x + x^4 \sin x$
   d) $\boxed{R'(x) = 4x^3 \cos x - x^4 \sin x}$
   e) Both a and b are acceptable answers

$R'(x) = 4x^3 \cos x - x^4 \sin x$

2. Find the derivative. $R(x) = \dfrac{\sin x}{x^3}$
   a) $R'(x) = \dfrac{-3x\sin x - \cos x}{x^3}$
   b) $\boxed{R'(x) = \dfrac{-3\sin x + x\cos x}{x^4}}$
   c) $R'(x) = \dfrac{3\sin x + x\cos x}{x^4}$
   d) $R'(x) = \dfrac{-3x^5 \sin x + \cos x}{x^3}$
   e) $R'(x) = \dfrac{3\sin x - x\cos x}{x^4}$

$R'(x) = \dfrac{\cos x(x^3) - 3x^2 \sin x}{(x^3)^2}$

$= \dfrac{\cancel{x^2}^{1}(x\cos x - 3\sin x)}{\cancel{x^6}\ x^4} = \dfrac{x\cos x - 3\sin x}{x^4}$

3. Find the derivative. $R(t) = \dfrac{6t}{t^5 + 9}$
   a) $R'(t) = \dfrac{6(9 + 4t^5)}{(t^5 + 9)^2}$
   b) $R'(t) = -\dfrac{6(9 + 6t^5)}{(t^5 + 9)^2}$
   c) $R'(t) = -\dfrac{6(9 + 5t^5)}{(t^5 + 9)^2}$
   d) $\boxed{R'(t) = \dfrac{6(9 - 4t^5)}{(t^5 + 9)^2}}$
   e) $R'(t) = \dfrac{6(-9 - 4t^5)}{(t^5 + 9)^2}$

$R'(t) = \dfrac{6(t^5+9) - 6t(5t^4)}{(t^5+9)^2}$

$R'(t) = \dfrac{6t^5 + 54 - 30t^5}{(t^5+9)^2}$

$= \dfrac{-24t^5 + 54}{(t^5+9)^2} = \dfrac{-6(4t^5 - 9)}{(t^2+9)^2}$

4. Find the derivative. $R(t)=\dfrac{\sin t}{t^2+2}$

$R'(t)= \dfrac{\cos t\,(t^2+2) - 2t(\sin t)}{(t^2+2)^2}$

a) $R'(t)=\dfrac{(2+t^2)\cos t-2t\sin t}{(t^2+2)}$

b) $\boxed{R'(t)=\dfrac{(2+t^2)\cos t-2t\sin t}{(t^2+2)^2}}$

c) $R'(t)=\dfrac{(2+t^2)\cos t+2t\sin t}{(t^2+2)^2}$

d) $R'(t)=\dfrac{(2-t^2)\cos t-2t\sin t}{(t^2+2)^2}$

e) $R'(t)=\dfrac{(2+t)\cos t-2t\sin t}{(t^2+2)^2}$

5. Write the equation of the tangent line to the graph of $f(x)=\dfrac{2-\cos x}{\sin x}$ at $x=\dfrac{3\pi}{2}$. $\boxed{y+2=1(x-3\pi/2)}$

$f'(x)= \dfrac{\sin x(\sin x) - \cos x(2-\cos x)}{(\sin x)^2}$

$f'(x)= \dfrac{\sin^2 x - 2\cos x + \cos^2 x}{\sin^2 x} = \dfrac{\overbrace{\sin^2 x+\cos^2 x}-2\cos x}{\sin^2 x}$

$f'(x)= \dfrac{1-2\cos x}{\sin^2 x}$

$f'(3\pi/2)=\dfrac{1-2(0)}{(-1)^2}=1$ $m=1$

$f(3\pi/2)=\dfrac{2-0}{-1}=-2$ $(\frac{3\pi}{2},-2)$

6. Find the derivative. $R(x)=x^4\tan x$

$R'(x)=4x^3\tan x + x^4\sec^2 x$

a) $R'(x)=-4x^3\tan x+x^3\sec^2 x$

b) $\boxed{R'(x)=4x^3\tan x+x^4\sec^2 x}$

c) $R'(x)=-x^3\sec^2 x+4x^3\tan x$

d) $R'(x)=3x^3\tan x+x^4\sec^2 x$

e) None of the above

7. Find the derivative. $P(x)=18x^3+5\sec(x)$

$P'(x)=54x^2+5\sec x\tan x$

a) $P'(x)=3x^2+5\sec(x)\tan(x)$

b) $P'(x)=54x^2+5\tan(x)$

c) $P'(x)=54x^2-5\sec(x)\tan(x)$

d) $P'(x)=3x^2+5\tan(x)$

e) $\boxed{P'(x)=54x^2+5\sec(x)\tan(x)}$

8. Find the derivative. $f(s)=5s^3\sin(s)+2s^7\cos(s)$

$f'(s)=15s^2\sin(s)+5s^3\cos(s)+14s^6\cos(s)-2s^7\sin(s)$

$f'(s)=(15s^2-2s^7)\sin s+(5s^3+14s^6)\cos(s)$

a) $\boxed{f'(s)=(15s^2-2s^7)\sin(s)+(5s^3+14s^6)\cos(s)}$

b) $f'(s)=(5s^3-14s^6)\sin(s)+(15s^2-2s^7)\cos(s)$

c) $f'(s)=-(15s^2-2s^7)\sin(s)+(5s^3+14s^6)\cos(s)$

d) $f'(s)=(15s^7-2s^2)\sin(s)+(5s^3+14s^6)\cos(s)$

e) $f'(s)=(15s^2-2s^7)\sin(s)-(5s^3+14s^6)\cos(s)$

9. Find the equation of the tangent line to the graph of f at the given point. $f(x)=(x-5)(x^2-5)$, at $(3,-8)$

a) $y+8=8(x-3)$
b) $y+8=32(x-3)$
c) $y-3=-8(x+8)$
d) $\boxed{y+8=-8(x-3)}$
e) $y-8=-8(x+3)$

$f'(x)=1(x^2-5)+2x(x-5)$

$f'(x)=x^2-5+2x^2-10x=3x^2-10x-5$

$f'(3)=-8$ $(3,-8)$ $m=-8$

$y+8=-8(x-3)$

10. Find the second derivative of the function. $g(s)=\dfrac{6s^2+7s-5}{s}$

a) $g''(s)=\dfrac{10}{s^3}$
b) $\boxed{g''(s)=-\dfrac{10}{s^3}}$
c) $g''(s)=\dfrac{5}{s^3}$
d) $g''(s)=-\dfrac{10}{s^2}$
e) $g''(s)=-\dfrac{s+10}{s^3}$

$g'(s)=\dfrac{(12s+7)(s)-1(6s^2+7s-5)}{s^2}$

$g'(x)=\dfrac{12s^2\cancel{+7s}-6s^2\cancel{-7s}+5}{s^2}=\dfrac{6s^2+5}{s^2}$

$g''(x)=\dfrac{12s(s^2)-2s(6s^2+5)}{(s^2)^2}=\dfrac{12s^3-12s^3-10s}{s^4}$

$g''(x)=\dfrac{-10\cancel{s}}{s^{\cancel{4}3}}=-\dfrac{10}{s^3}$

11. Find the second derivative of the function. $H(s)=s^4\sin(s)$

a) $\boxed{H''(s)=12s^2\sin(s)+8s^3\cos(s)-s^4\sin(s)}$
b) $H''(s)=12s^2\sin(s)+4s^3\cos^2(s)-s^4\sin(s)$
c) $H''(s)=12s^2\sin(s)+s^4\sin(s)$
d) $H''(s)=-12s^2\cos(s)$
e) $H''(s)=12s^6\sin(s)+8s^3\cos(s)$

$H'(s)=4s^3\sin(s)+s^4\cos(s)$

$H''(s)=12s^2\sin(s)+4s^3\cos(s)+4s^3\cos(s)-s^4\sin(s)$

$H''(s)=12s^2\sin(s)+8s^3\cos(s)-s^4\sin(s)$

12. Mrs. Canonaco is standing on a 20 ft balcony throwing water balloons with an initial velocity of 14 feet per second at her students on the ground.

down!

a) Write the position function.

$s(t)=-16t^2-14t+20$

b) What is the velocity after 0.5 seconds?

$v(t)=-32t-14$ $v(\tfrac{1}{2})=-32(\tfrac{1}{2})-14=\boxed{-30\text{ ft/sec}}$

c) What is the velocity after 0.75 seconds?

$v(\tfrac{3}{4})=-32(\tfrac{3}{4})-14=\boxed{-38\text{ ft/sec}}$

d) What is the acceleration after 0.75 seconds?

$a(t)=-32$ $a(\tfrac{3}{4})=\boxed{-32\text{ ft/sec}^2}$

e) Is the speed of the balloon increasing or decreasing at 0.75 seconds? Justify your answer!

Speed is increasing since $v(t)<0$ and $a(t)<0$ (same sign!)

13. Answer each of the following

a. Find the derivative of $y = 2\cot x - 4\sec x$.

$y' = -2\csc^2 x - 4\sec x \tan x$

b. Write the equation of the tangent line at $x = \frac{\pi}{4}$.

$y(\frac{\pi}{4}) = 2\cot\frac{\pi}{4} - 4\sec\frac{\pi}{4}$
$= 2(1) - 4\sqrt{2} = 2 - 4\sqrt{2}$
$(\frac{\pi}{4}, 2-4\sqrt{2})$

$y'(\frac{\pi}{4}) = -2\csc^2\frac{\pi}{4} - 4\sec\frac{\pi}{4}\tan\frac{\pi}{4}$
$= -2(\sqrt{2})^2 - 4(\sqrt{2})(1)$
$= -4 - 4\sqrt{2}$ $\quad m = -4 - 4\sqrt{2}$

$\boxed{y - 2 + 4\sqrt{2} = (-4 - 4\sqrt{2})(x - \frac{\pi}{4})}$

14. Write the equation of the tangent line to the graph when $x = \frac{\pi}{2}$

$$h(x) = \frac{3\cos x - 1}{\sin x}$$

$h(\pi/2) = \frac{3\cos\frac{\pi}{2} - 1}{\sin\frac{\pi}{2}} = \frac{3(0) - 1}{1} = -1$ $\quad (\pi/2, -1)$

$h'(x) = \frac{-3\sin x(\sin x) - \cos x(3\cos x - 1)}{(\sin x)^2} = \frac{\overbrace{-3\sin^2 x - 3\cos^2 x}^{-3(\sin^2 x + \cos^2 x)} + \cos x}{\sin^2 x}$

$= \frac{-3 + \cos x}{\sin^2 x}$ OR $\frac{\cos x - 3}{\sin^2 x}$ $\quad m = -3$

$h'(\pi/2) = \frac{\cos\pi/2 - 3}{\sin^2(\pi/2)} = \frac{0-3}{1^2} = -3$

$\boxed{y + 1 = -3(x - \pi/2)}$

15. Find the second derivative of $f(x) = 3\csc x$.

$f'(x) = -3\csc x \cot x$

$f''(x) = 3\csc x \cot x \cdot \cot x + (-3\csc x)(-\csc^2 x)$
$= \underline{3\csc x} \cot^2 x + \underline{3}\,\underline{\csc^3 x}$
$\boxed{3\csc x(\cot^2 x + \csc^2 x)}$